# RISK AVERSE MECHANISMS IN THE BRAZILIAN POWER SYSTEM

LACIAM
Latin American Congress on Industrial and Applied Mathematics
2023

Cepel | A pesquisa que constrói o futuro

André Luiz Diniz

Jan 31th, 2023

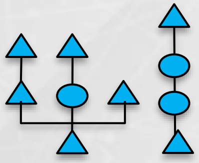
# Power Generation Planning and Operation Overview

**The main objective is to minimize:**

- **Thermal Generation costs +**
- **Risk Measure (CVaR)**

**Subject to:**

- **Cascaded hydro plants in several river basins**
- **Pumping stations connecting different rivers**
- **Many hydro constraints**



Pumped-storage

Reservoirs

Hydro Plants

- **thermal unit commitment constraints**
- **anticipated dispatch requirement of LNG units**

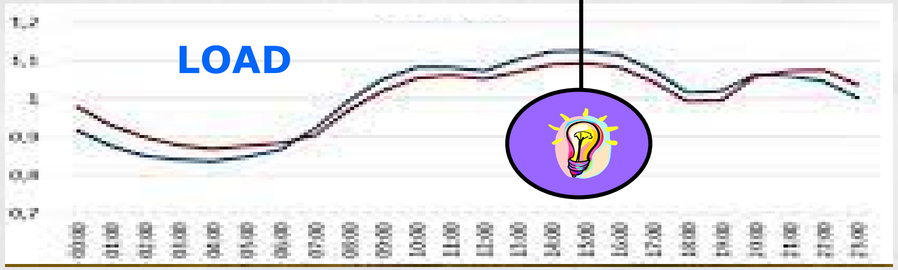
Thermal Plants

- **Intermittent generation (new renewables)**
- **other fixed generations**

Other sources

**TRANSMISSION**

- **Interconnection limits among areas**
- **electrical losses in major interconnections**
- **line flow limits constraints (DC power flow)**




- **MT/LT: Monthly/weekly profiles in several load blocks, per system area**
- **ST: hourly load profiles in each bus**

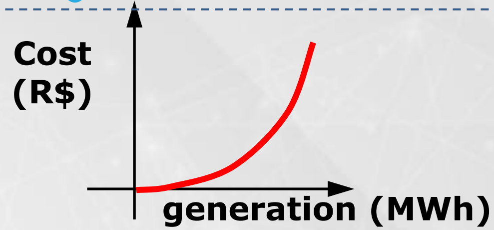
**TARGET: to obtain an "optimal" Policy for planning purposes, to set the dispatch of hydro/thermal plants and to establish market prices**

# Cost Information for Decision Making

## THERMAL PLANTS

- Explicit representation of thermal generation costs, as well as startup/shutdown costs

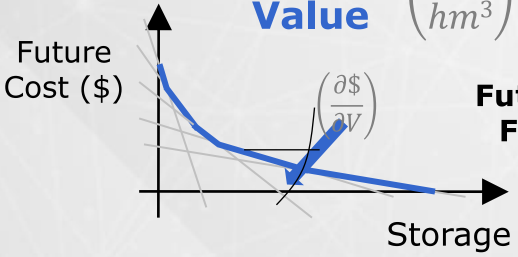
$$\left(\frac{\$}{MWh}\right)$$



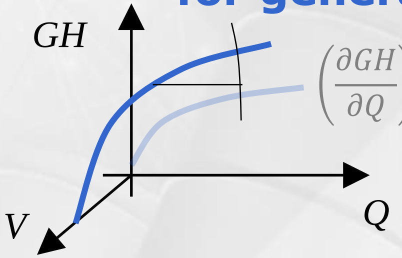

## HYDRO PLANTS

- Combination of water values and plant efficiency yields hydro generation costs

**Generation Costs** $\left(\frac{\$}{MWh}\right)$ = **Water Value** $\left(\frac{\$}{hm^3}\right)$ × **Water consumption for generation** $\left(\frac{\partial GH}{\partial Q}\right)$ $\left(\frac{hm^3}{MWh}\right)$




**Future Cost Function**



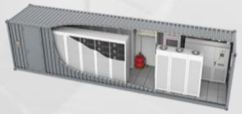

## NEW RENEWABLES (WIND, SOLAR...)

Taking advantage as much as possible of "free" generation

**Wind, PV/CSP power plants**

## ADDITIONAL SYSTEM COMPONENTS

Mitigate the uncertainty/intermittency of new renewable Generation to maximize available energy



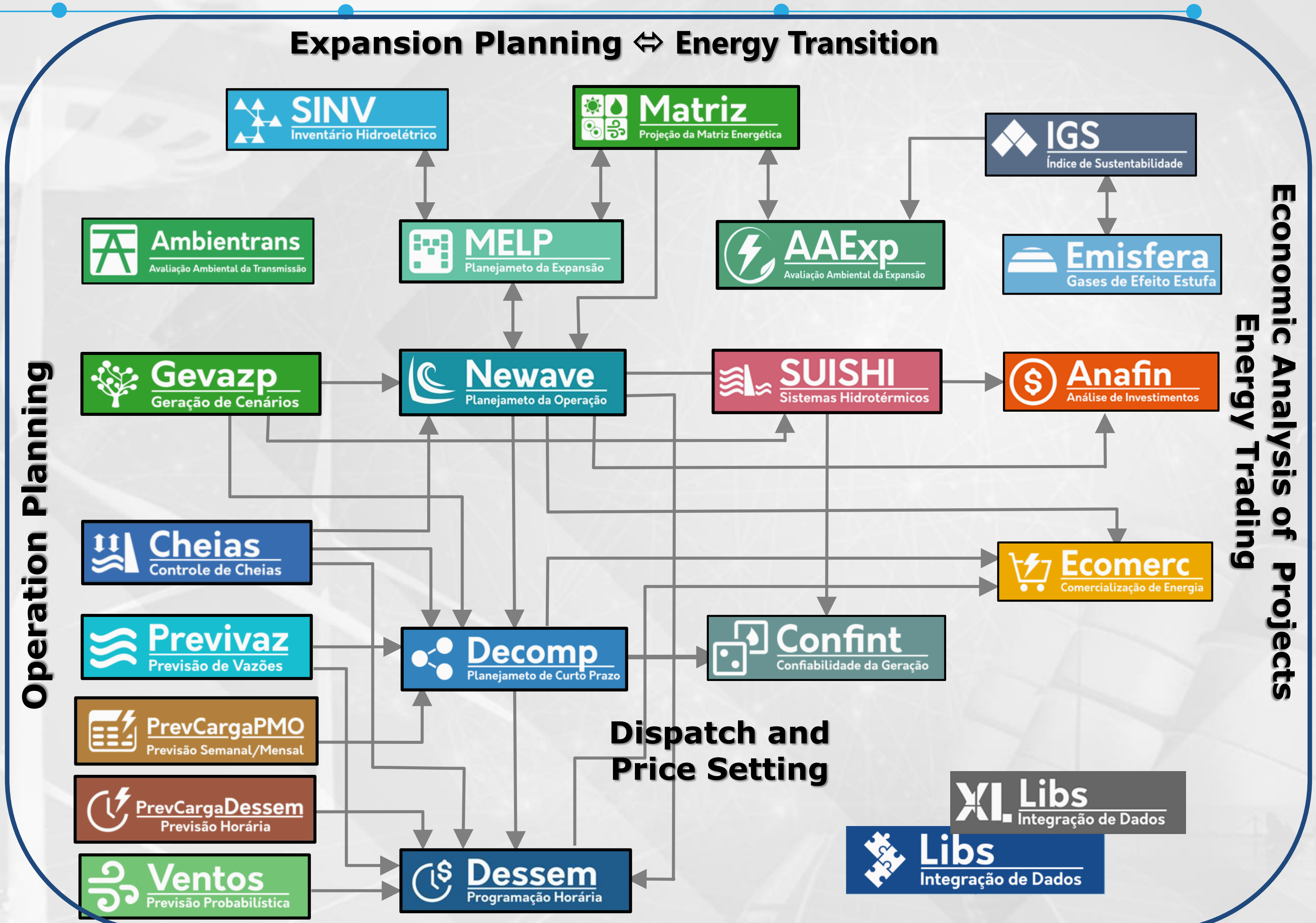
**Energy Storage**

**Pumped Storage**

**Demand Response**

# Optimization Models for Energy Planning Developed by CEPEL

Cepel A pesquisa que constrói o futuro




**Developed by CEPEL, collaborating with scientific community**

**Validated in working groups in by ONS, CCEE, EPE, MME, ANEEL, as well as task forces with most power system utilities**

**Approved for official use by the regulatory agency** ANEEL

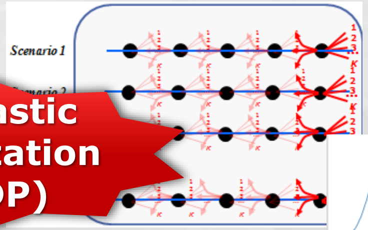
**used for**
**System planning** epe
**System dispatch** ONS
**Setting of market prices** ccee

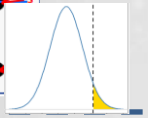
# Models for risk-averse energy Planning, Hydrothermal-wind Scheduling and Price Setting

Cepel | A pesquisa que constrói o futuro

**Newave** – Planejameto da Operação

**Long term: optimal policy (FCFs)**



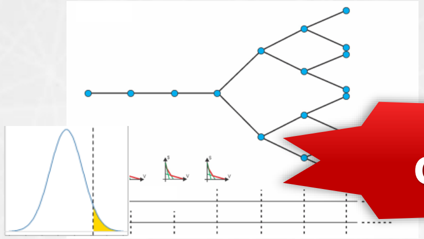



Stochastic Optimization (SDDP)

CVaR Risk Averse Mechanism

**Decomp** – Planejameto de Curto Prazo

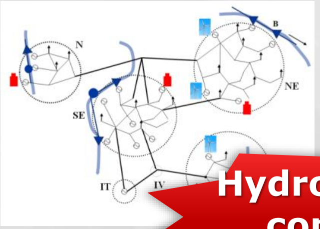
**Mid term: Policy refinement**



Stochastic Optimization (DDP)

**Dessem** – Programação Horária

**day ahead dispatch/pricing**



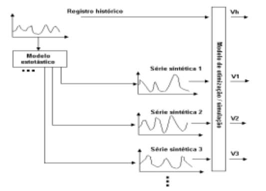

Hydrothermal unit commitment, (MILP)

**SUISHI** – Sistemas Hidrotérmicos

**Hydrothermal simulation**

Nonlinear (Heuristic Rules)

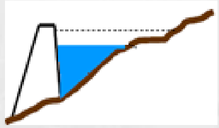
**Gevazp** – Geração de Cenários




**Generation of inflow/wind scenarios**

Par(p)-A model

**Cheias** – Controle de Cheias

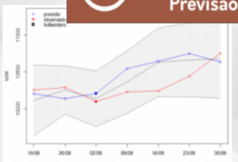


**Flood Prevention and Control**

Stochastic analysis, climatic phenomena

**PrevCargaPMO** – Previsão

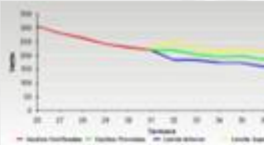
**PrevCargaDessem** – Previsão Horária



**Monthly / Weekly / hourly load forecast**

Dynamic Regression Models, SVM

**Previvaz** – Previsão de Vazões



**Daily/Weekly inflow forecast**

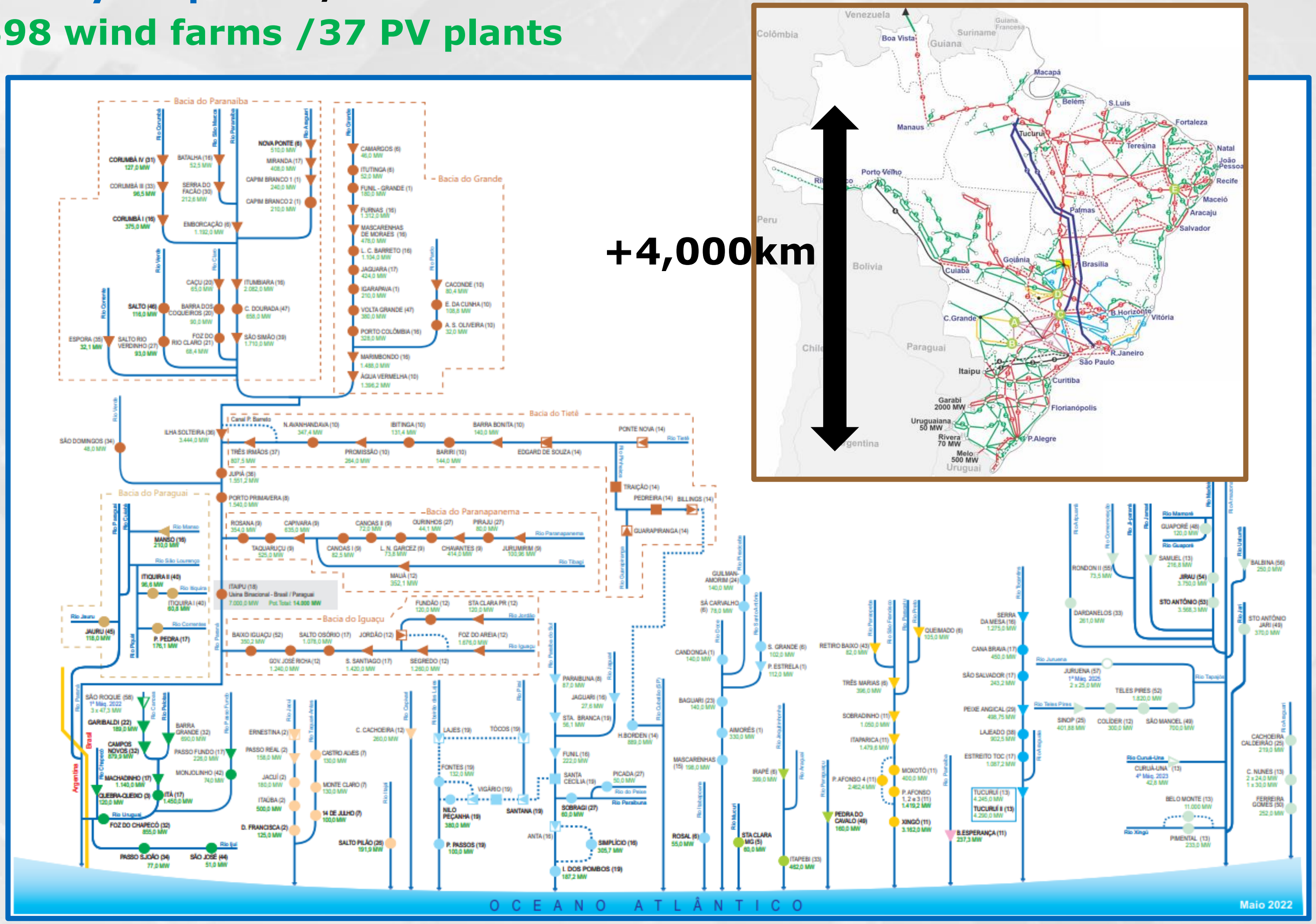
Statistical + rainfall-information

# Brazilian Interconnected System Centrally Dispatched by the Brazilian ISO (ONS)

Cepel | A pesquisa que constrói o futuro

**162 hydro plants / 386 thermal units** **+9,000buses / +13,000 transmission lines**

**398 wind farms / 37 PV plants**




source: www.ons.org.br

## HYDRO (61.6%)

| 2022 | 2026 |
|---|---|
| 109.058 MW (61,6%) | 109.868 MW (55,2%) |

## THERMAL (13.9%)

| | 2022 | 2026 |
|---|---|---|
| GAS/LNG | 15.459 MW (8,7%) | 20.820 MW (10,5%) |
| OIL/DIESEL | 4.198 MW (2,4%) | 4.241 MW (2,1%) |
| COAL | 3.017 MW (1,7%) | 3.017 MW (1,5%) |
| NUCLEAR | 1.990 MW (1,1%) | 1.990 MW (1,0%) |

## WIND/SOLAR (15.4%)

| | 2022 | 2026 |
|---|---|---|
| Wind | 22.214 MW (12,5%) | 29.505 MW (14,8%) |
| Solar | 5.552 MW (3,1%) | 12.556 MW (6,3%) |

## BIOMASS+ OTHERS (8.9%)

| | 2022 | 2026 |
|---|---|---|
| Biomass | 14.989 MW (8.5%) | 16.264 MW (8,2%) |
| Others | 645 MW (0,4%) | 851 MW (0,4%) |

| TOTAL | 2022 | 177.122 MW | 2026 | 199.112 MW |
|---|---|---|---|---|

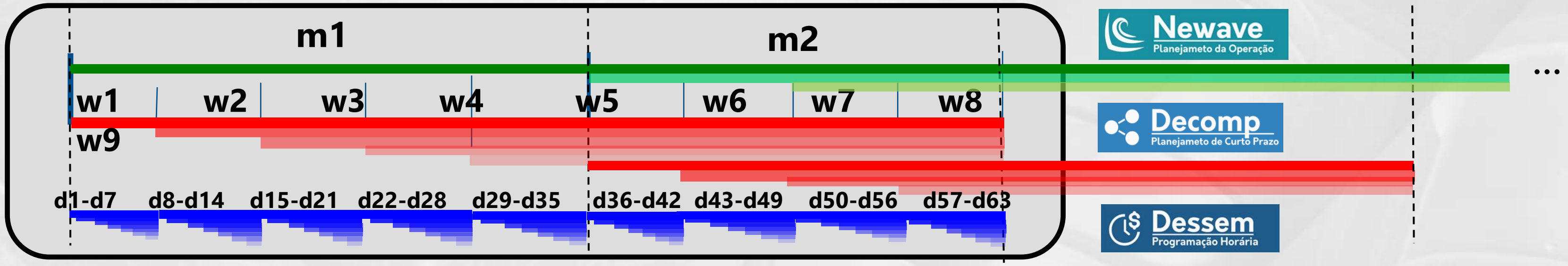
# Cost Minimization Generation Planning with CVaR Risk Averse Criterion - Rolling Horizon Scheme

| | Frequency | Horizon | Discretization |
|---|---|---|---|
| long | Monthly | 10 years | Monthly |
| mid | Weekly | 2 months | weekly |
| short | Daily | 7 days | up to half-an-hour |

Newave (Planejameto da Operação) → FCF → Decomp (Planejameto de Curto Prazo) → FCF → Dessem (Programação Horária)

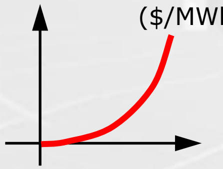
| Uncertainty | System Modeling | Solution |
|---|---|---|
| Stochastic, CVar | Aggregate reservoirs, tie lines | SDDP |
| Stochastic, CVar | Individual plants, tie lines, | DDP |
| Deterministic | unit commitment, DC Power Flow | MILP |

## Rolling Horizon Scheme

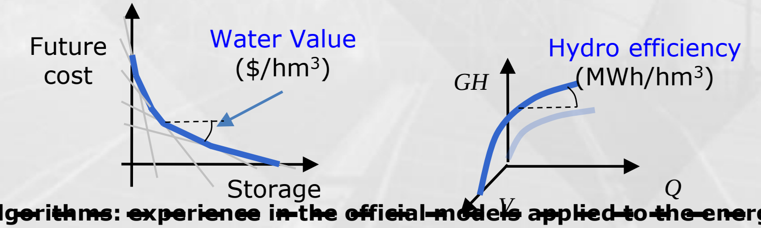



## Cost Information

### Thermal units



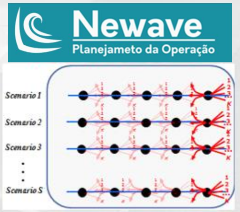

### Hydro Plants




### New Renewable generation

- use "free" generation as much as possible
- Generation is curtailed if necessary

# DAY-AHEAD Scheduling and Hourly Pricing in Brazil

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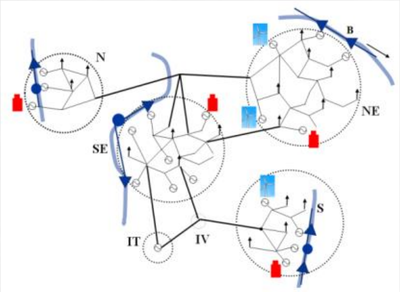
**Dessem**
Programação Horária




**Official use started on Jan 1st, 2020**

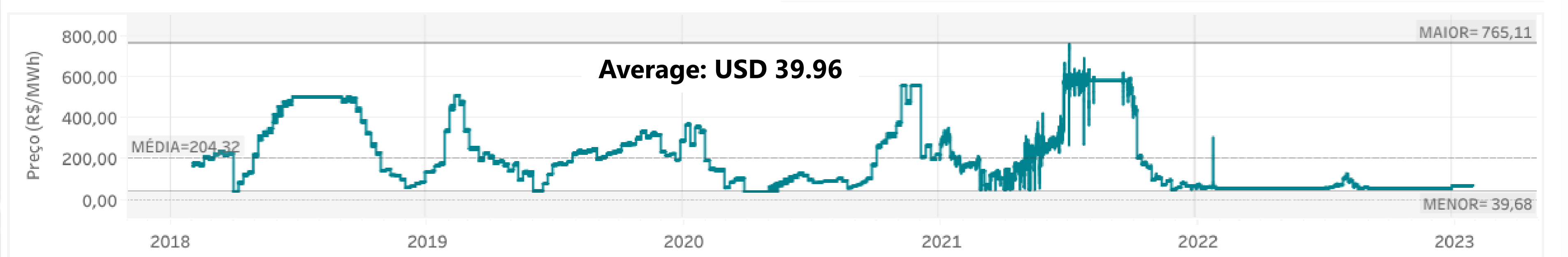
ONS
Operador Nacional do Sistema Elétrico

**Official use started on Jan 1st, 2021**

ccee

**Hourly Prices- Jan 30th, 2023 – 8am-9am**


**Hourly Prices – last 6 months**




**Source: CCEE web site (https://www.ccee.org.br/precos/painel-preços)**

# Main References for the Models

Newave
Planejameto da Operação

Twenty Years of Application of Stochastic Dual Dynamic Programming in Official and Agent Studies in Brazil – Main Features and Improvements on the NEWAVE Model

M.E.P. Maceira[1,2] D.D.J. Penna[1] A.L. Diniz[1,2] R.J. Pinto[1] A.C.G. Melo[1,2] C.V. Vasconcellos[1] C.B. Cruz[1]
[1]CEPEL –Electric Energy Research Center [2]UERJ- State Univesity of Rio de Janeiro
Rio de Janeiro, Brazil
{elvira, debora, diniz, rjpinto, albert, cesarluis, criscruz}@cepel.br

**Maceira, Penna et al, PSCC 2018**

Decomp
Planejameto de Curto Prazo

Short/Mid-Term Hydrothermal Dispatch and Spot Pricing for Large-Scale Systems - the Case of Brazil

André Luiz Diniz[1,2], Fernanda da Serra Costa[1,2]
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**Diniz, Costa et al, PSCC 2018**

Dessem
Programação Horária

Electric Power Systems Research 189 (2020) 106709
Contents lists available at ScienceDirect
Electric Power Systems Research
journal homepage: www.elsevier.com/locate/epsr
ELSEVIER

Hourly pricing and day-ahead dispatch setting in Brazil: The dessem model

T.N. Santos[a,b], A.L. Diniz[a,b,*], C.H. Saboia[a,b], R.N. Cabral[a,b], L.F. Cerqueira[a,b]

**Santos, Diniz et al, EPSR, 2020**

SINTEF

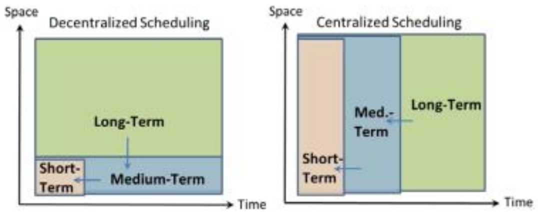
2020:00757- Unrestricted

Report

Scheduling Toolchains in Hydro-Dominated Systems

Evolution, Current Status and Future Challenges for Norway and Brazil

**Author(s)**
Arild Helseth
Albert Cordeiro Geber de Melo



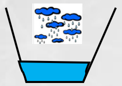

SINTEF Energy Research
Markets
2020-08-10

**Helseth, Melo, Sintef Report, 2020**

# Computation of the Operation Policy by SDDP

# Operation Policy: Future Cost Function (FCF) at each decision stage

**Decision maker´s dilemma**

| Current Decision | Future Inflows | Outcome |
|---|---|---|
|  | | |
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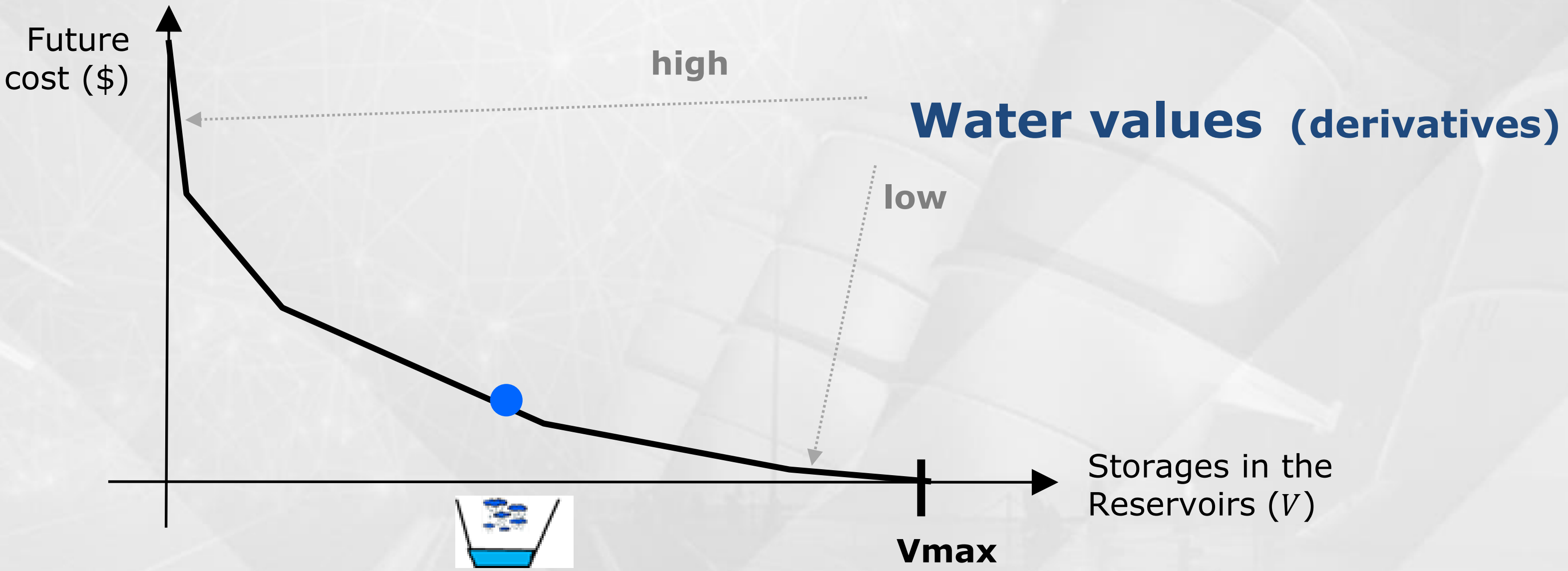

# Operation Policy: Future Cost Function (FCF) at each decision stage

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| Current Decision | Future Inflows | Outcome |
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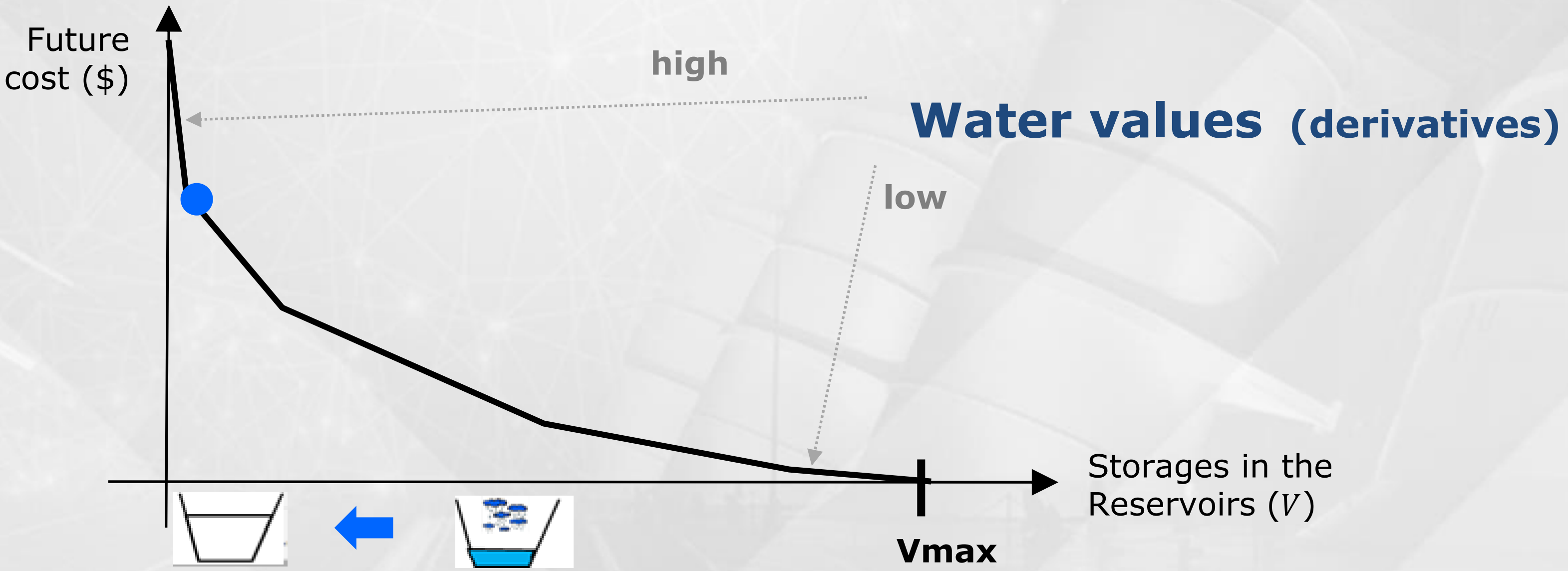
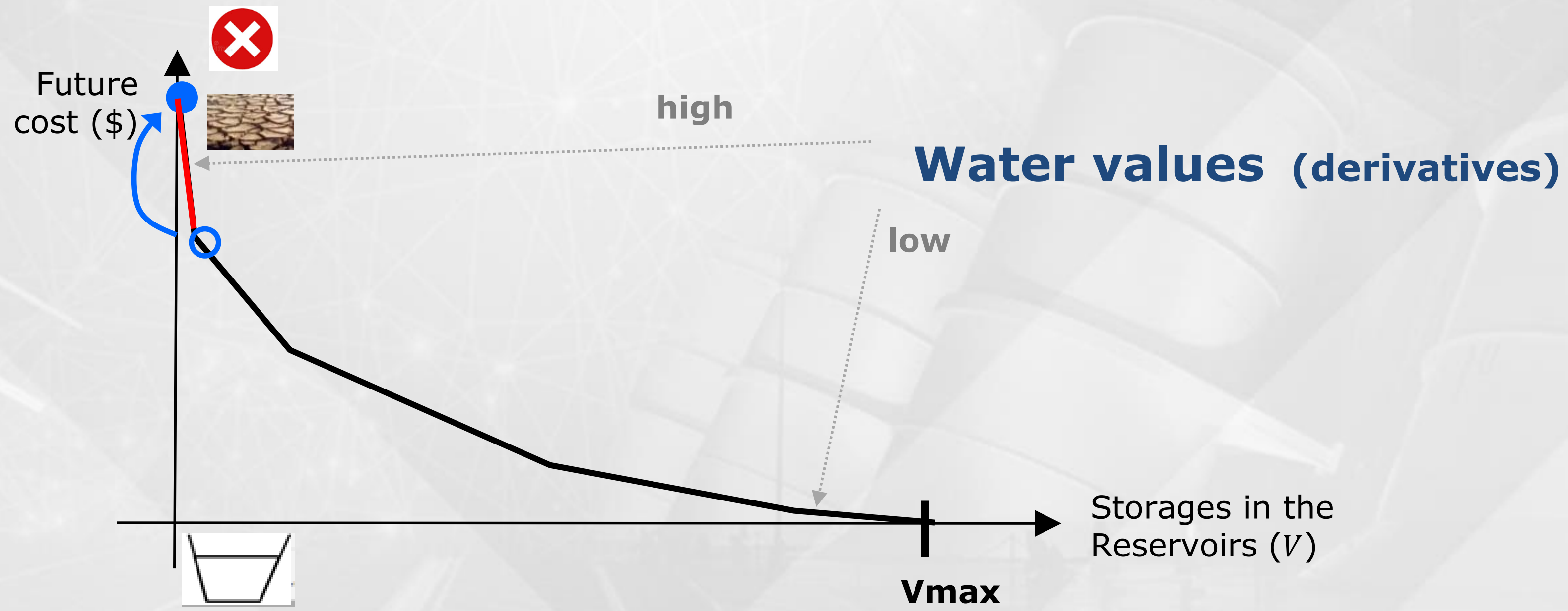
| use **Hydro** Generation (lower costs) | | |
| | | |
| | | |
| | | |



# Operation Policy: Future Cost Function (FCF) at each decision stage

**Decision maker´s dilemma**

| Current Decision | Future Inflows | Outcome |
|---|---|---|
| use Hydro Generation (lower costs) | Low | Higher costs, rationing |
| | | |
| | | |
| | | |



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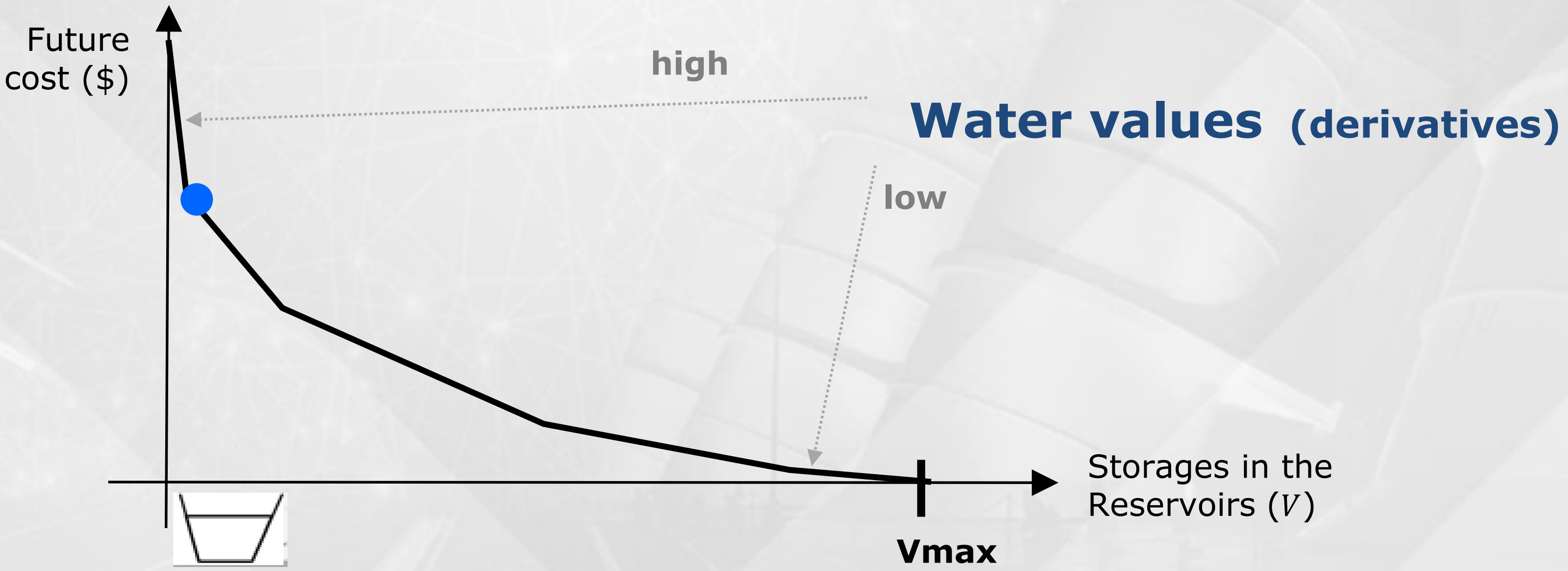
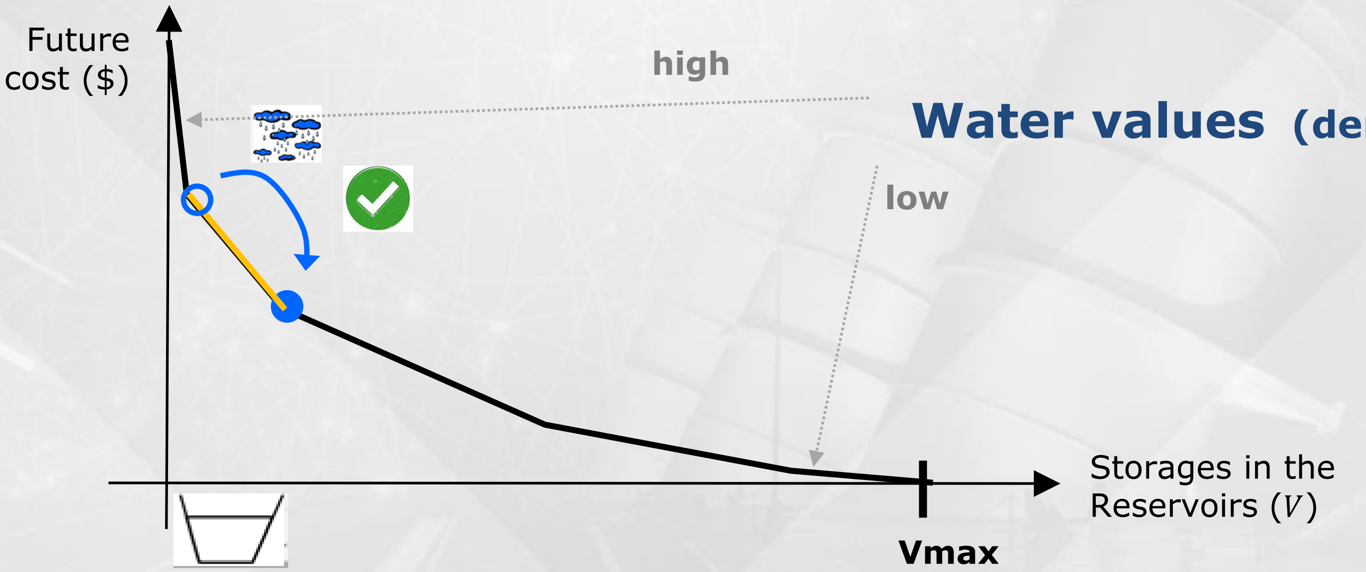


# Operation Policy: Future Cost Function (FCF) at each decision stage

**Decision maker´s dilemma**

| Current Decision | Future Inflows | Outcome |
|---|---|---|
| use **Hydro** Generation (lower costs) | Low | Higher costs, rationing |
| | High | Economic operation |
| | | |
| | | |



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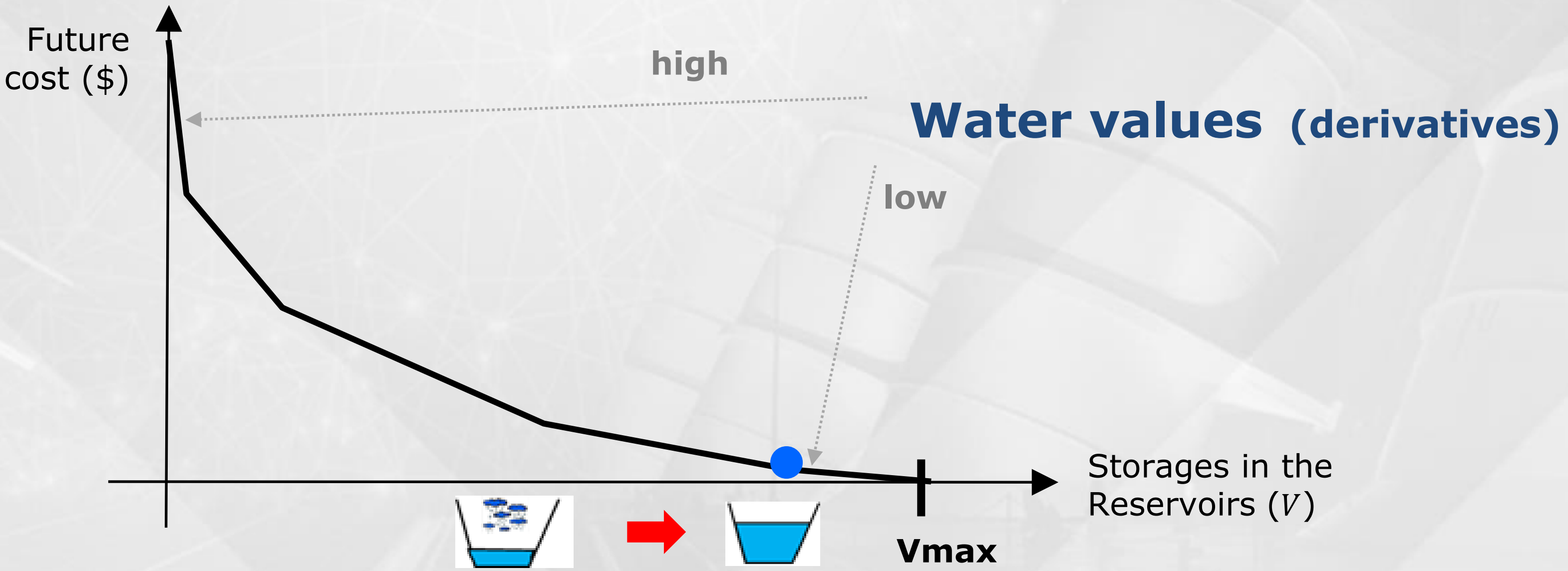
Cepel A pesquisa que constrói o futuro

**Decision maker´s dilemma**

| Current Decision | Future Inflows | Outcome |
|---|---|---|
| use Hydro Generation (lower costs) | Low | Higher costs, rationing |
| | High | Economic operation |
| Use Thermal Generation (higher costs) | | |
| | | |



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**Decision maker´s dilemma**

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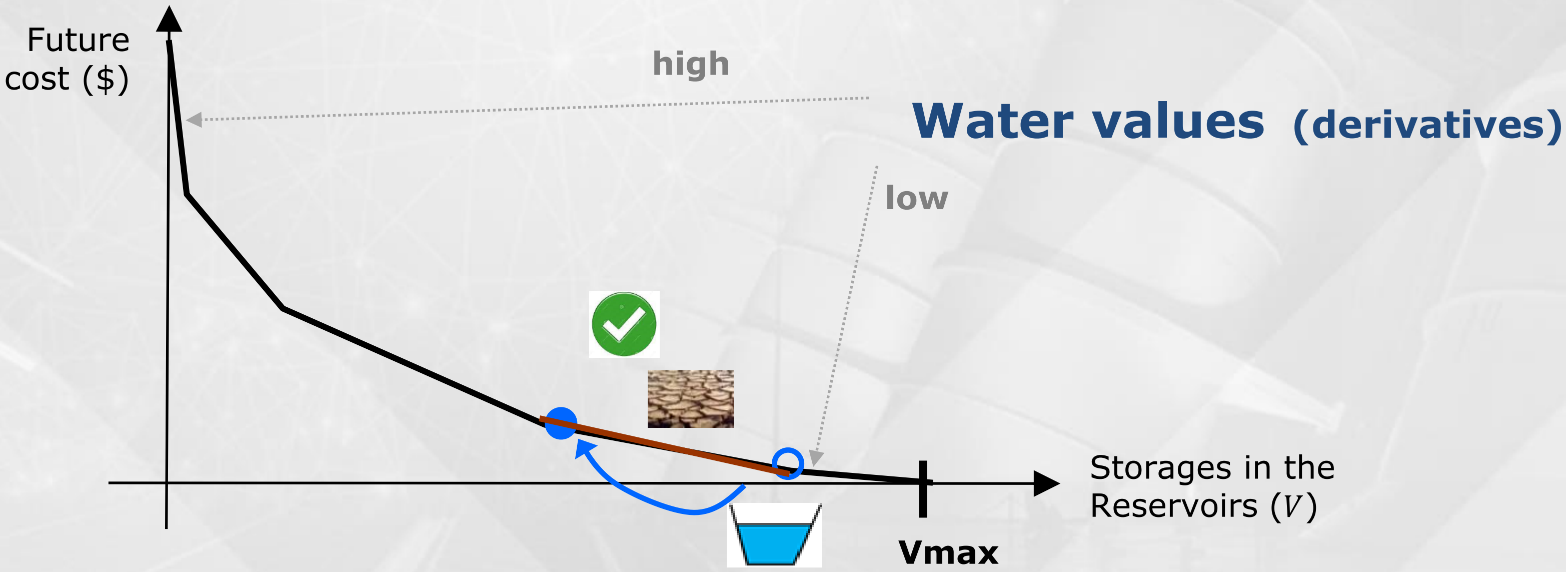
| | High | Economic operation |
| Use Thermal Generation (higher costs) | Low | Economic operation |



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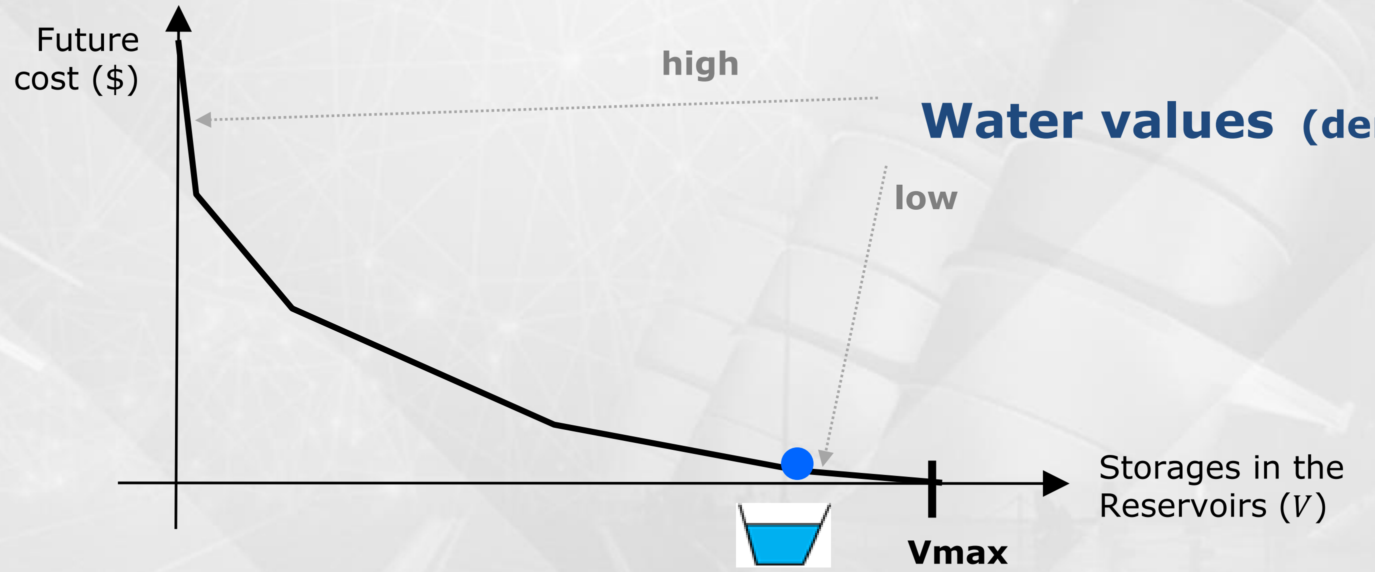

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**Decision maker´s dilemma**

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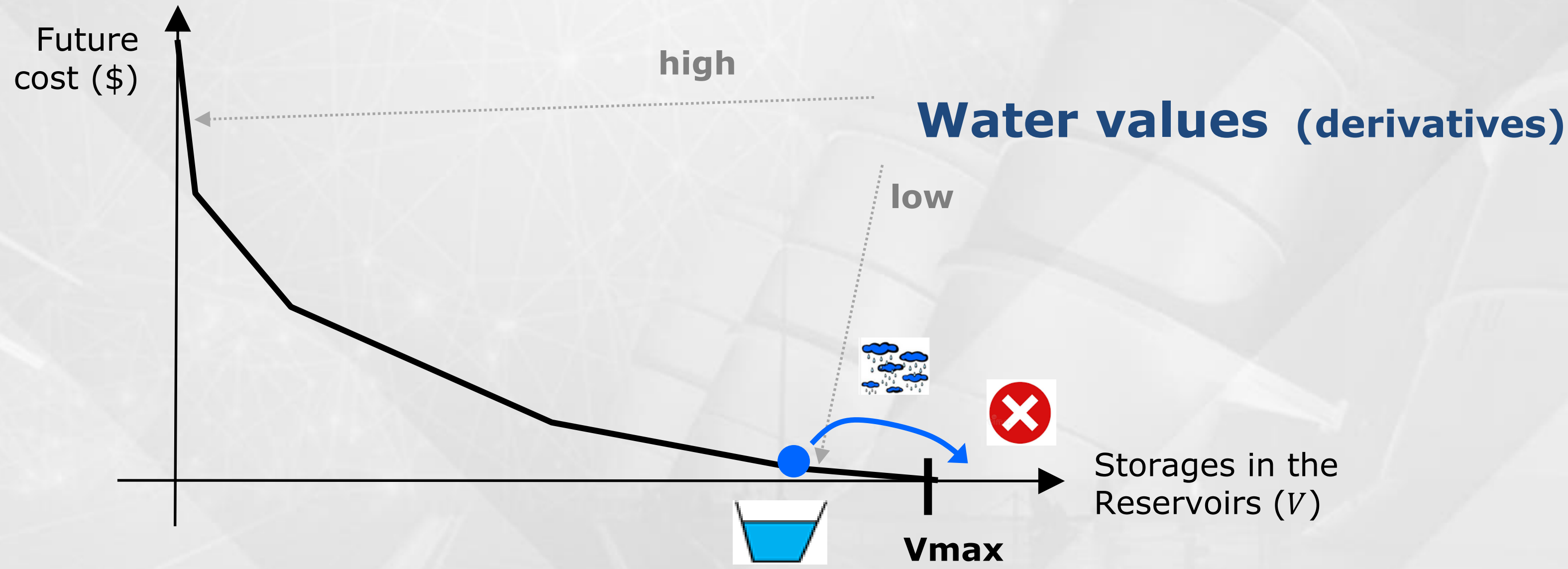
| | High | Economic operation |
| Use Thermal Generation (higher costs) | Low | Economic operation |
| | high | Unnecessary spillage |



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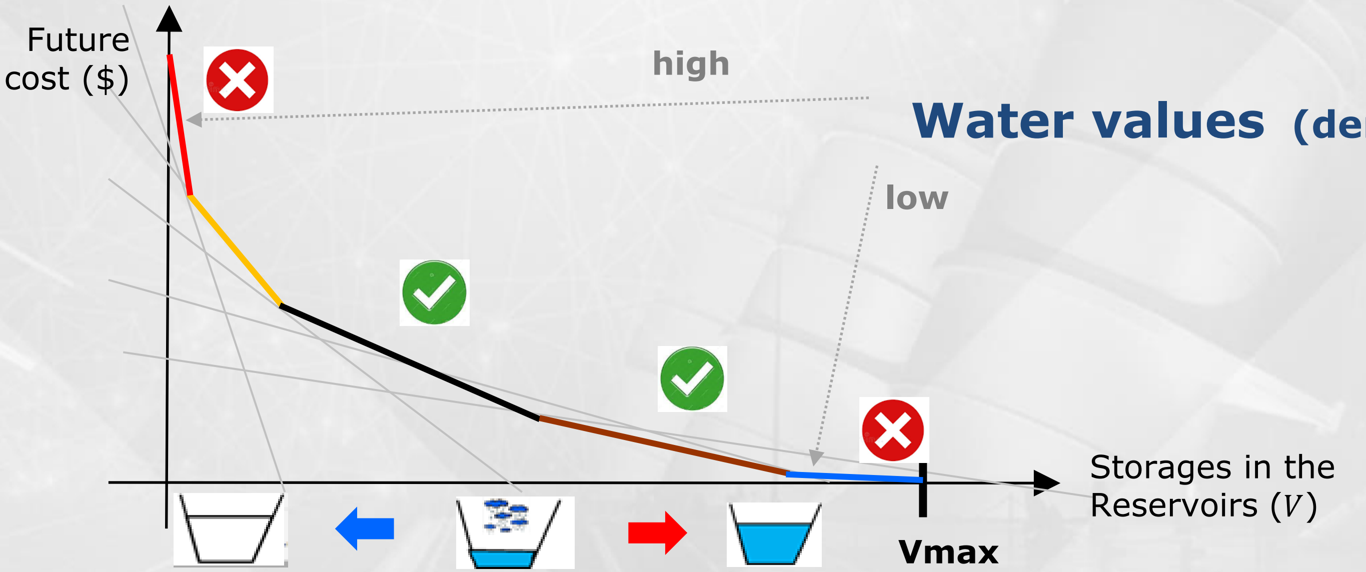


# How to obtain the Operation Policy?

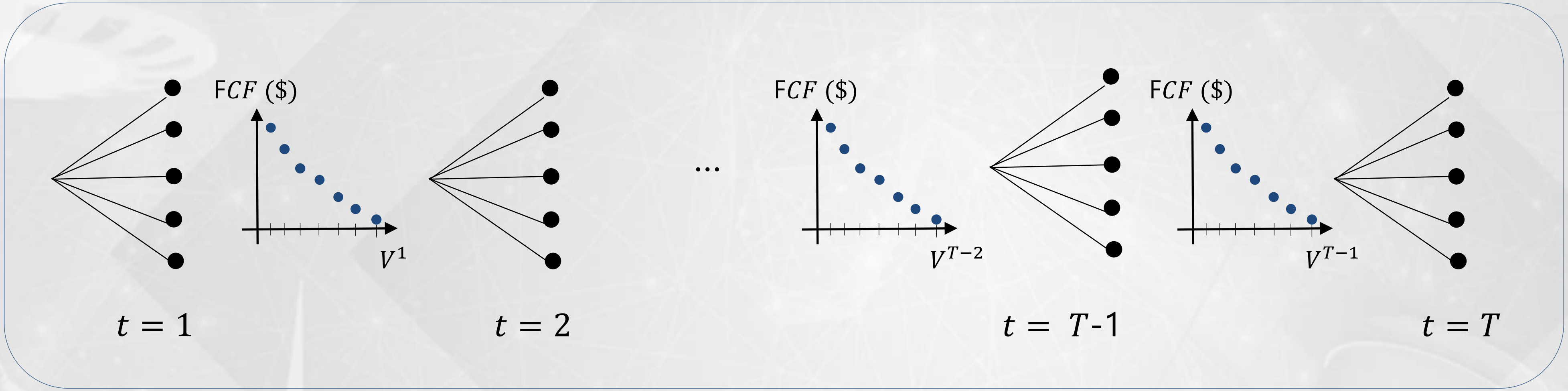
## Theoretical Background

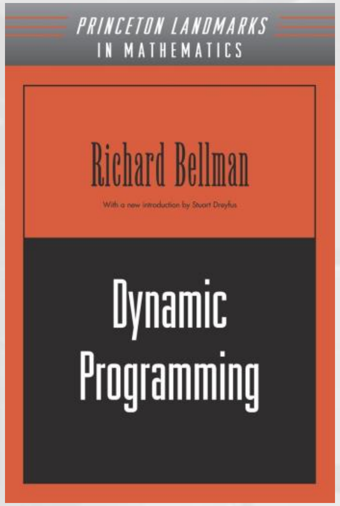
- **1962:** Bellman´**s (Stochastic) Dynamic Programming** (**SDP**)
- **1969:** Slyke & Wets L-shaped Method (2-stage problem) + Benders cuts

  **1985:** Birge´s **Dual** Dynamic Programming (**DDP**) – multistage problem
- **1991:** Pereira´s **Sampling**-based DDP (**SDDP** – Stochastic Dual Dynamic Programming)

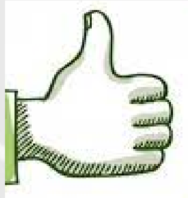
**The FCFs consist in Recourse Functions obtained as an output of Dynamic Programming-based optimization strategies**

# 1962: Bellman´s (Stochastic) Dynamic Programming (SDP)

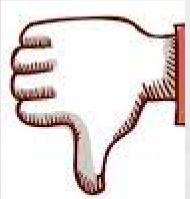



- Discretization of system states (storages at each stage
- Backward pass to build the FCFs for each stage



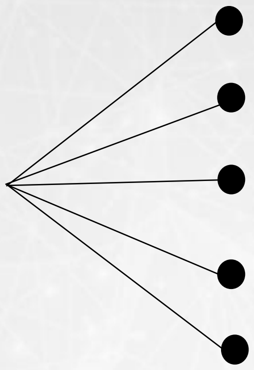
**Allows any cost function/constraints (nonconvex problems)**



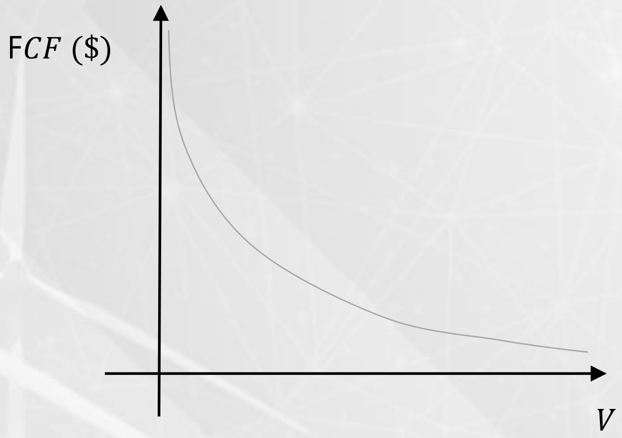
**Curse of dimensionality**

# 1969: Slyke & Wets L-shaped Method (2-stage)

1st stage



2nd stage




Numerische Mathematik 4, 238—252 (1962)

**Partitioning procedures for solving mixed-variables programming problems***

By

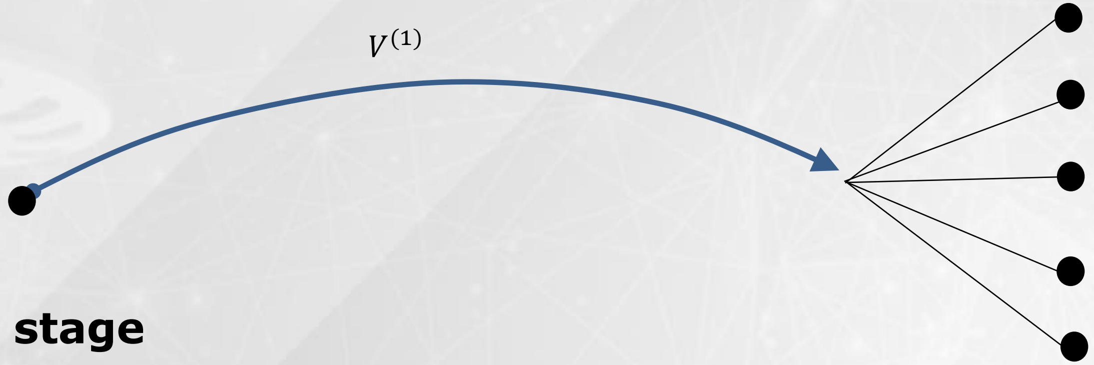
J. F. BENDERS**

SIAM J. APPL. MATH.
Vol. 17, No. 4, July 1969

**L-SHAPED LINEAR PROGRAMS WITH APPLICATIONS TO OPTIMAL CONTROL AND STOCHASTIC PROGRAMMING***

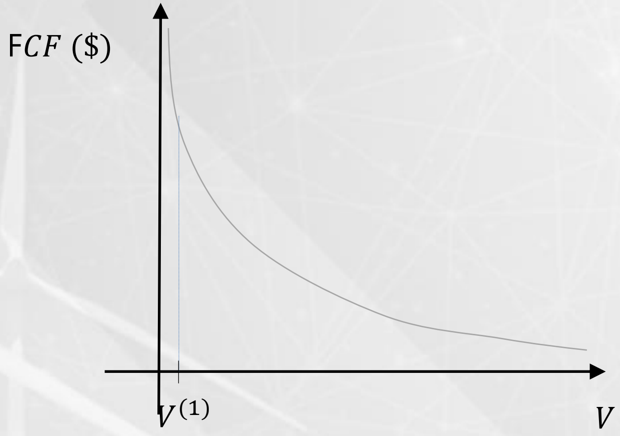
R. M. VAN SLYKE† AND ROGER WETS‡

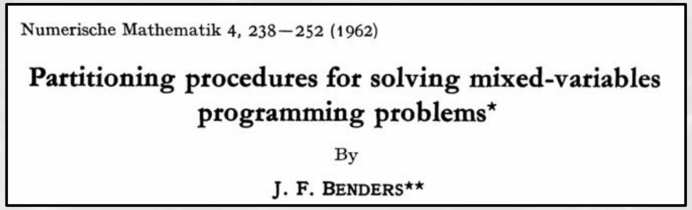
# 1969: Slyke & Wets L-shaped Method (2-stage)




**1st stage**

**2nd stage**



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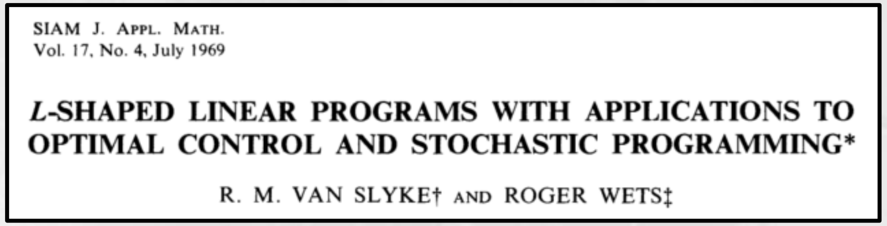

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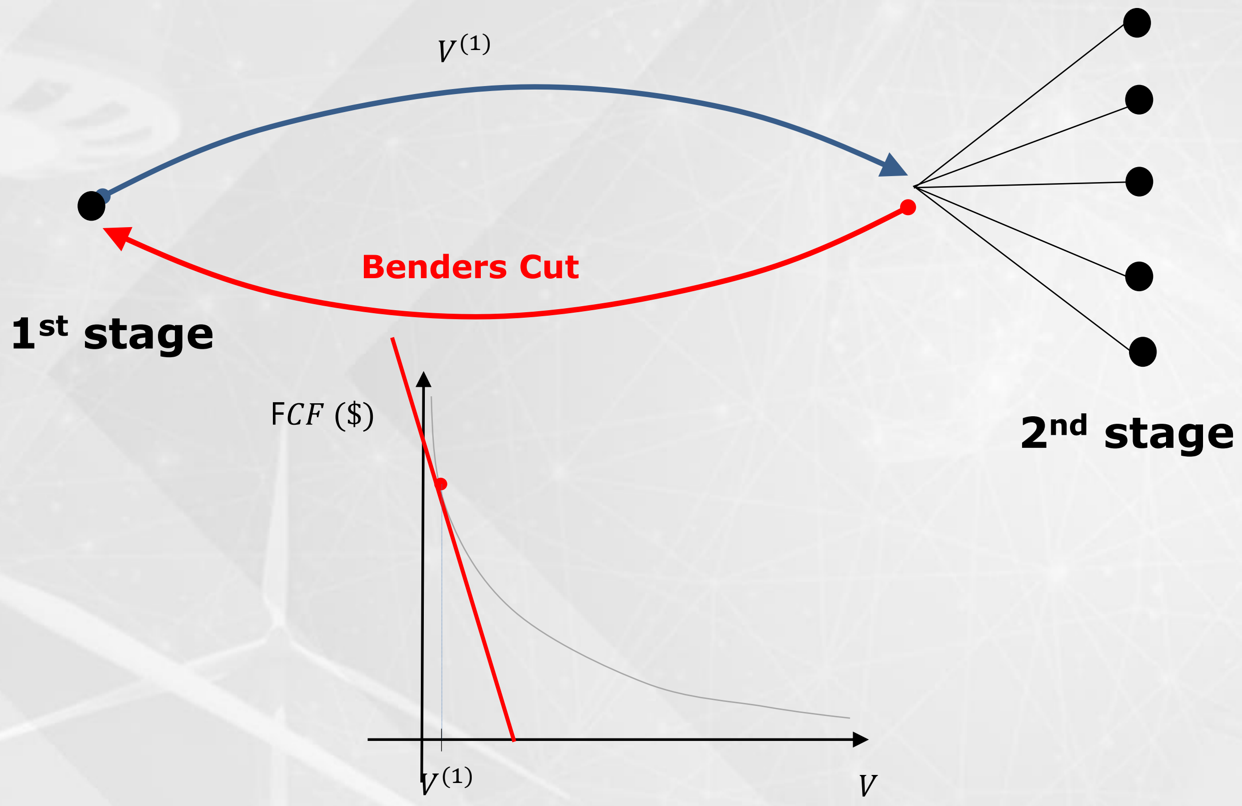


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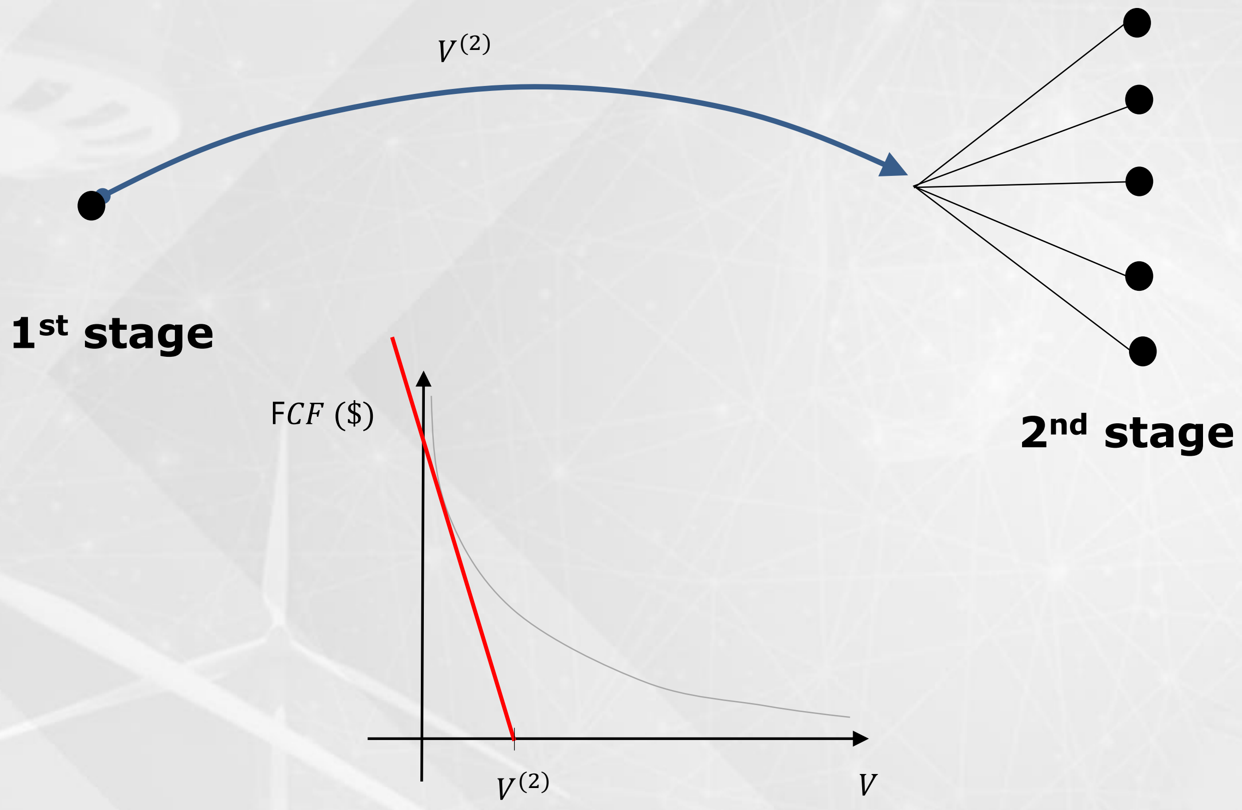


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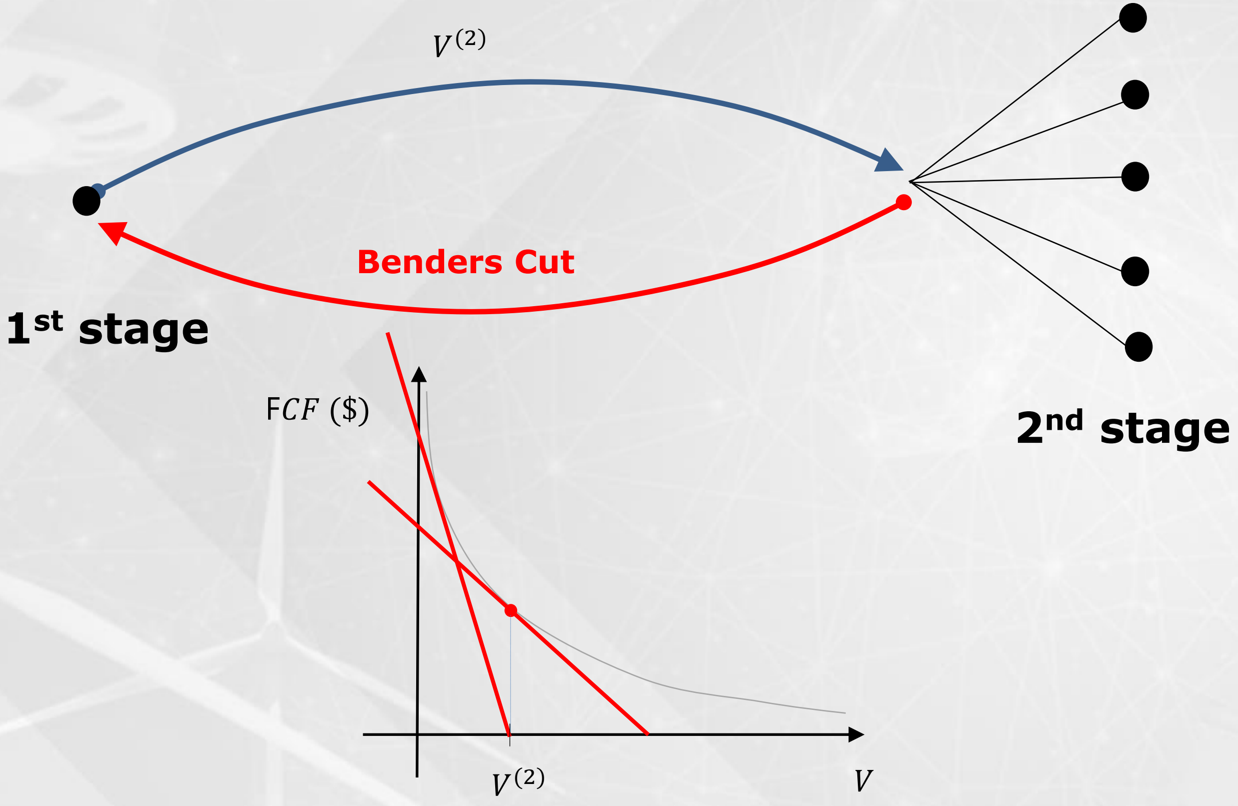


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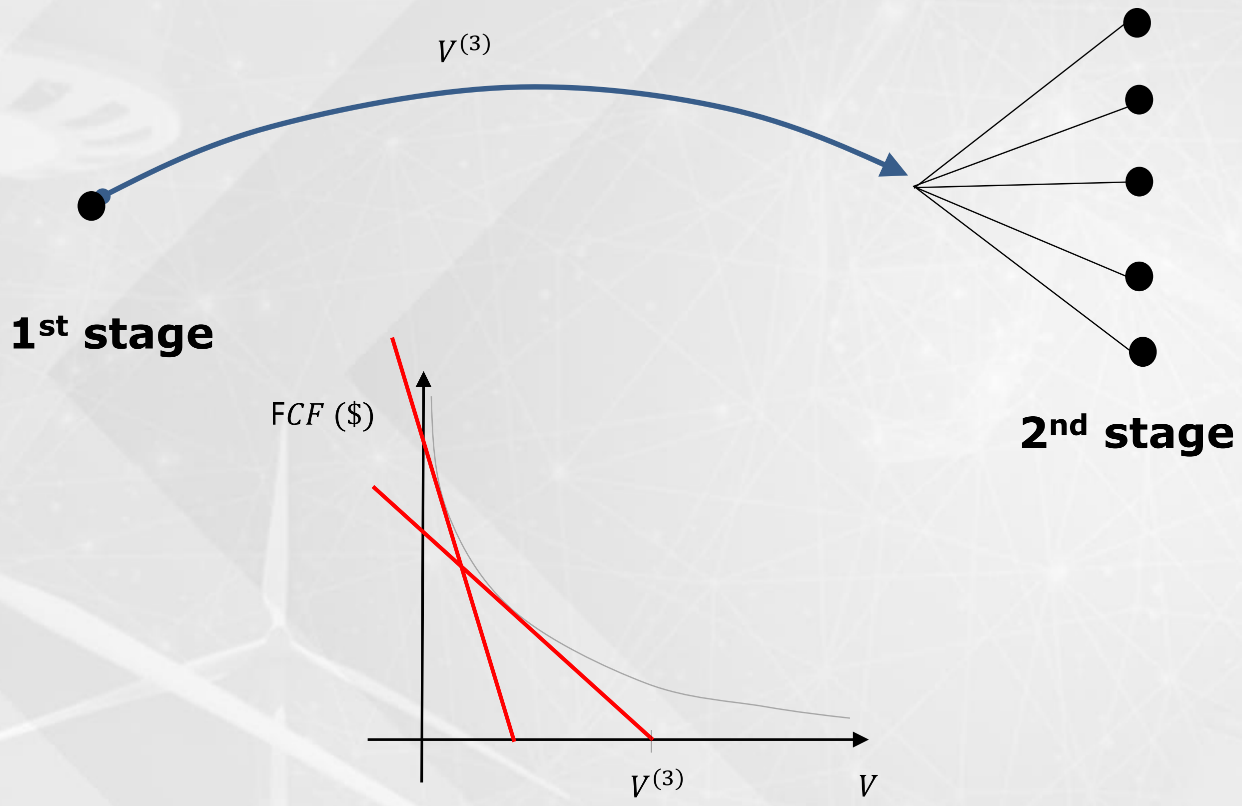


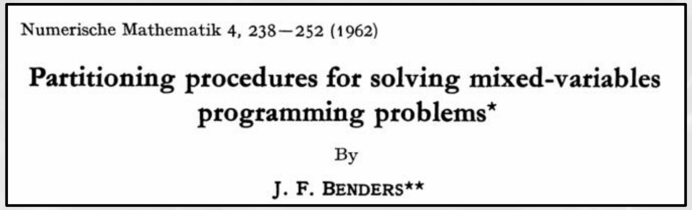
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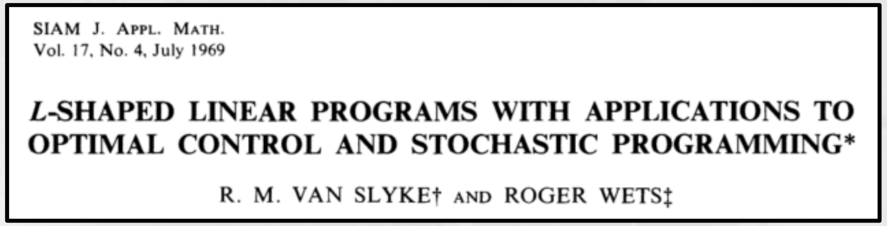
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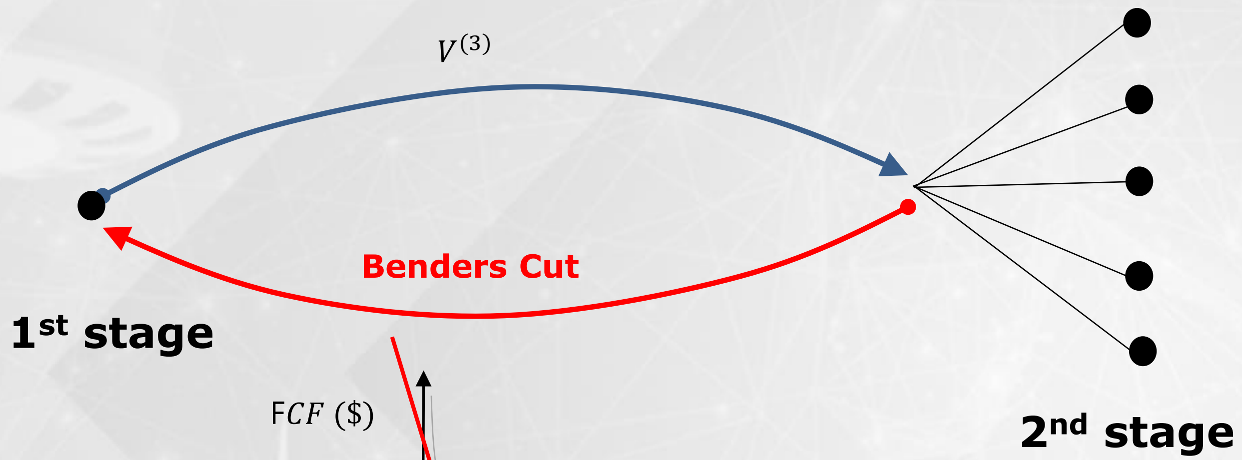


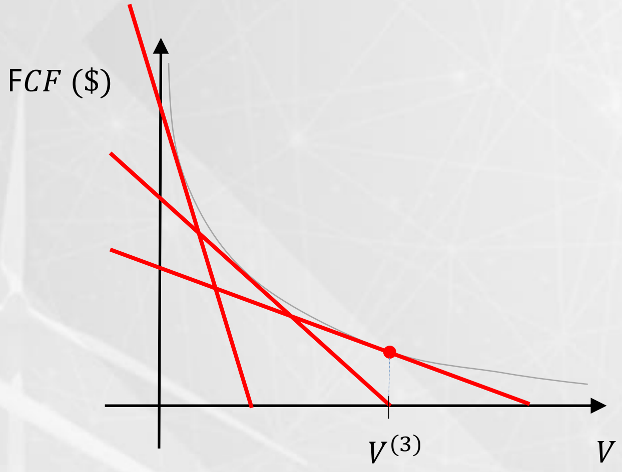


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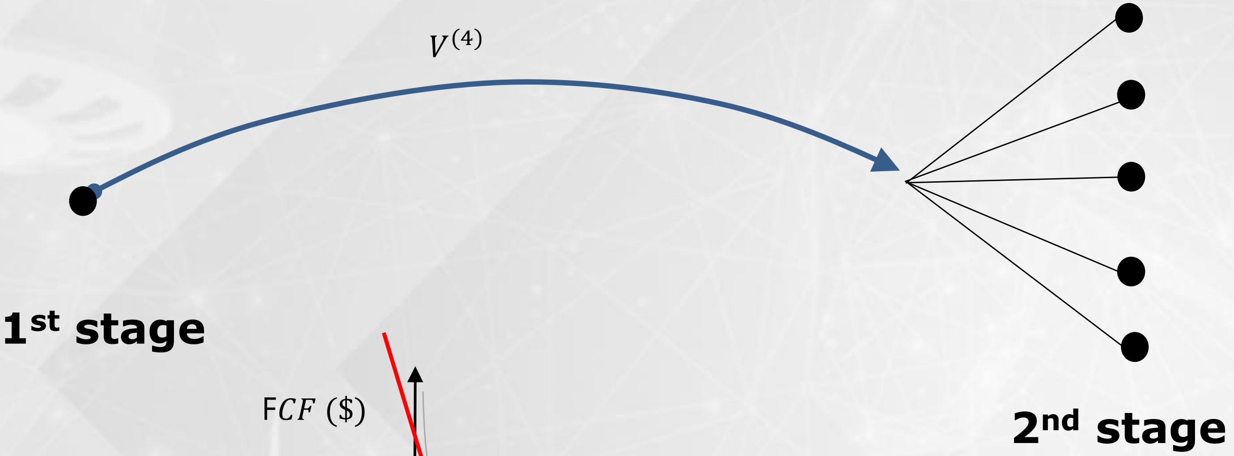


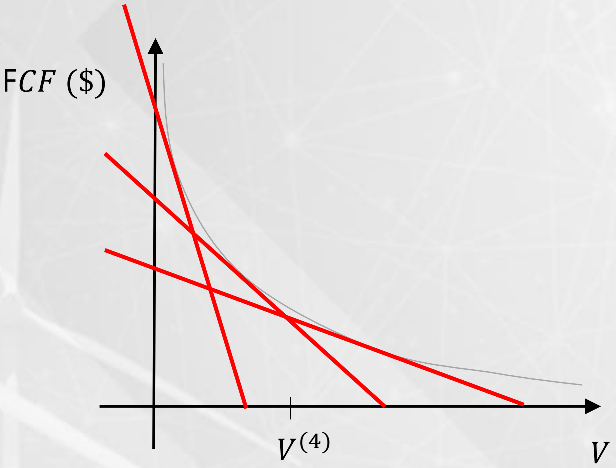


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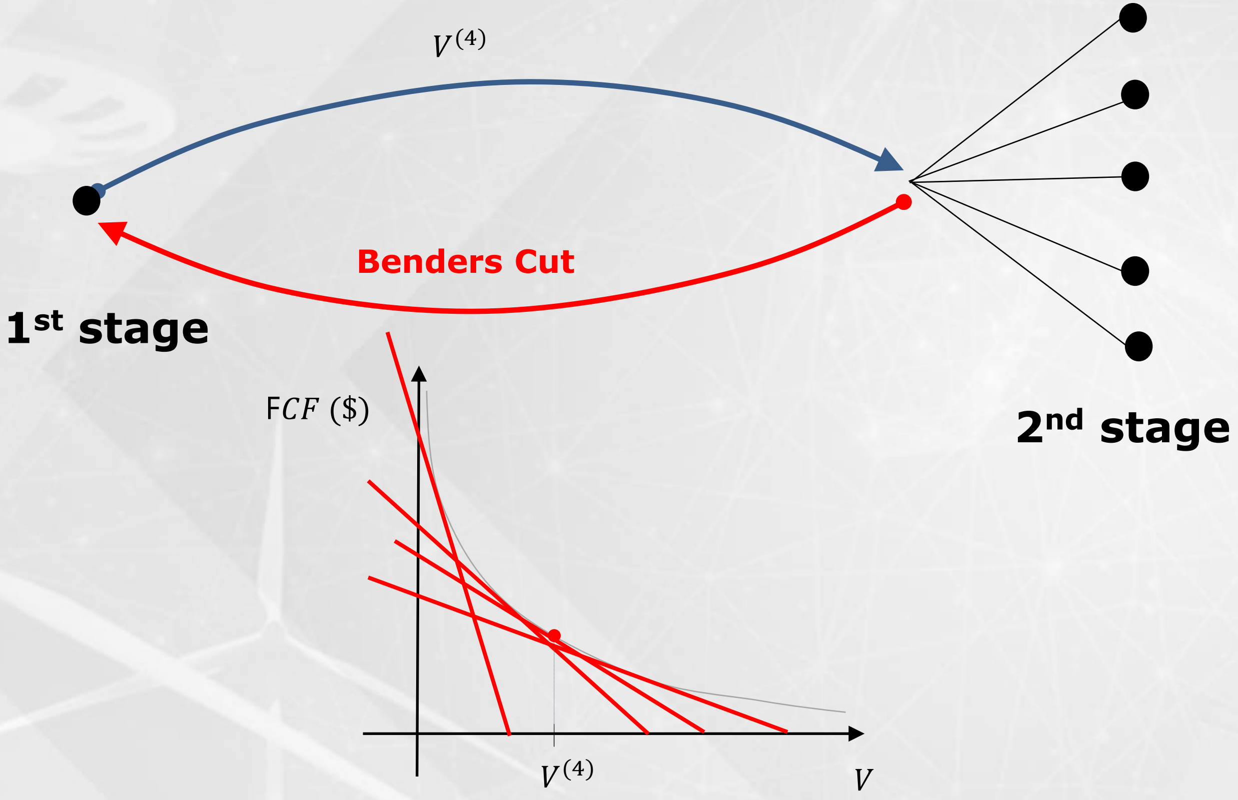


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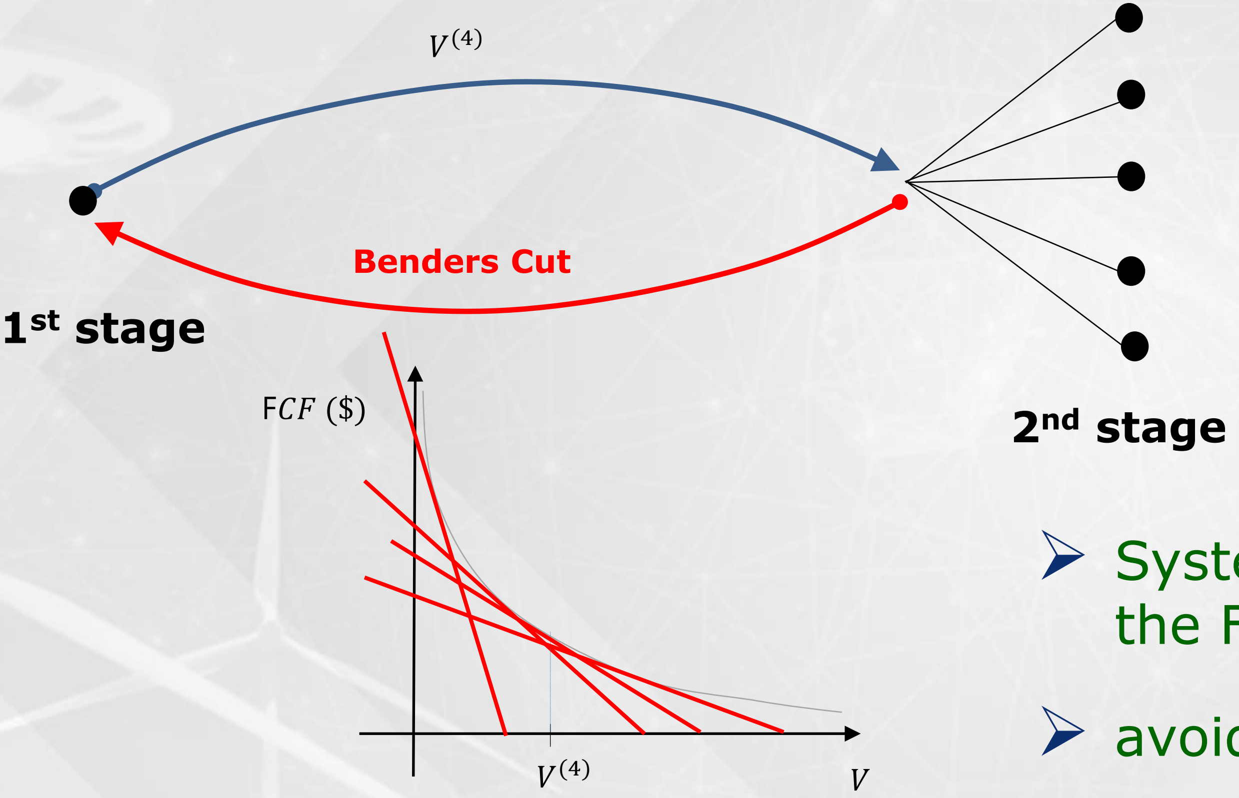


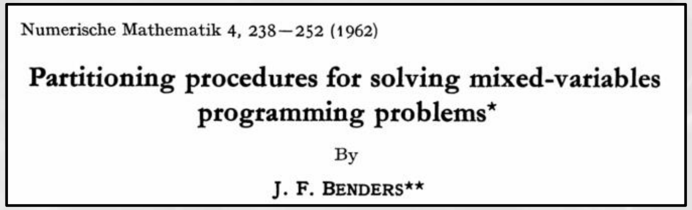
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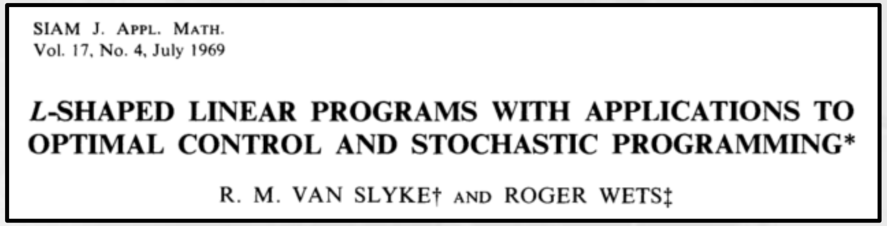
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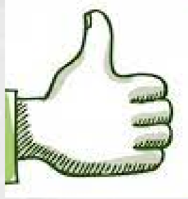
- System states where to approximate the FCF are obtained iteratively
- avoids the curse of dimensionality



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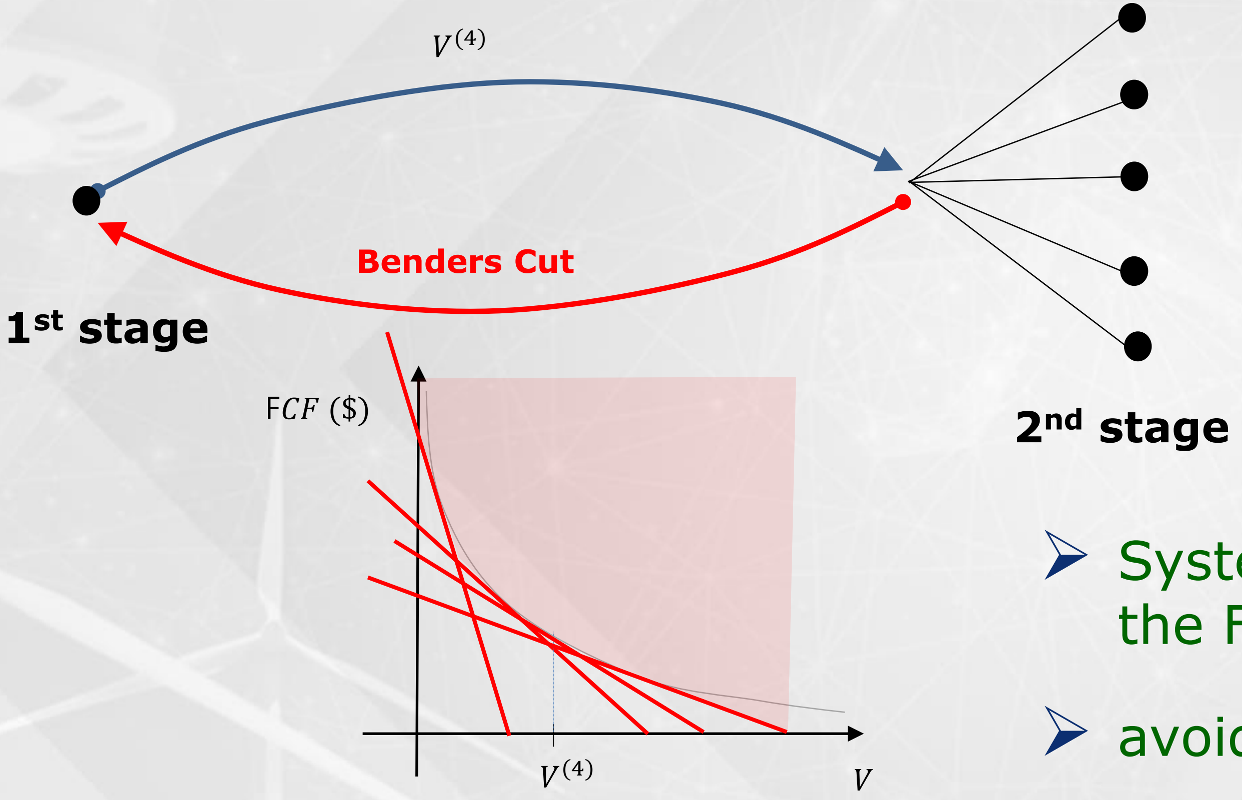


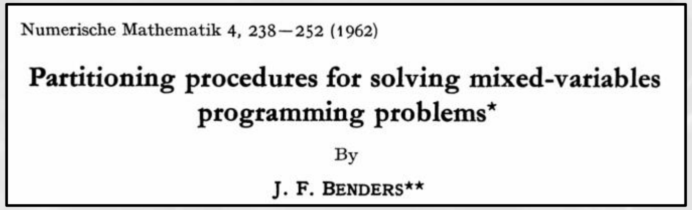
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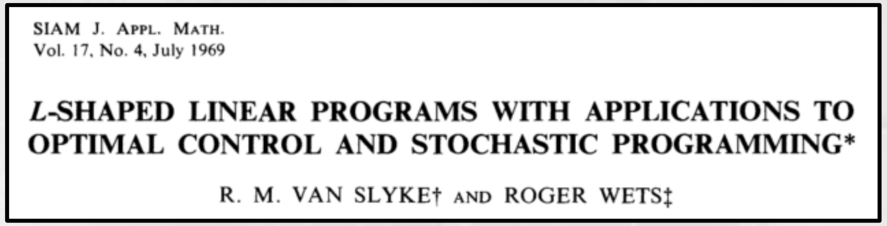
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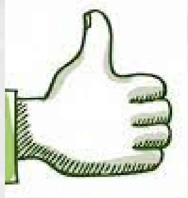

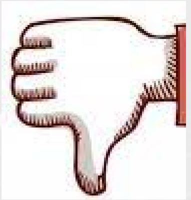

**Lower Piecewise Linear Approximation (PWL) requires convexity of the FCF**

**Second-stage subproblems need to be convex**

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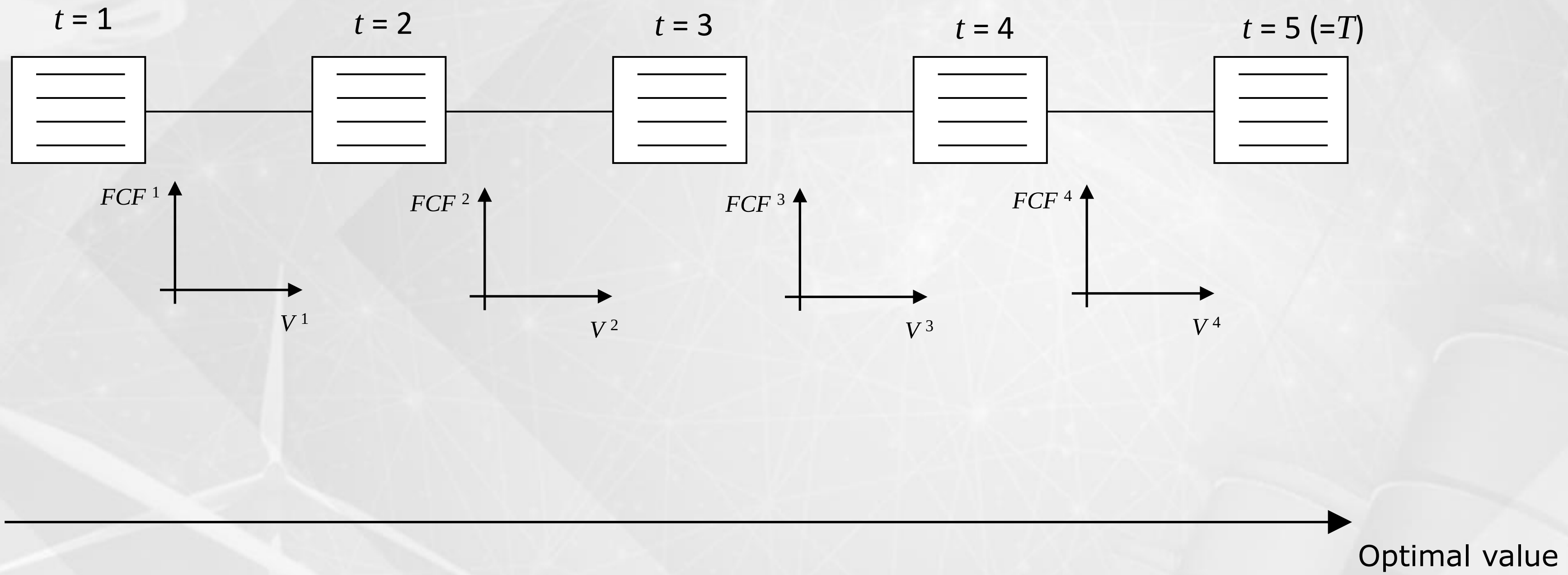


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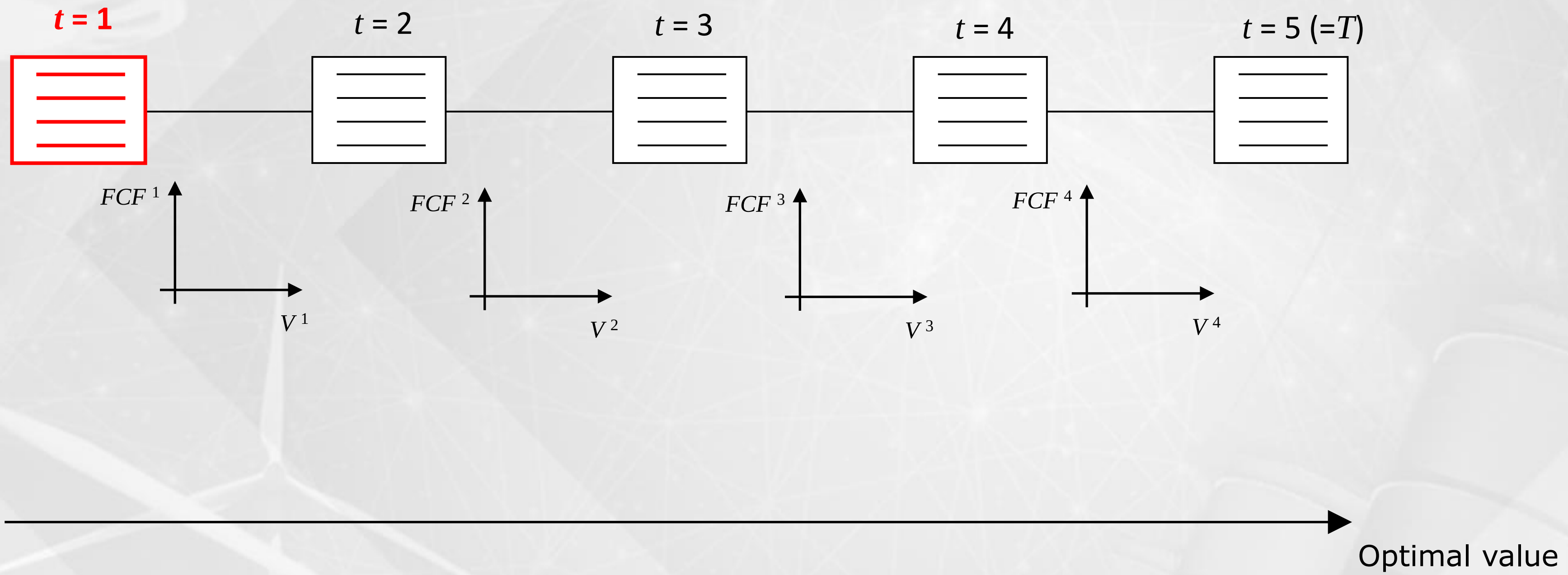


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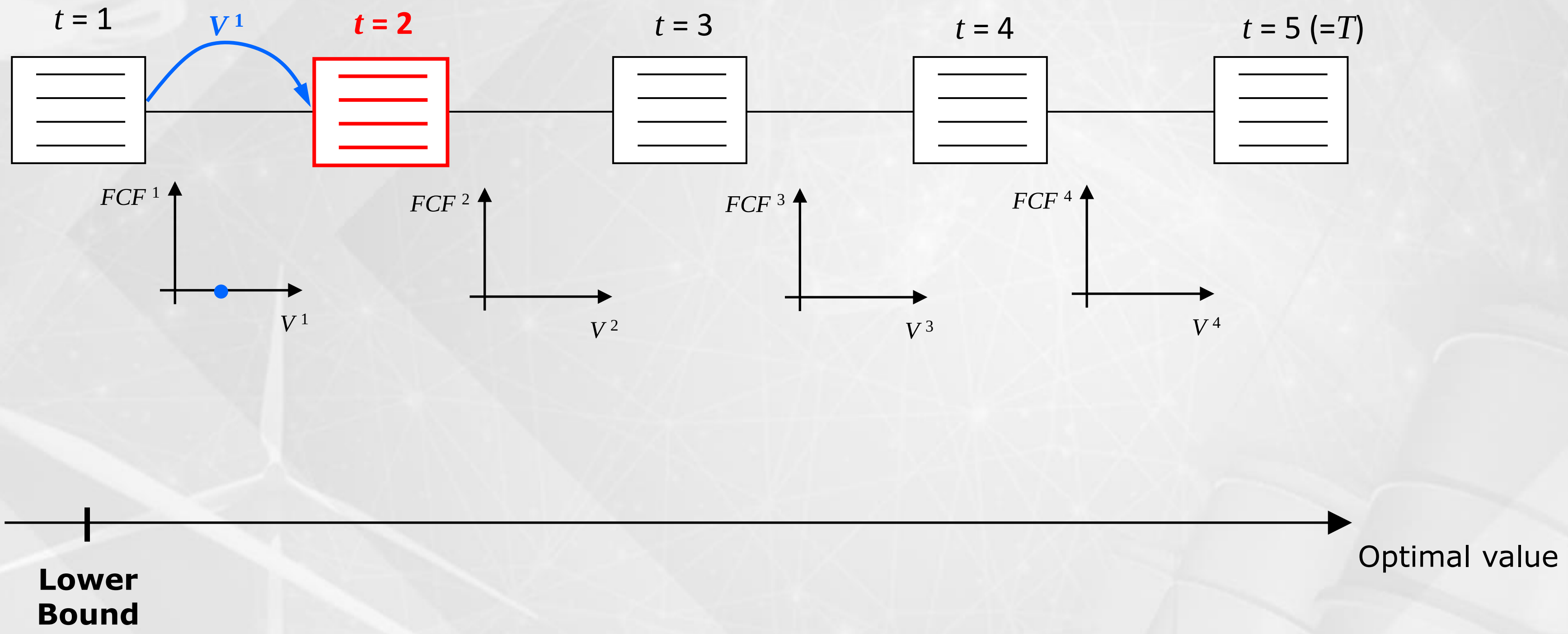


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Cepel A pesquisa que constrói o futuro

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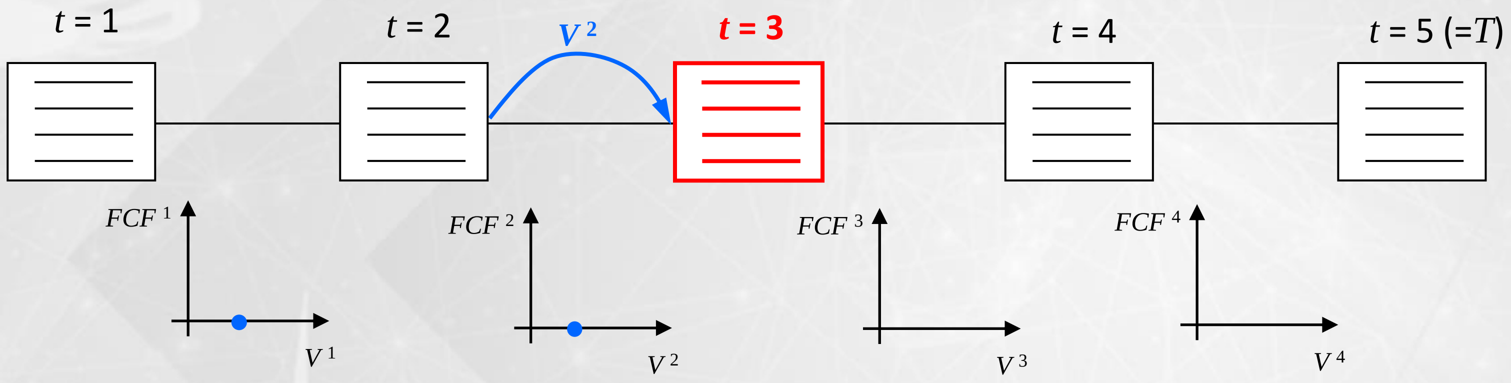


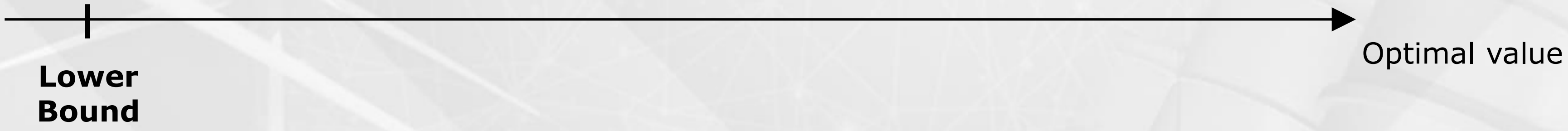


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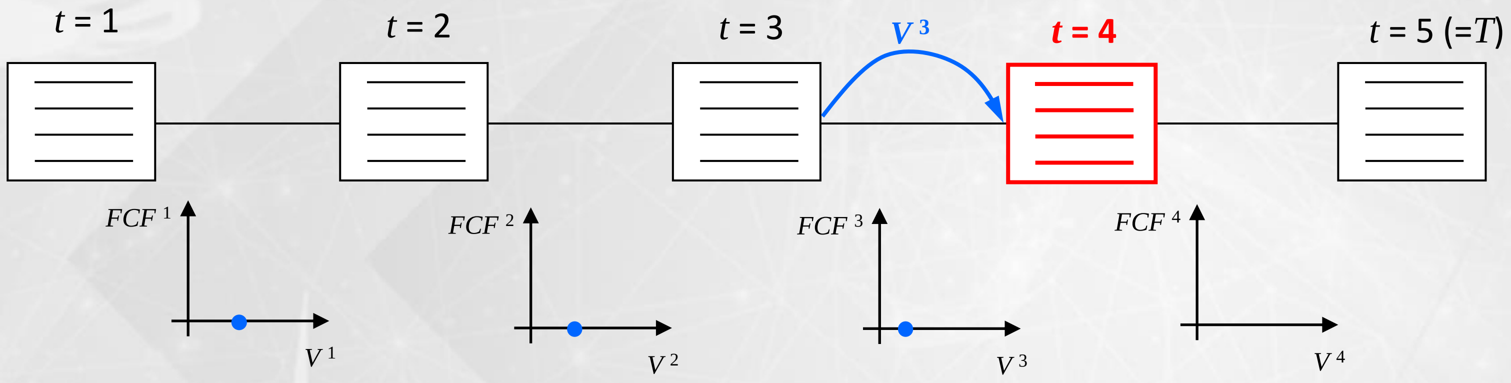



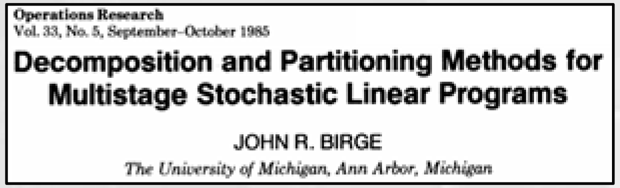
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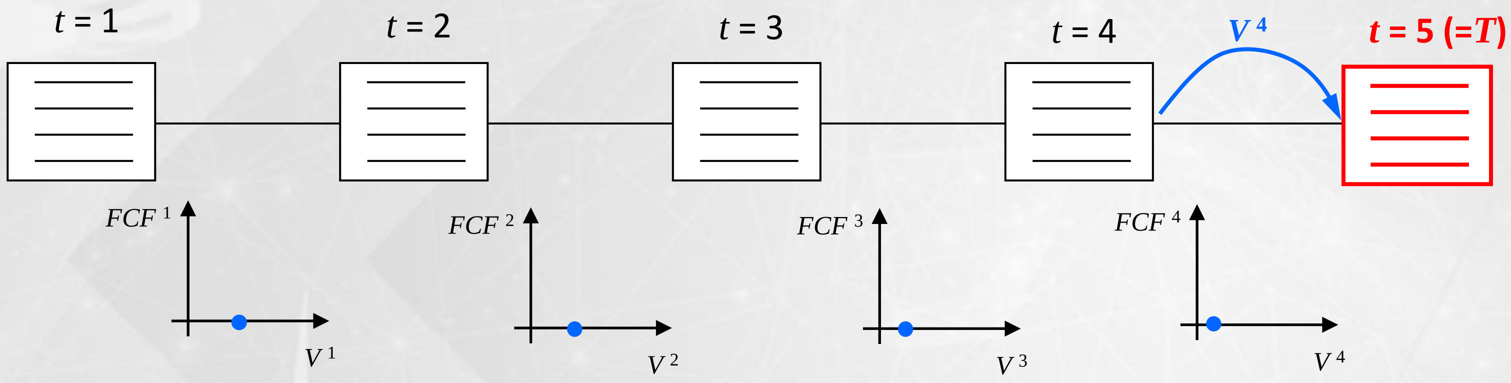


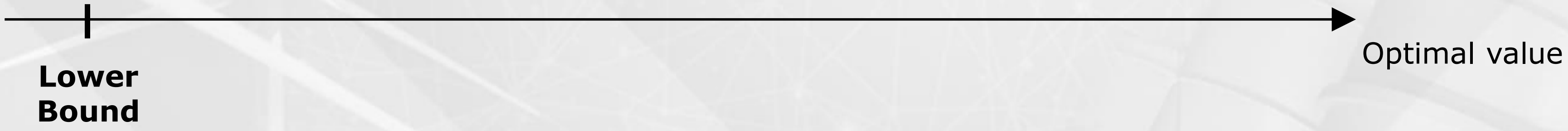


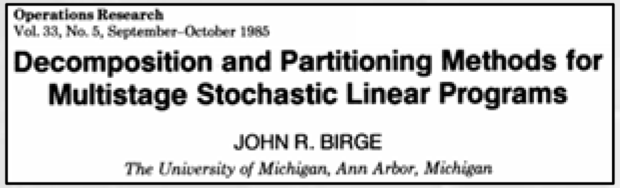
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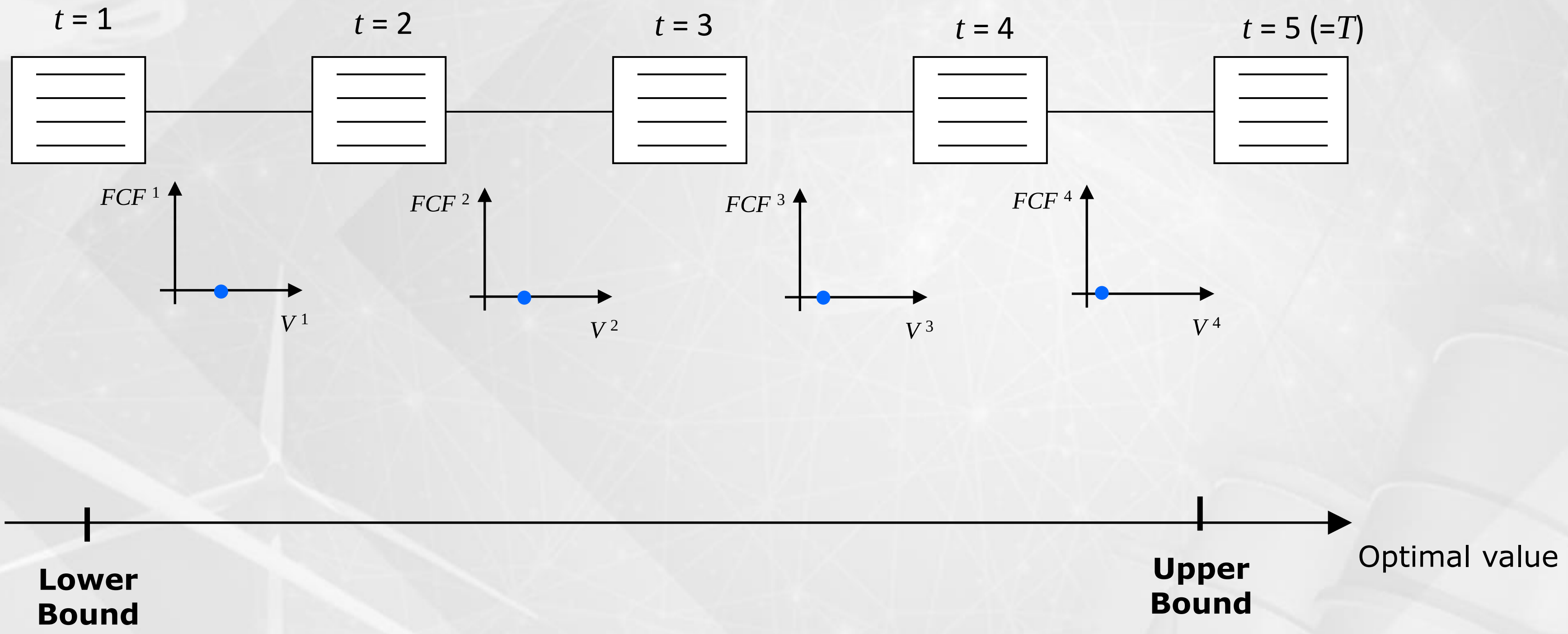


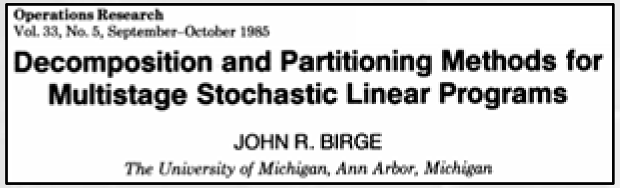

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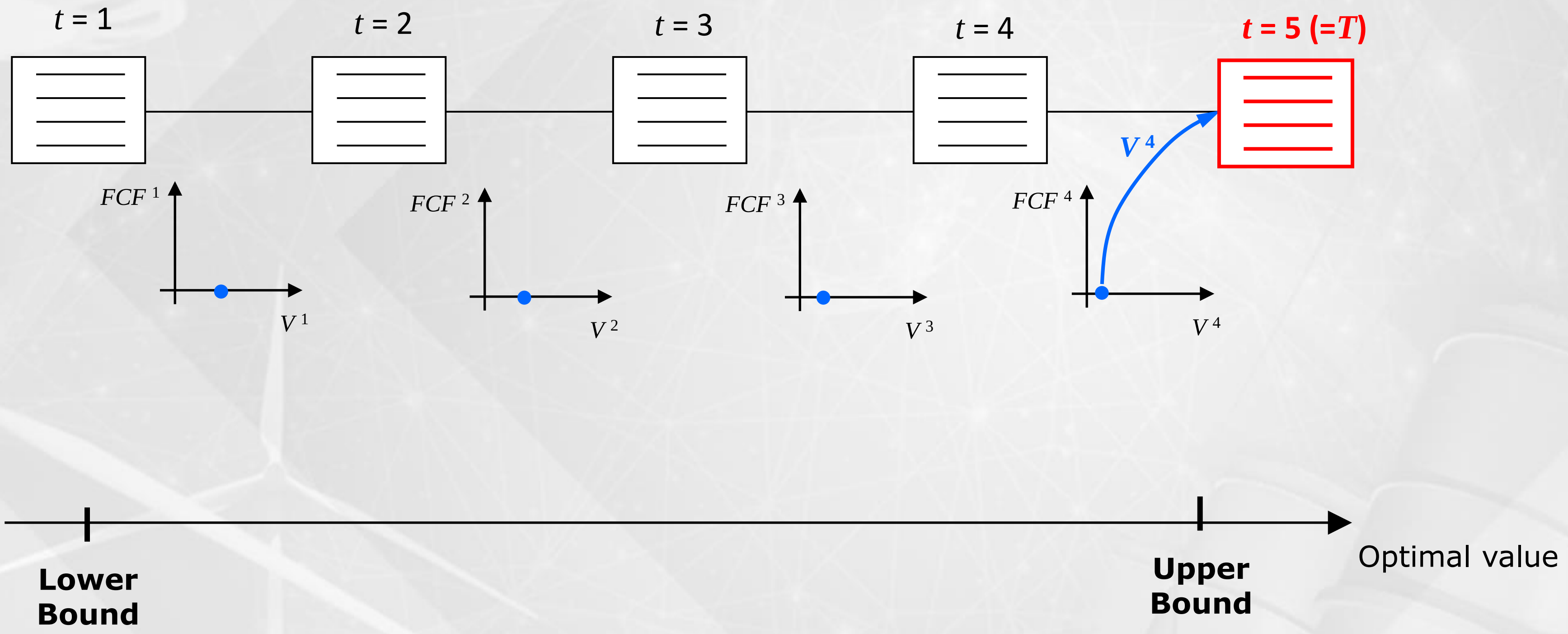


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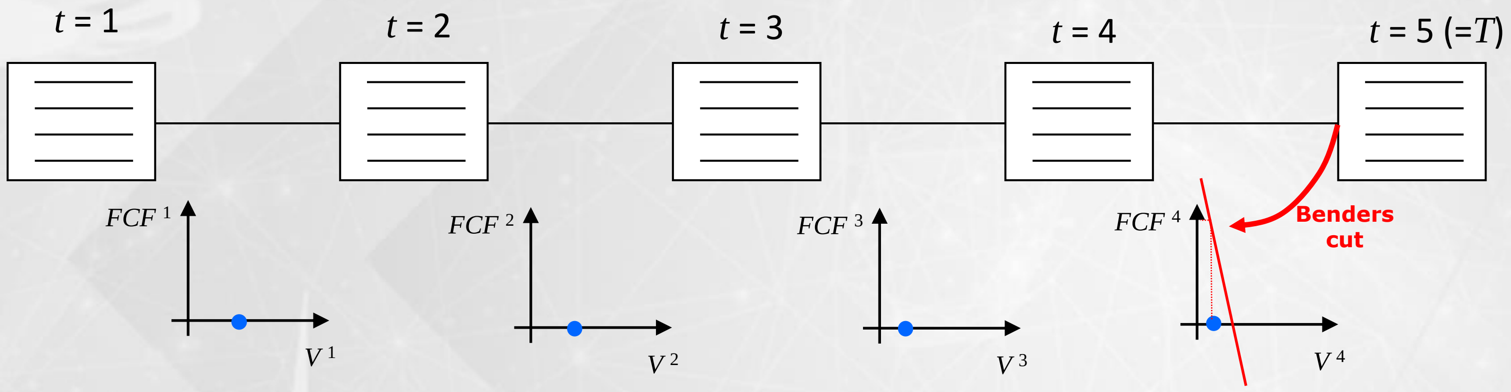


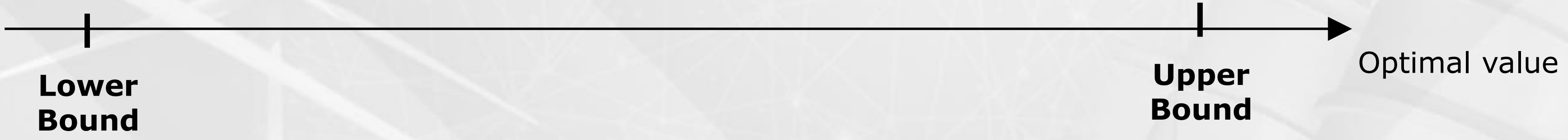


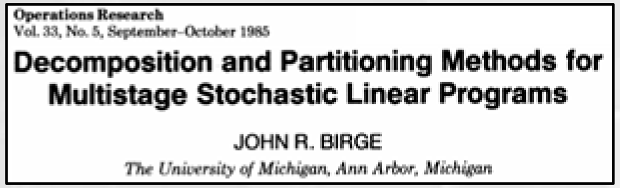

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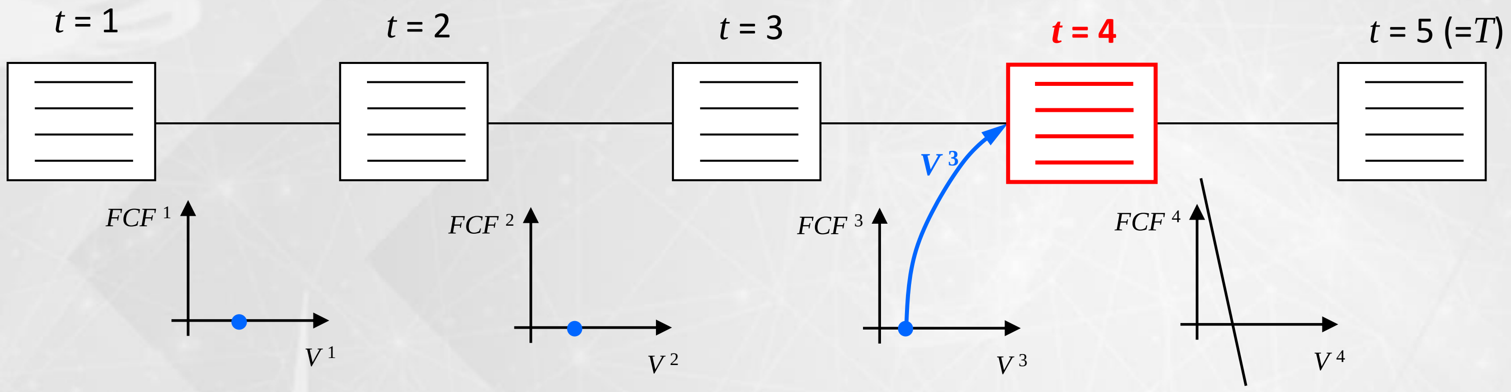


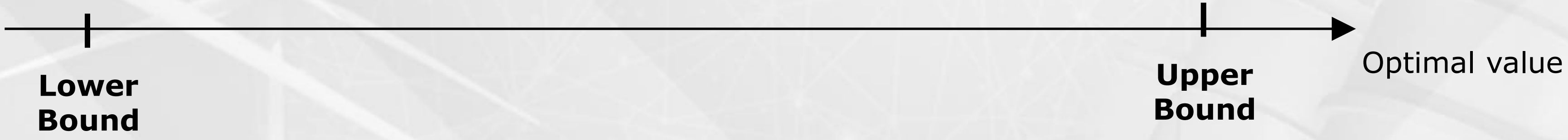


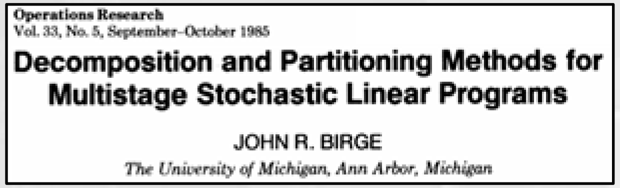

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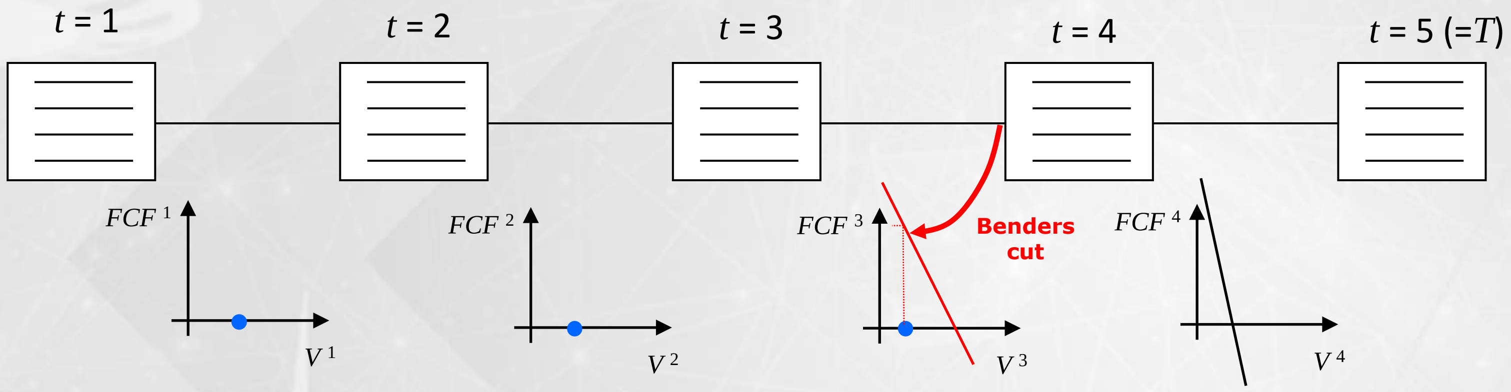


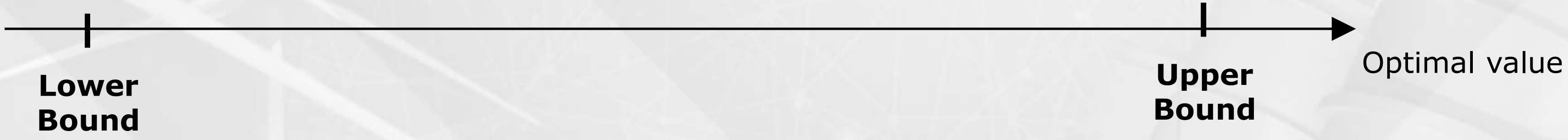


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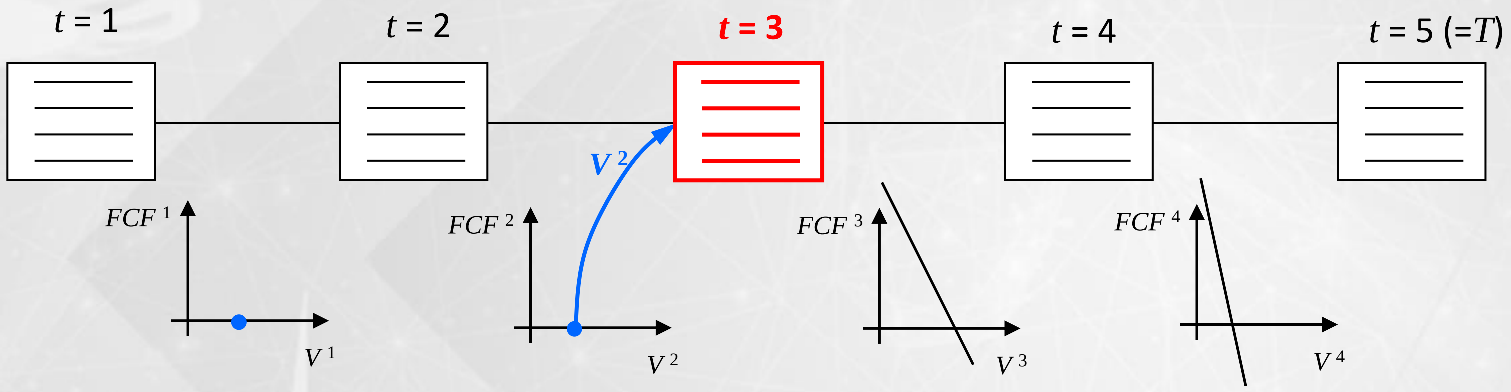


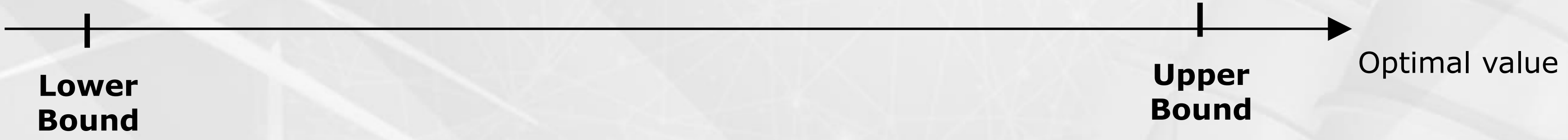


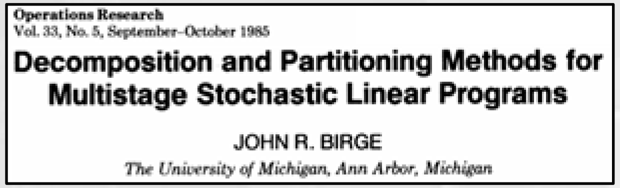
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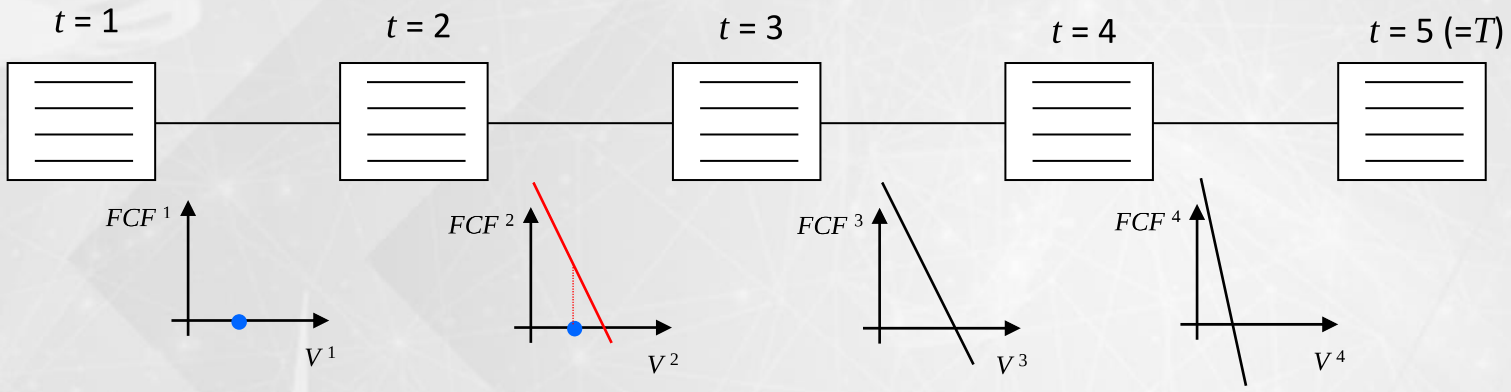


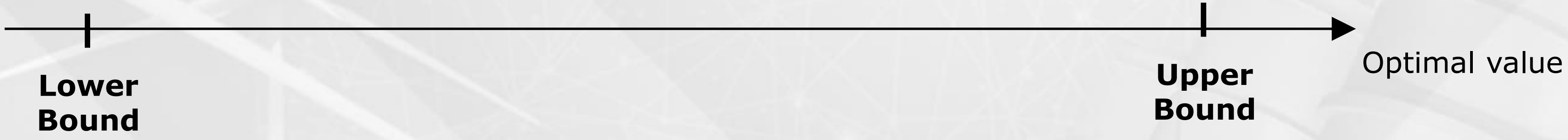


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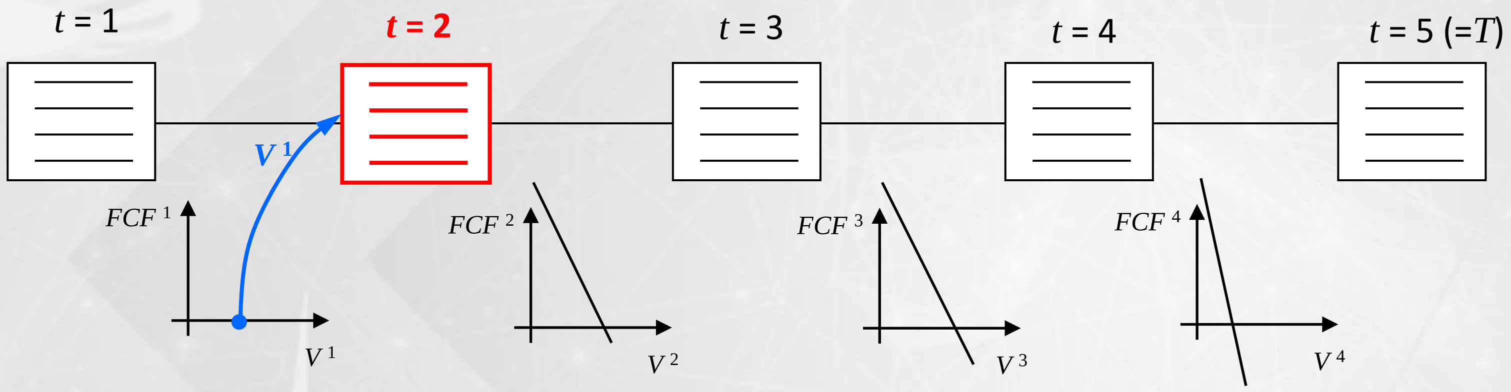


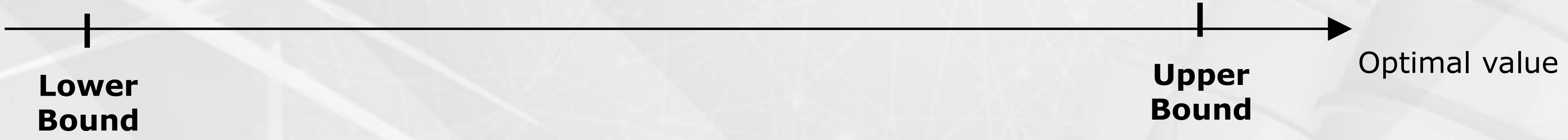


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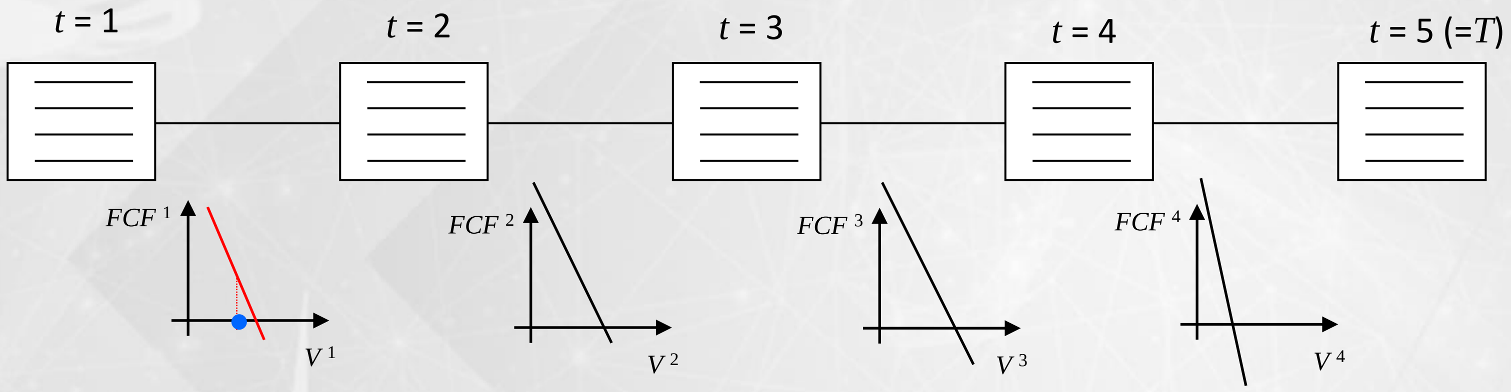


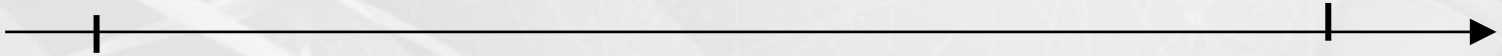


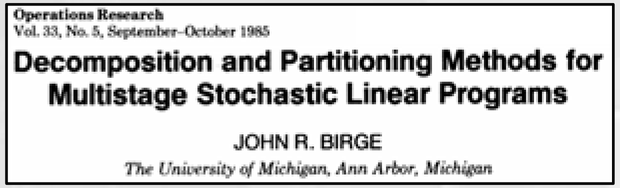

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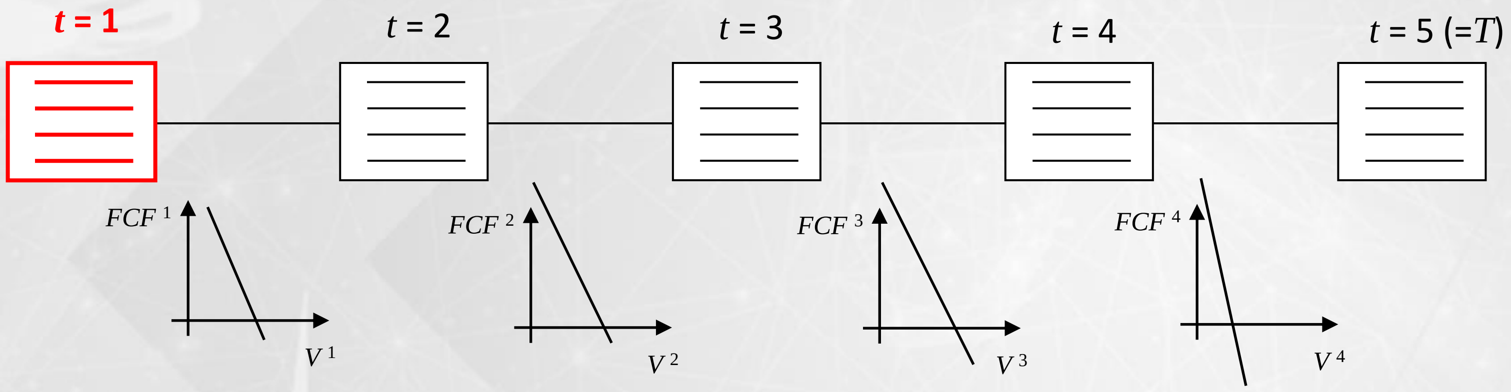


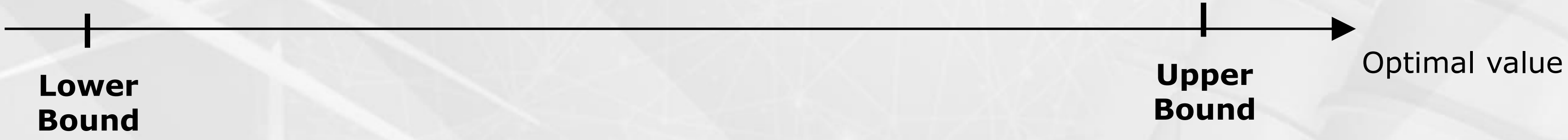


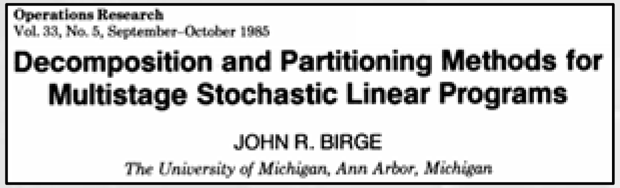
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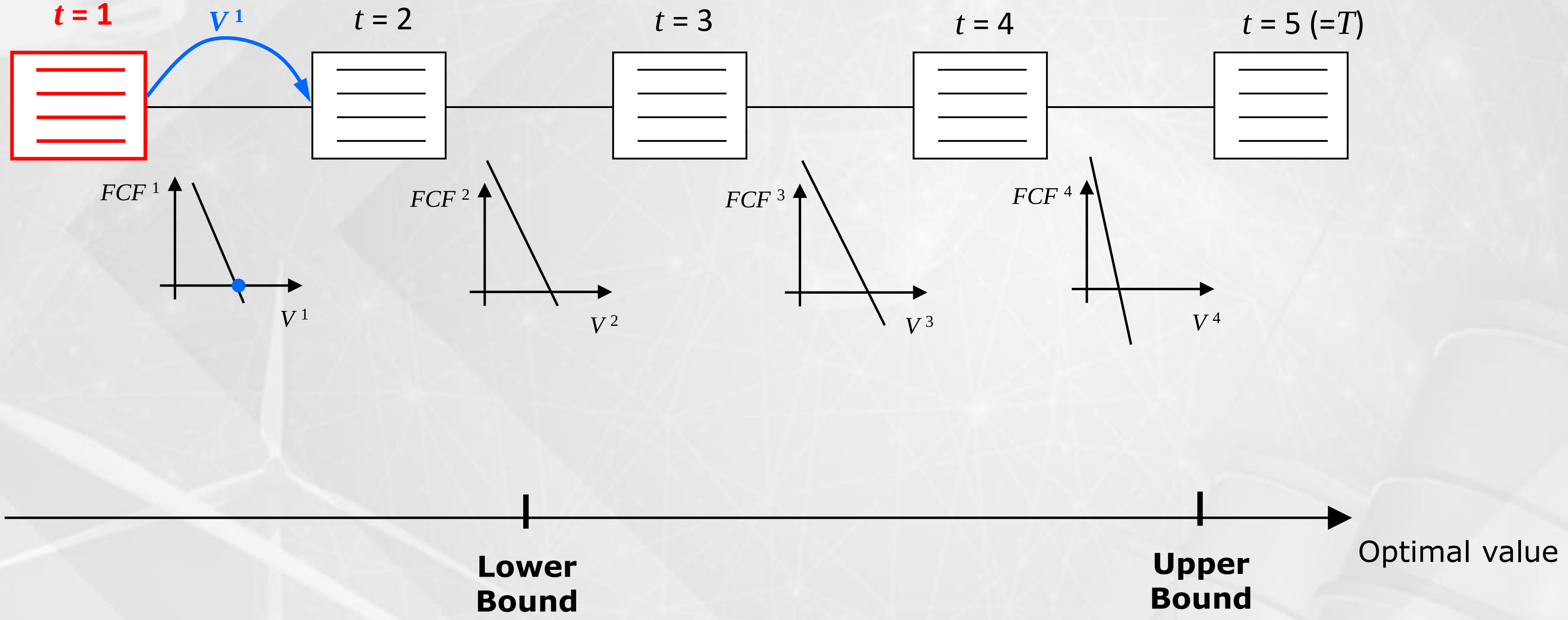


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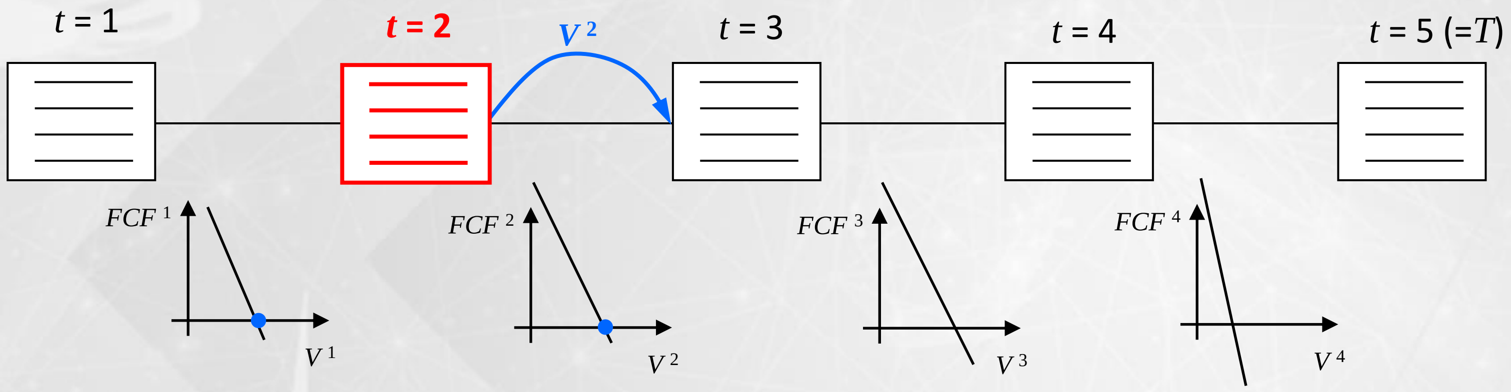



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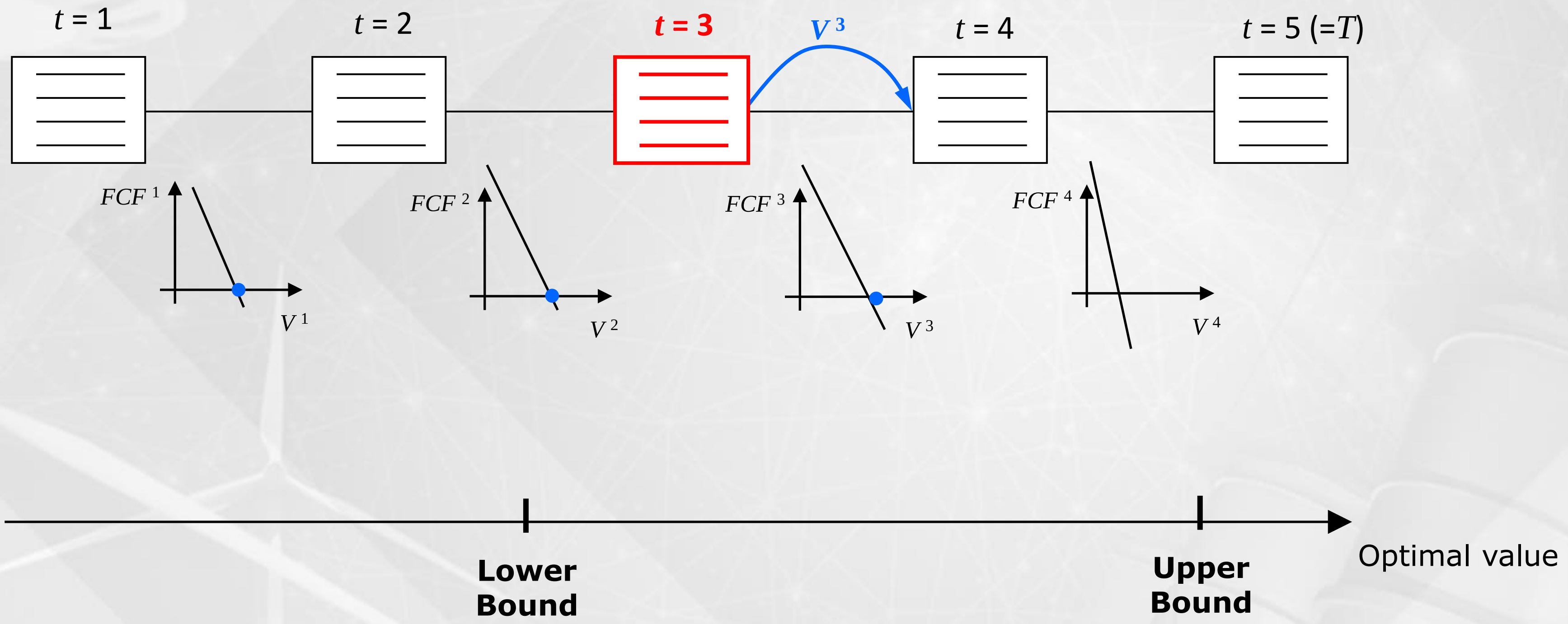


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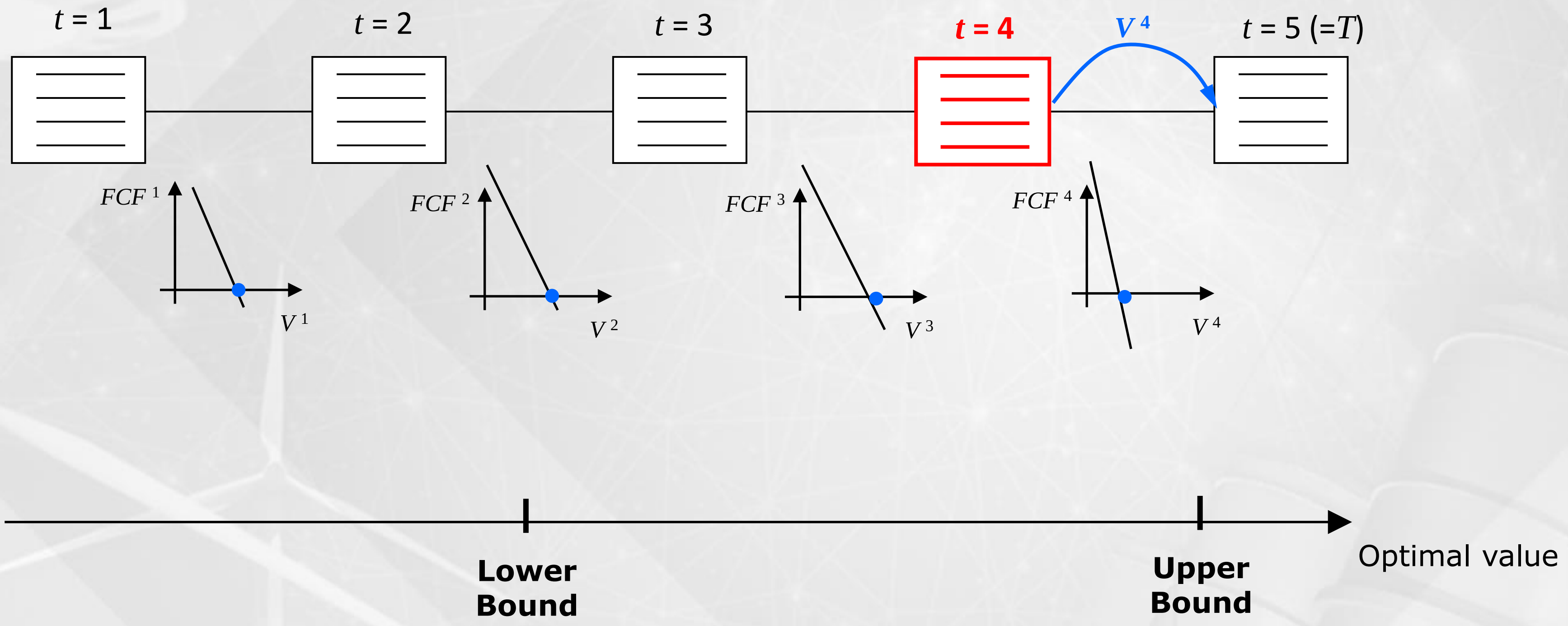


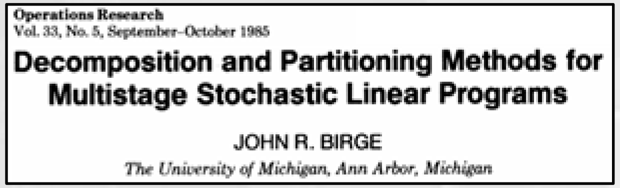

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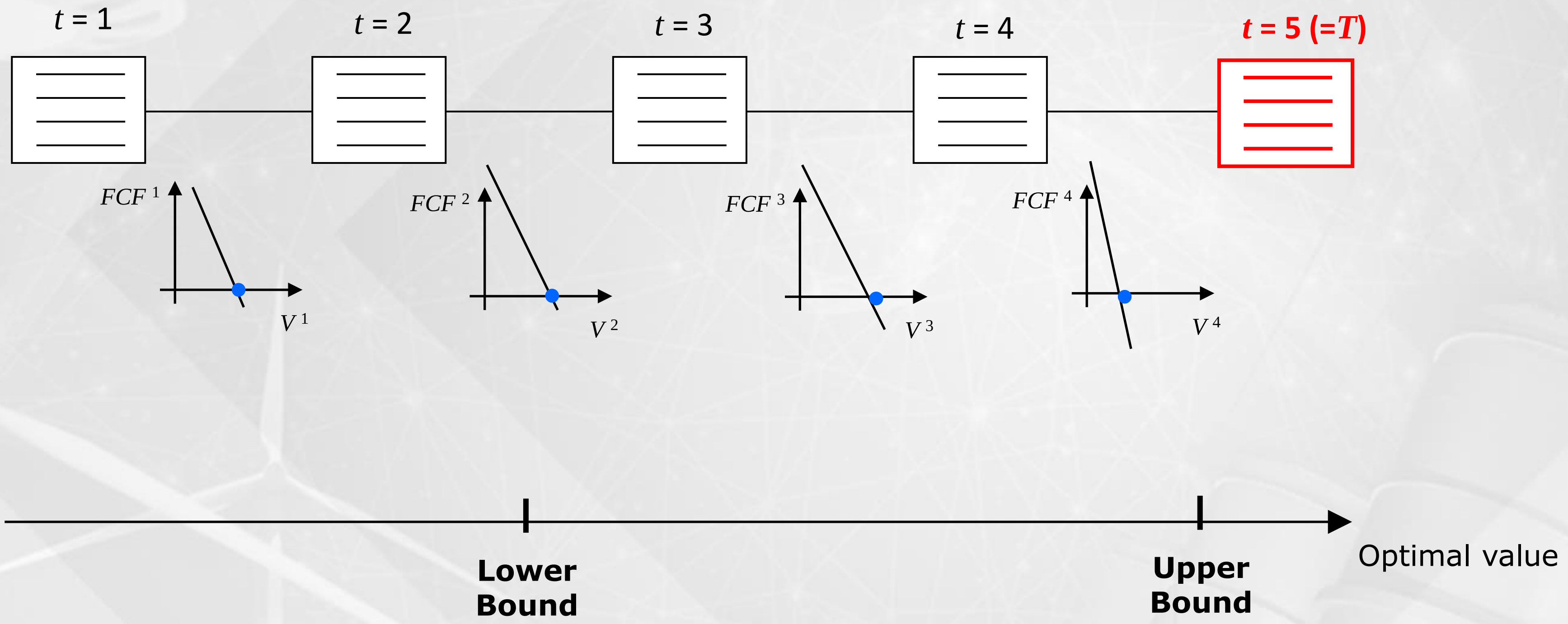


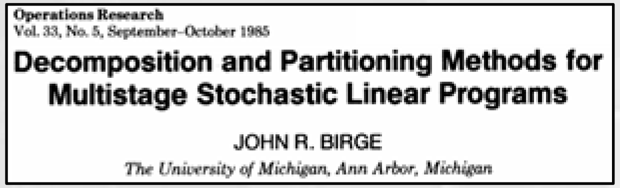

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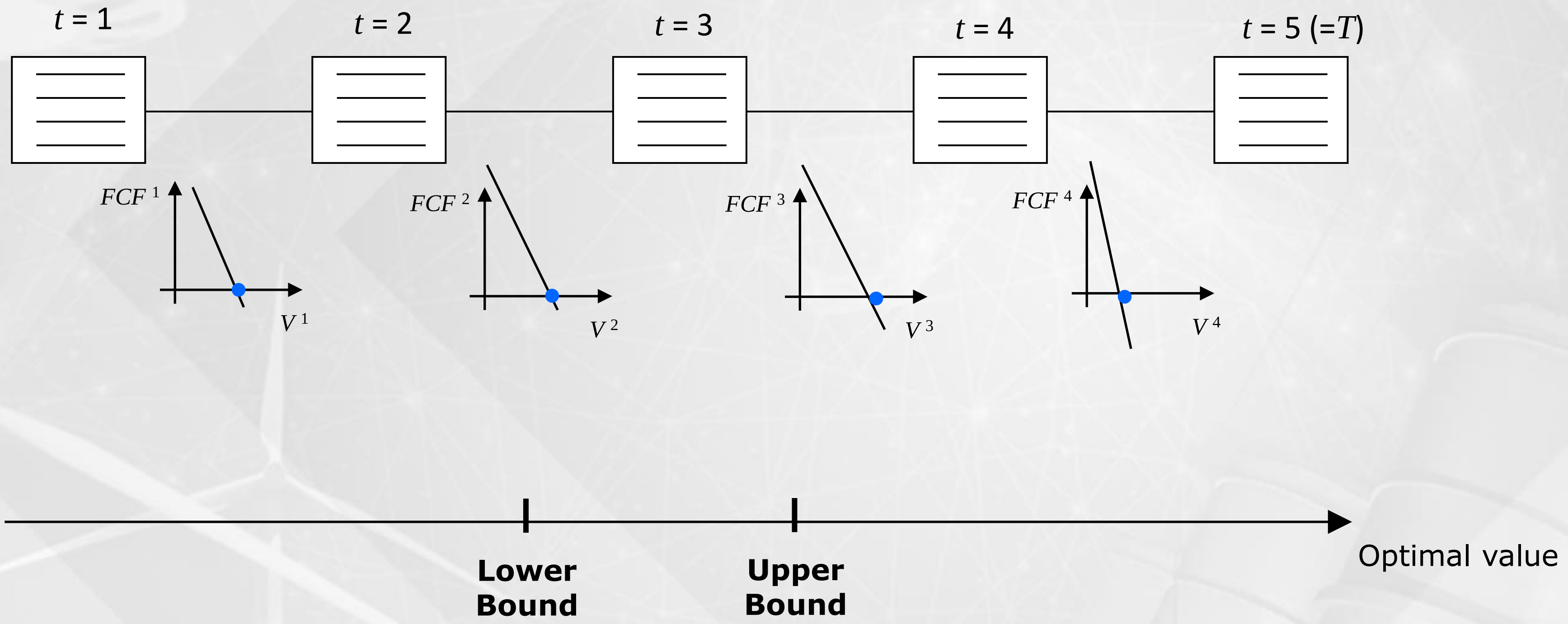


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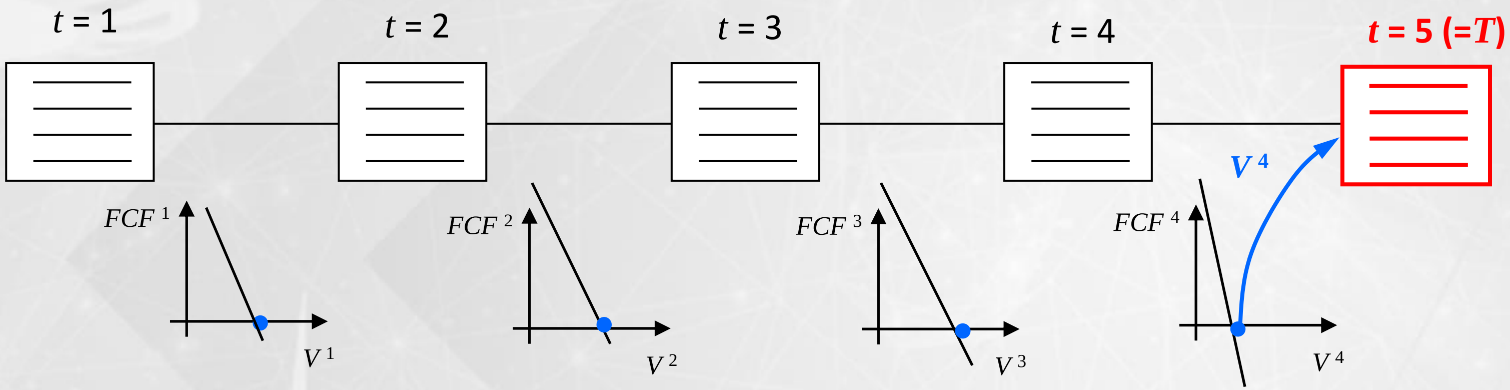



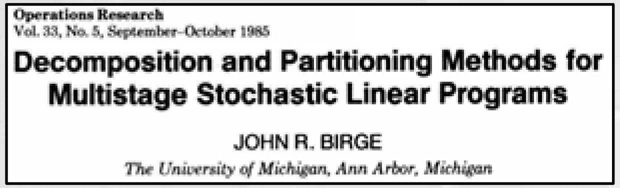
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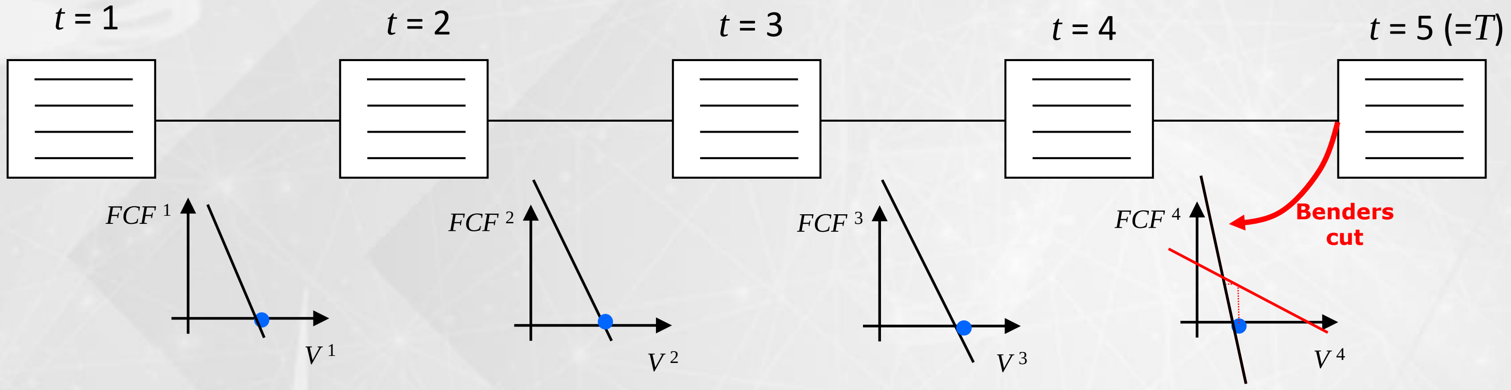



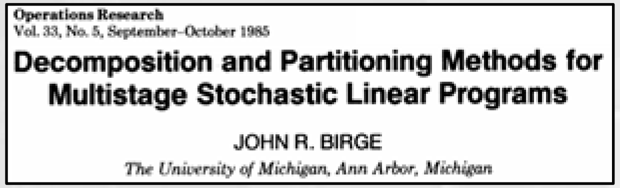

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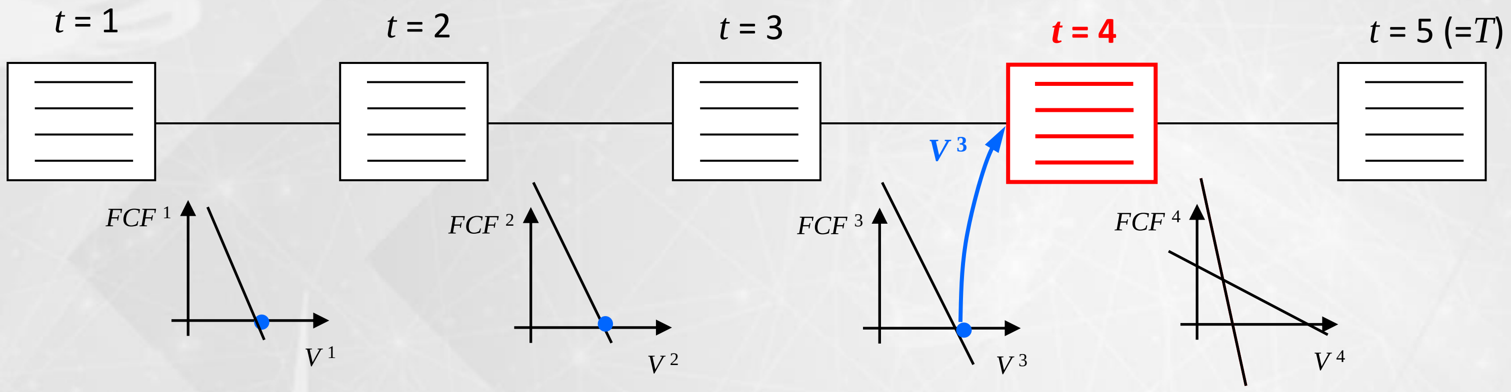


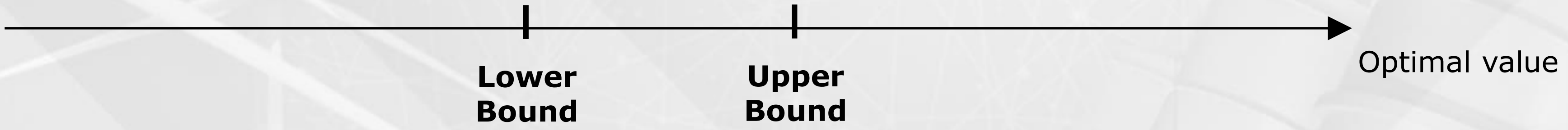


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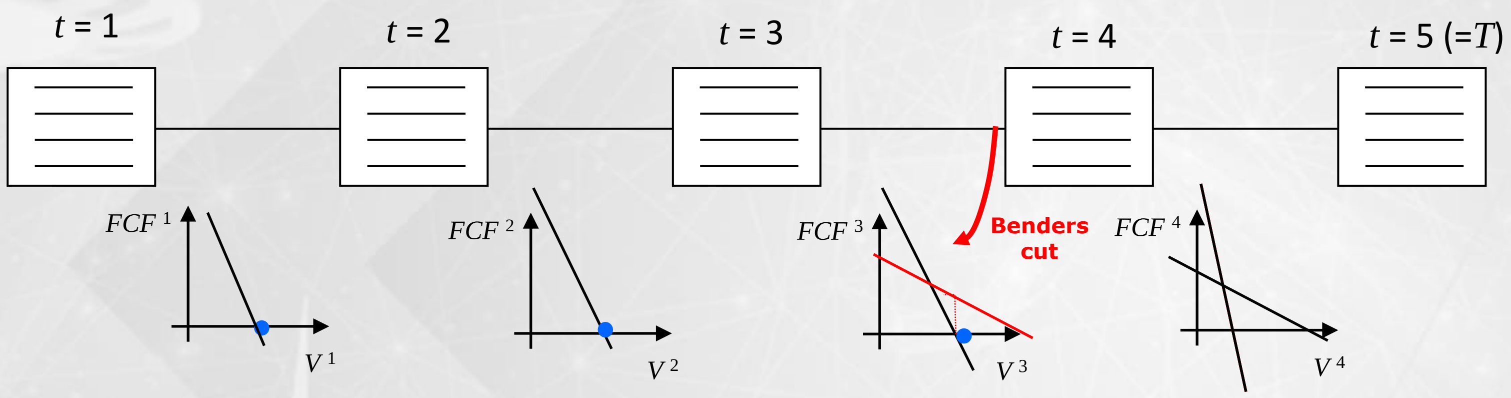



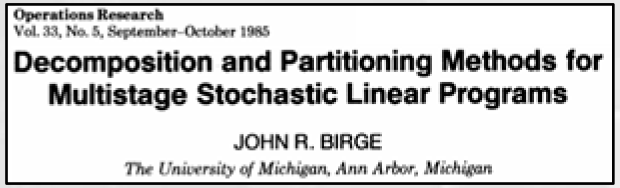
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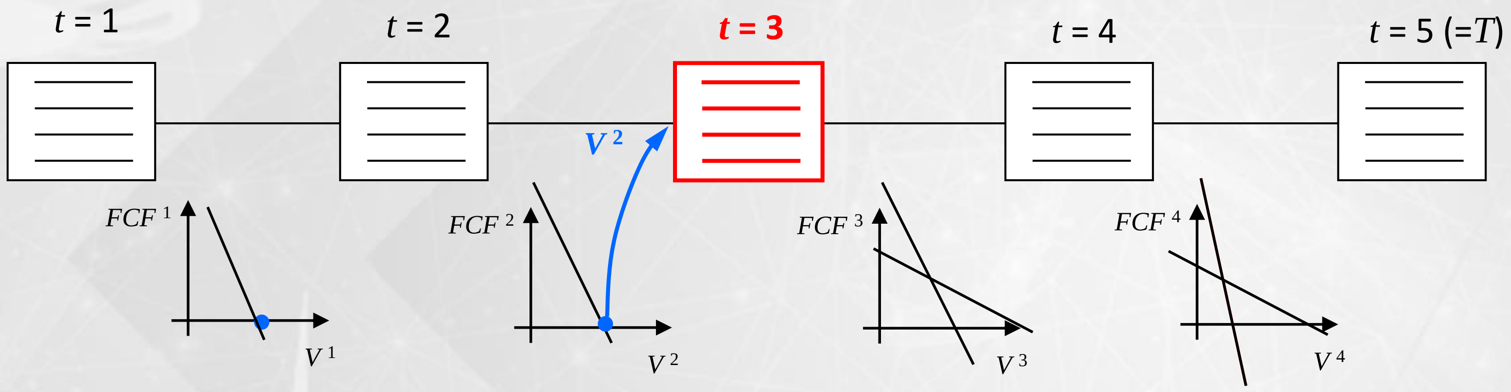


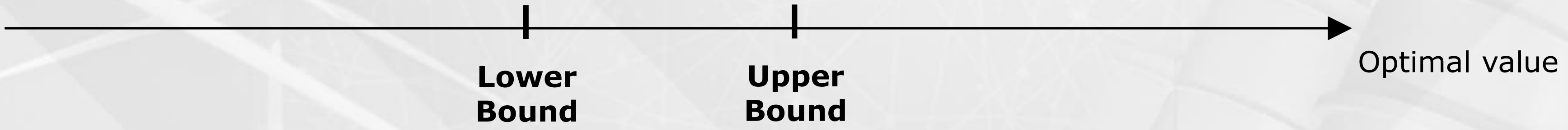


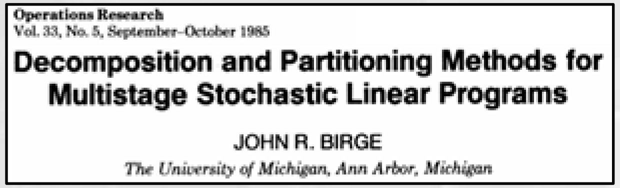

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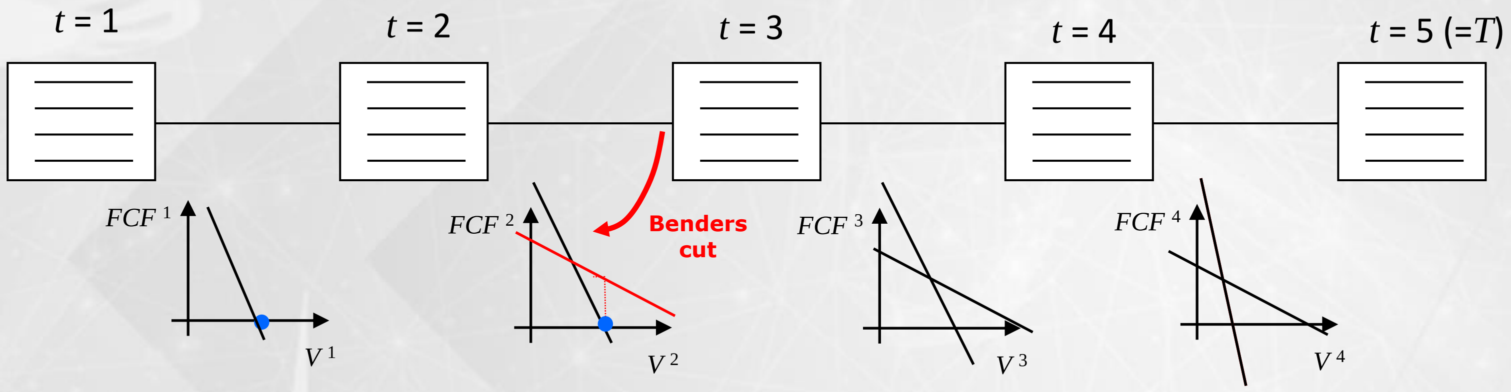


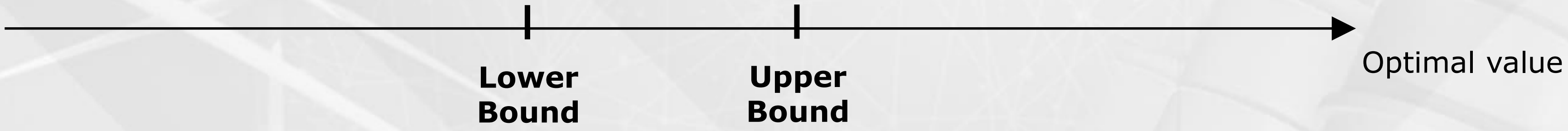


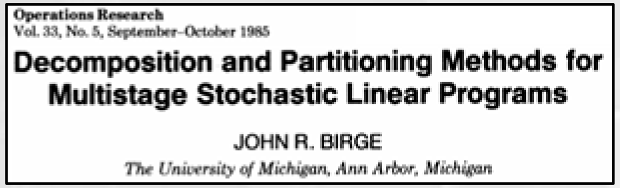
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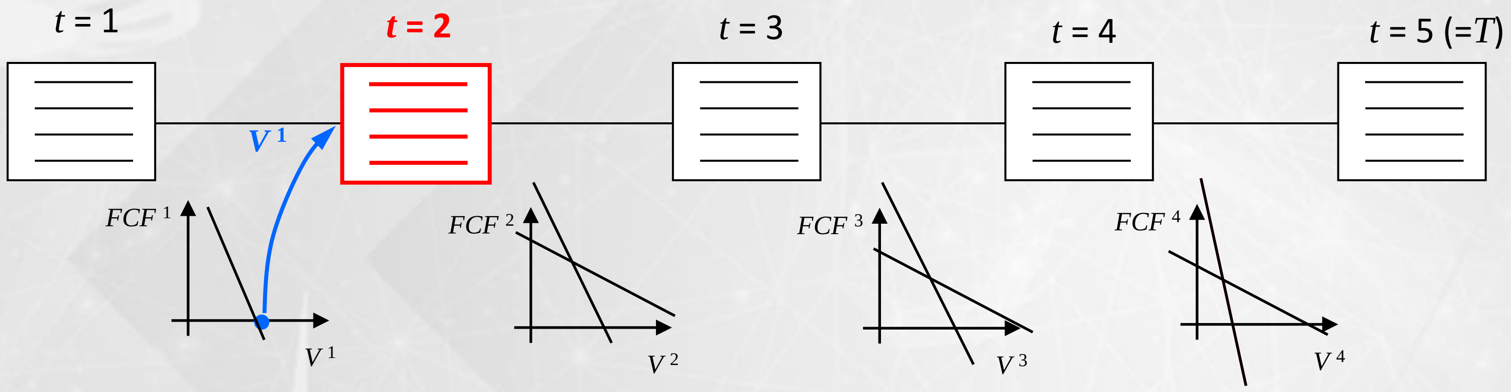


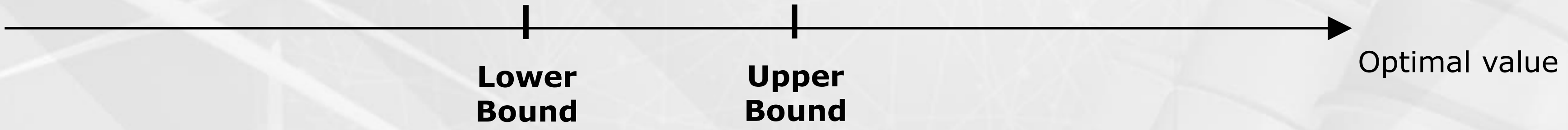



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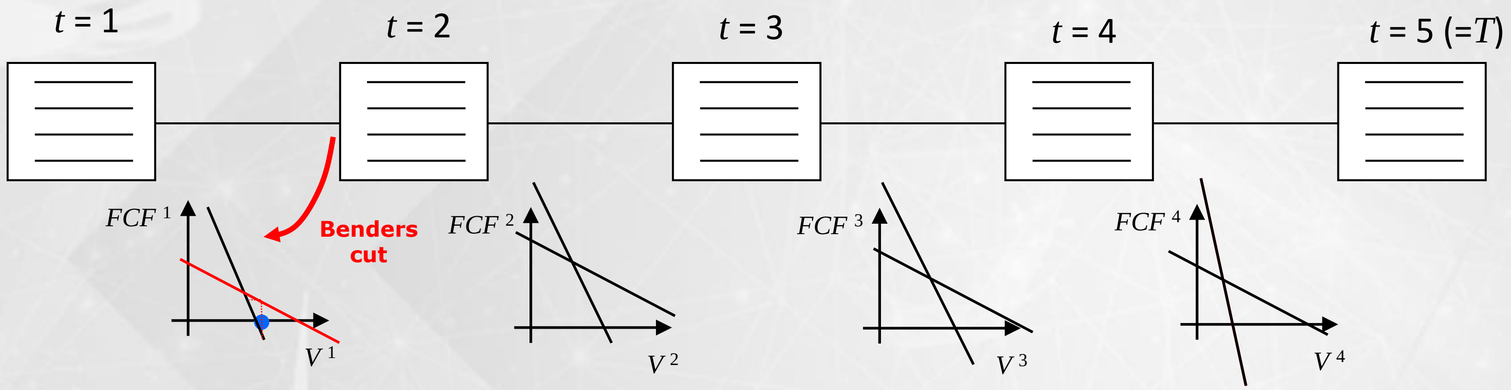


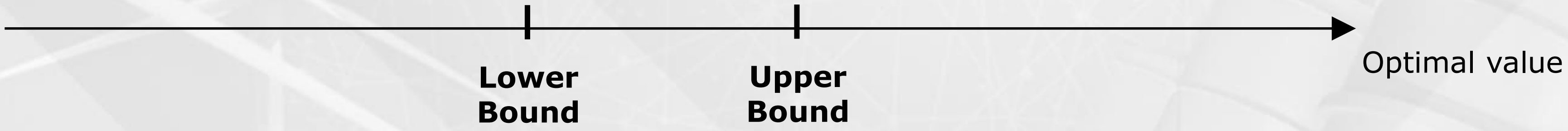


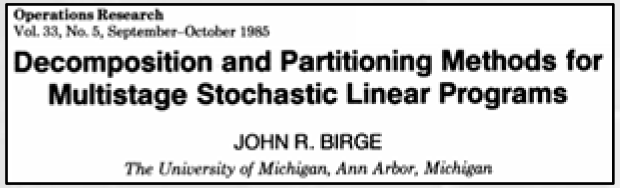
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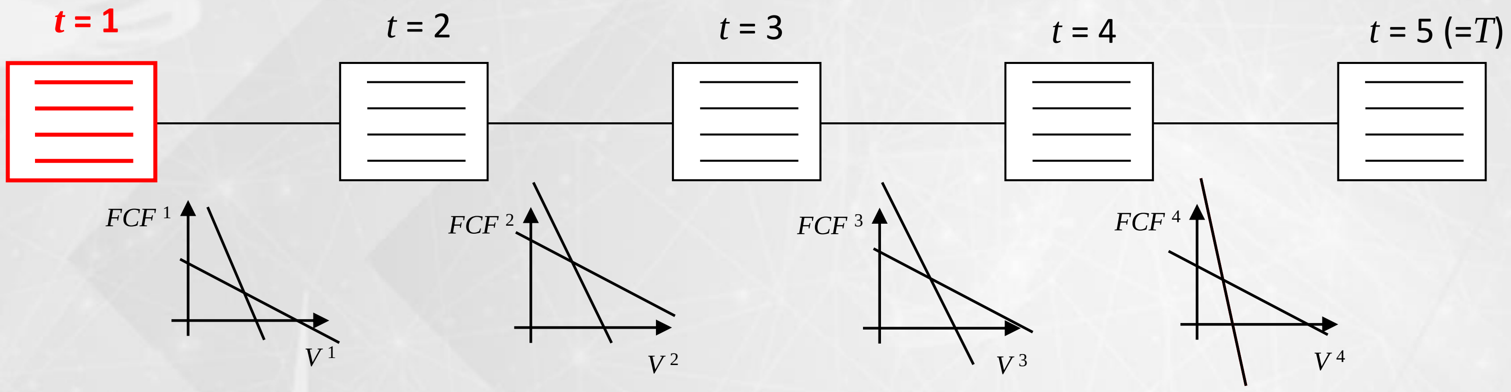


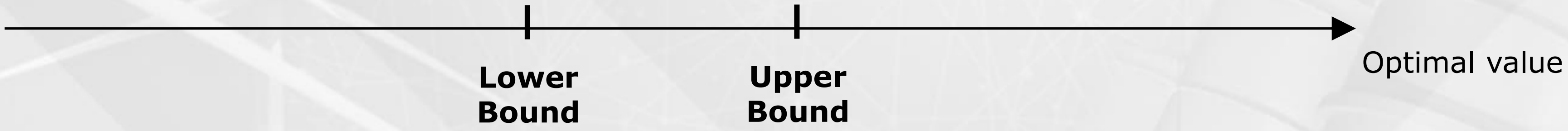


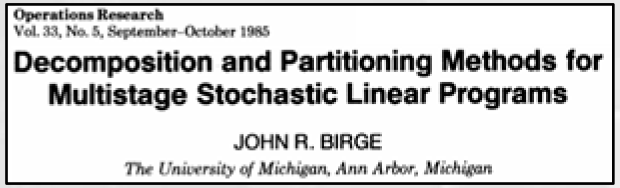

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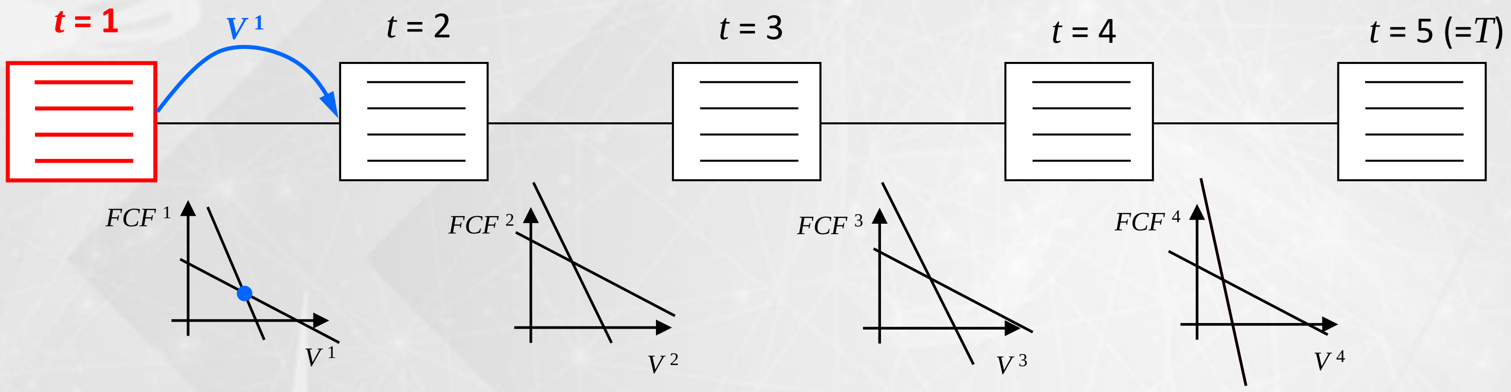
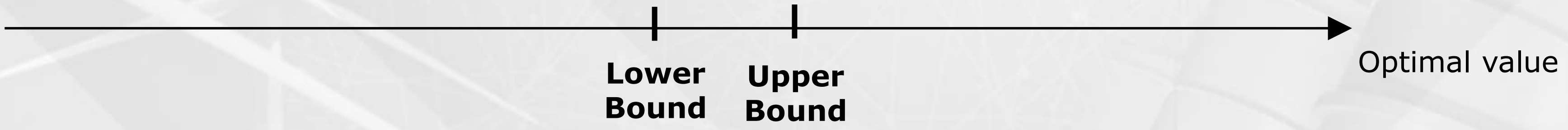
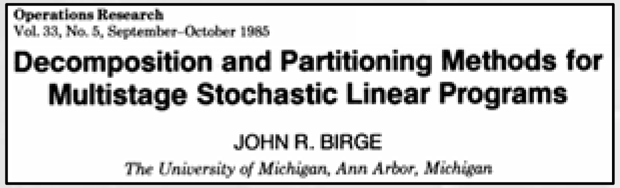
M. V. F. Pereira and L. M. V. G. Pinto
*CEPEL, Centro de Pesquisas de Energia Elétrica, Rio de Janeiro, Brazil*

# 1985: Dual Dynamic Programming (DDP)

## 3rd iteration - Forward Pass





Operations Research
Vol. 33, No. 5, September–October 1985

**Decomposition and Partitioning Methods for Multistage Stochastic Linear Programs**

JOHN R. BIRGE

*The University of Michigan, Ann Arbor, Michigan*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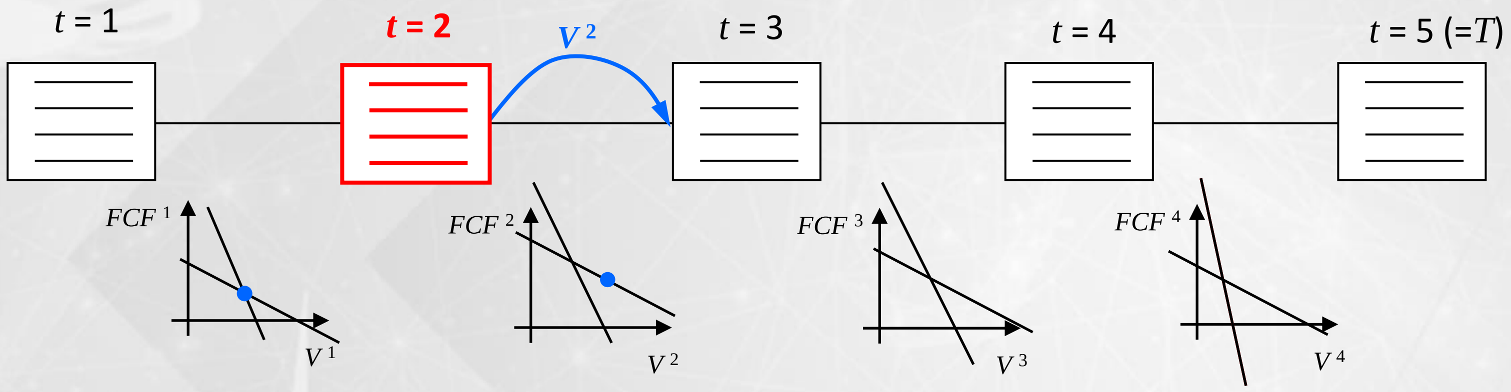


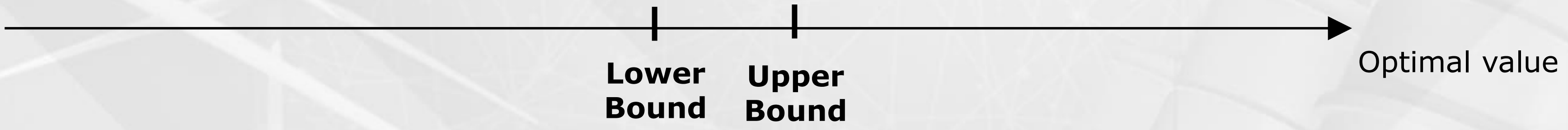



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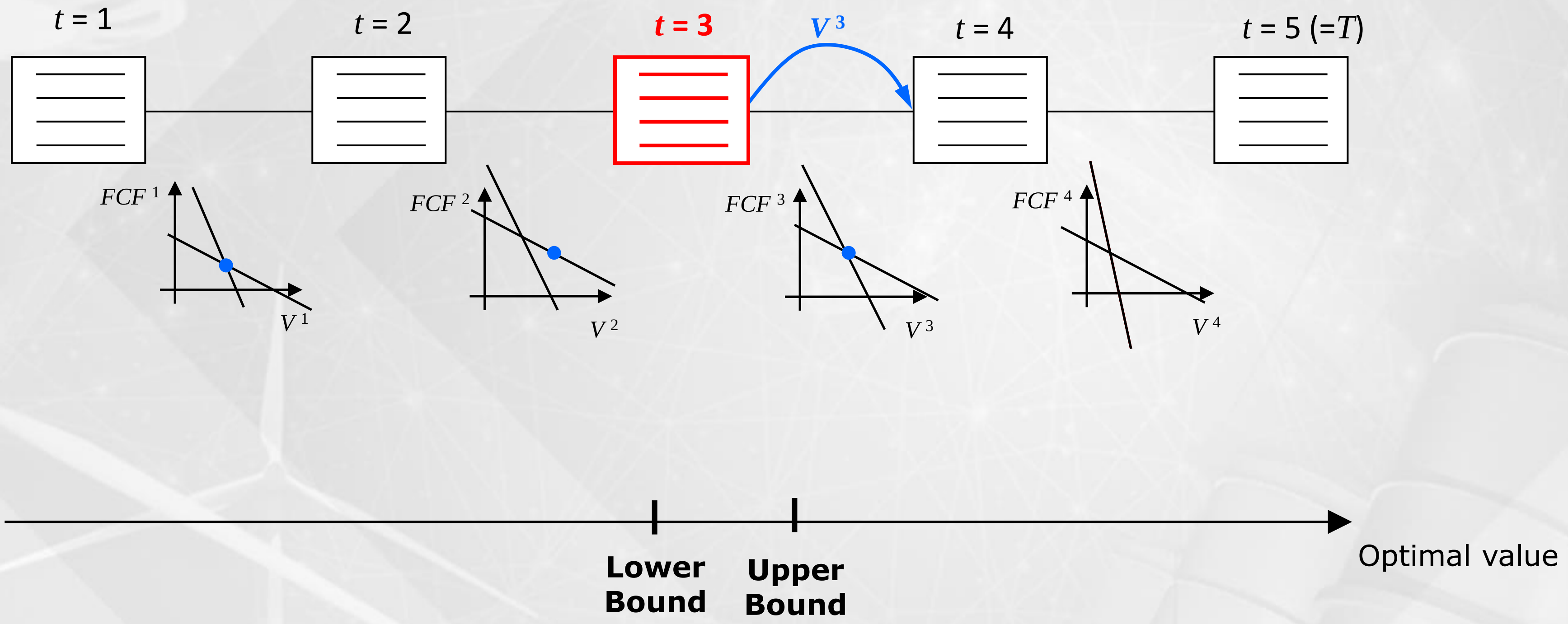


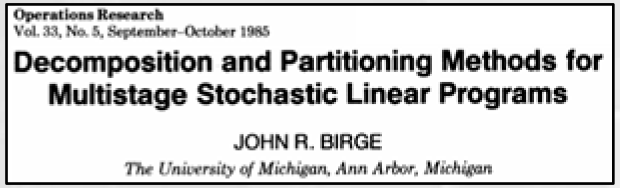

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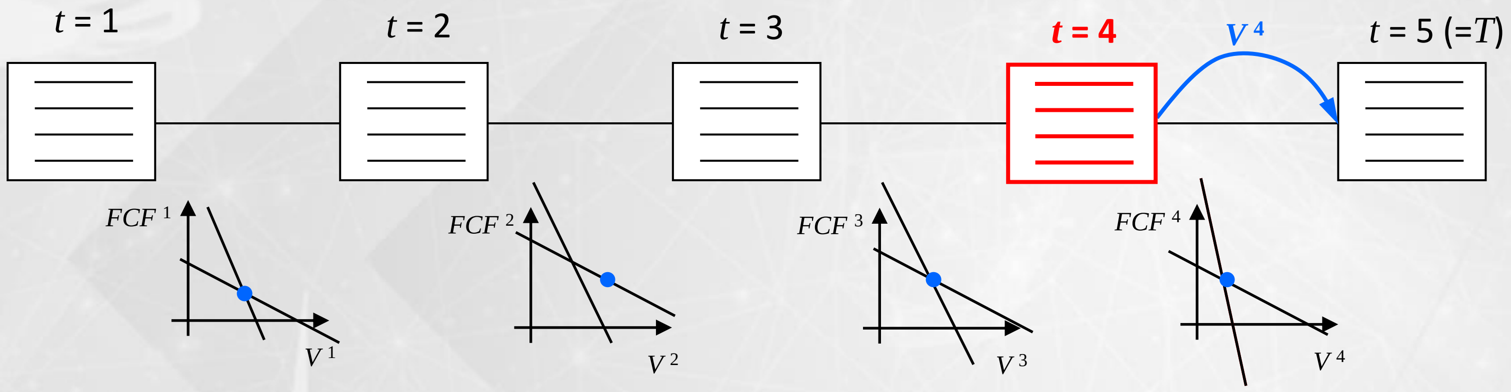


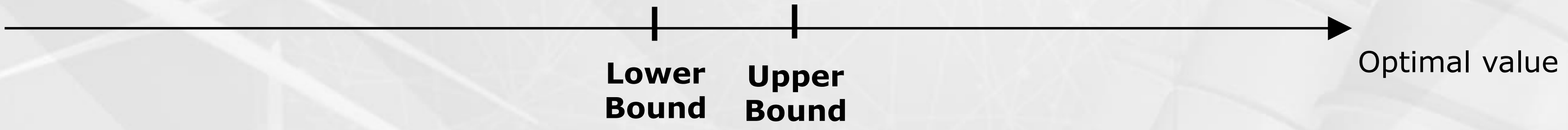


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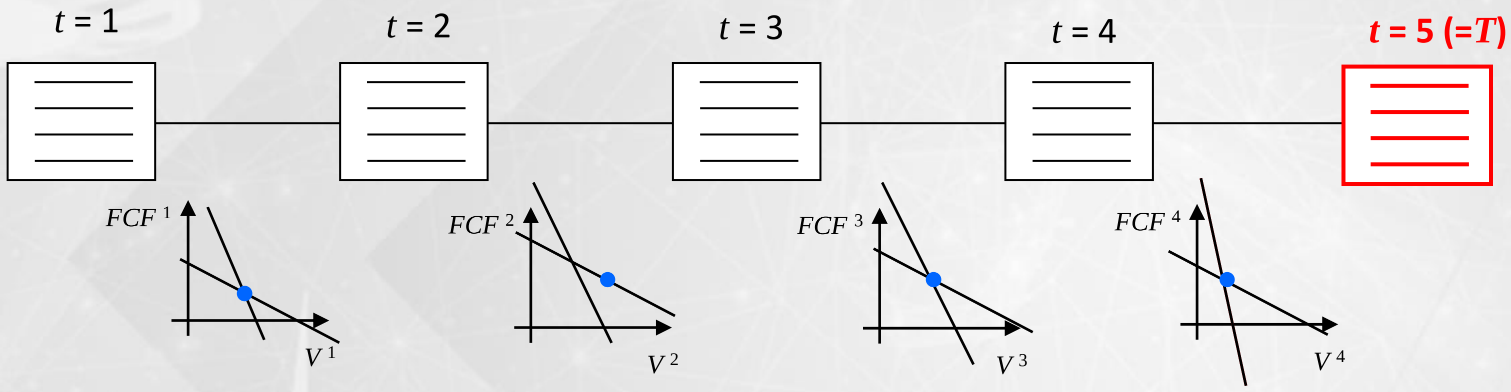
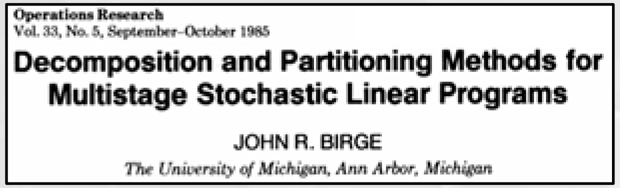




Optimal value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Vol. 33, No. 5, September-October 1985

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*CEPEL, Centro de Pesquisas de Energia Elétrica, Rio de Janeiro, Brazil*

# 1985: Dual Dynamic Programming (DDP)

Cepel | A pesquisa que constrói o futuro

Operations Research
Vol. 33, No. 5, September–October 1985

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*CEPEL, Centro de Pesquisas de Energia Elétrica, Rio de Janeiro, Brazil*

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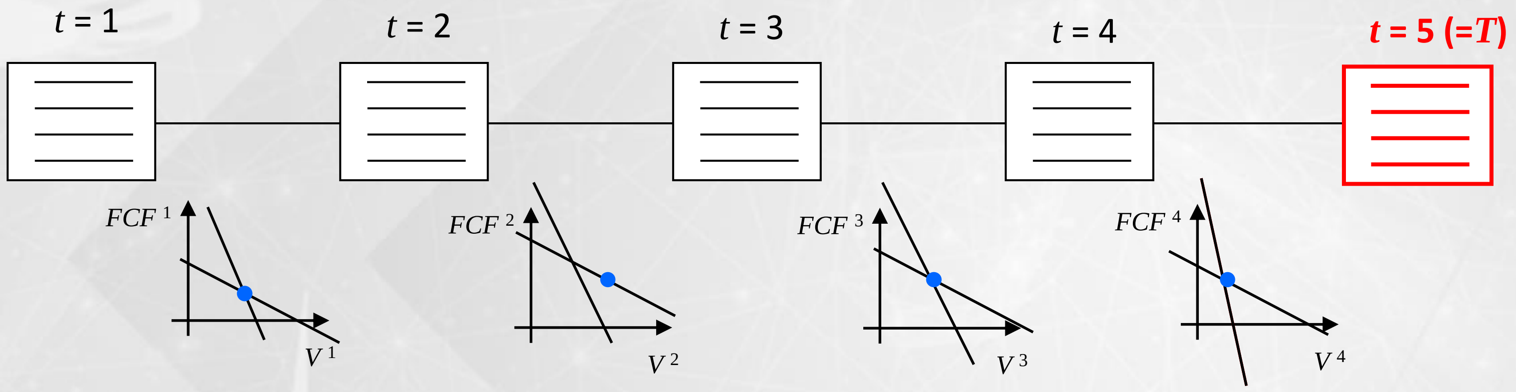
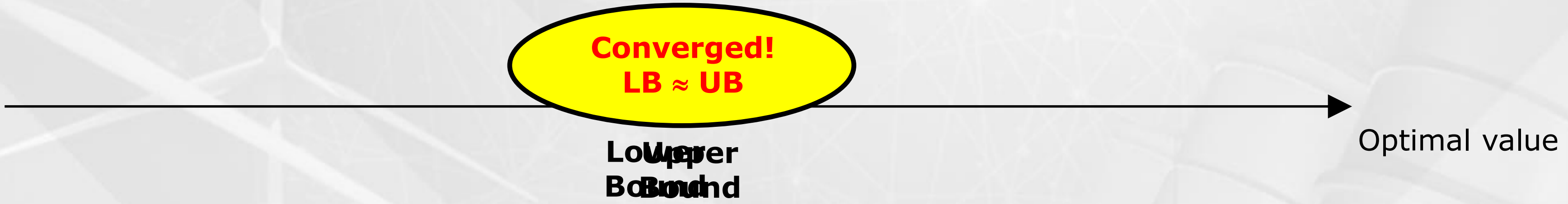
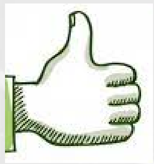
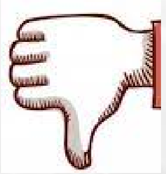


- System states where to approximate the FCF are obtained iteratively
- avoids the curse of dimensionality

- Lower PWL approximations requires convexity of the FCF
- Only 1st stage subproblem can be non-convex

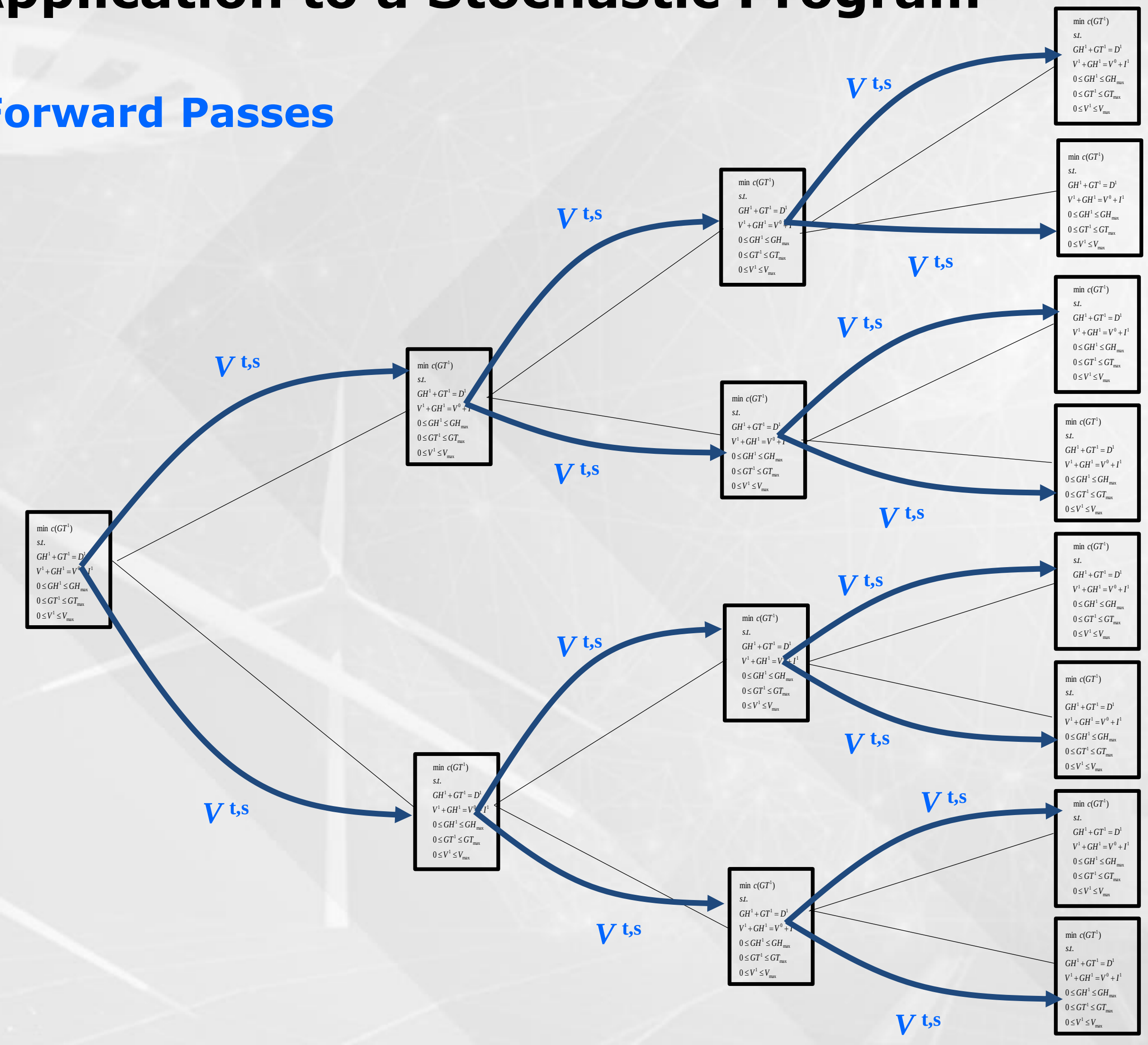


# 1985: Dual Dynamic Programming (DDP)

Cepel | A pesquisa que constrói o futuro

## Application to a Stochastic Program

### Forward Passes




Operations Research
Vol. 33, No. 5, September-October 1985

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*CEPEL, Centro de Pesquisas de Energia Elétrica, Rio de Janeiro, Brazil*

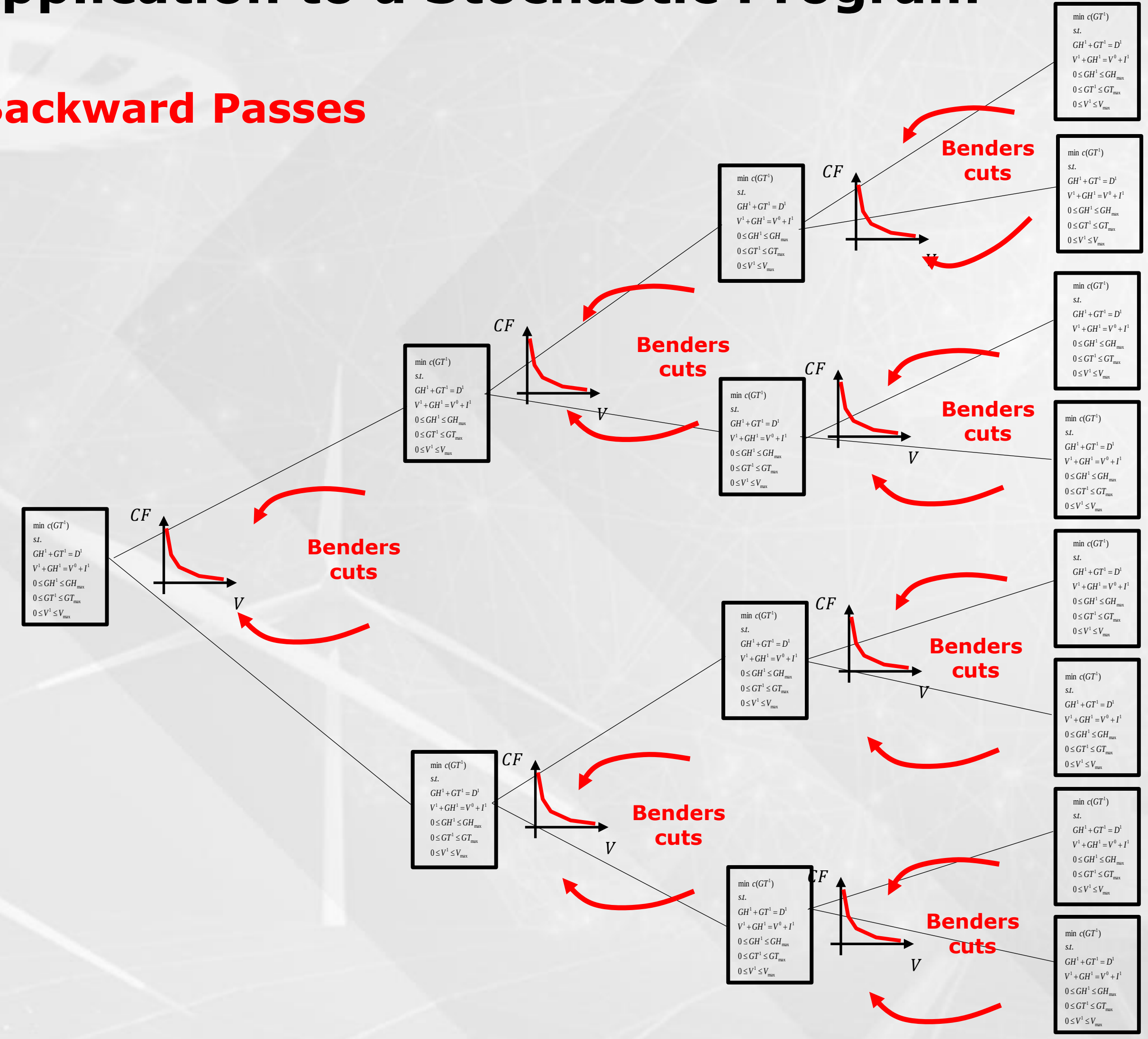
- **System states are not discretized (avoids curse of dimensionality)**
- **Any scnenario generator can be used**
- **Complete scenario tree must be set ("curse of scenarity")**
- **Same convexity requirements of the L-shaped method**

# 1985: Dual Dynamic Programming (DDP)

Cepel A pesquisa que constrói o futuro

## Application to a Stochastic Program

**Backward Passes**




Operations Research
Vol. 33, No. 5, September–October 1985

**Decomposition and Partitioning Methods for Multistage Stochastic Linear Programs**

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Stochastic Optimization of a Multireservoir Hydroelectric System: A Decomposition Approach

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*CEPEL, Centro de Pesquisas de Energia Elétrica, Rio de Janeiro, Brazil*

- System states are not discretized (avoids curse of dimensionality)
- Any scnenario generator can be used
- Complete scenario tree must be set ("curse of scenarity")
- Same convexity requirements of the L-shaped method
- Yields an optimal <u>solution</u> but not a policy for simulation purposes

**Suitable for mid term planning**

# 1991: Stochastic Dual Dynamic Programming (SDDP)

Cepel | A pesquisa que constrói o futuro

- Only a sample of the complete scenario tree is traversed during forward passes → generate many system states
- At each given state, all single stage scenarios are solved in backward passes → Build policy function approximations
- Set of descendant nodes of of each stage have the same pdf (random variables + probabilities) → **Cut sharing**
- Several paths are traversed in each iteration, with resampling of forward scenarios → **Resampling**

Mathematical Programming 52 (1991) 359–375

Multi-stage stochastic optimization applied to energy planning

M.V.F. Pereira and L.M.V.G. Pinto

*Electric Engineering Department, Catholic University of Rio de Janeiro, P.O. Box 38063, Gavea, 22452 Rio de Janeiro, RJ, Brazil*

Mathematical Programming 75 (1996) 241–256

Cut sharing for multistage stochastic linear programs with interstage dependency

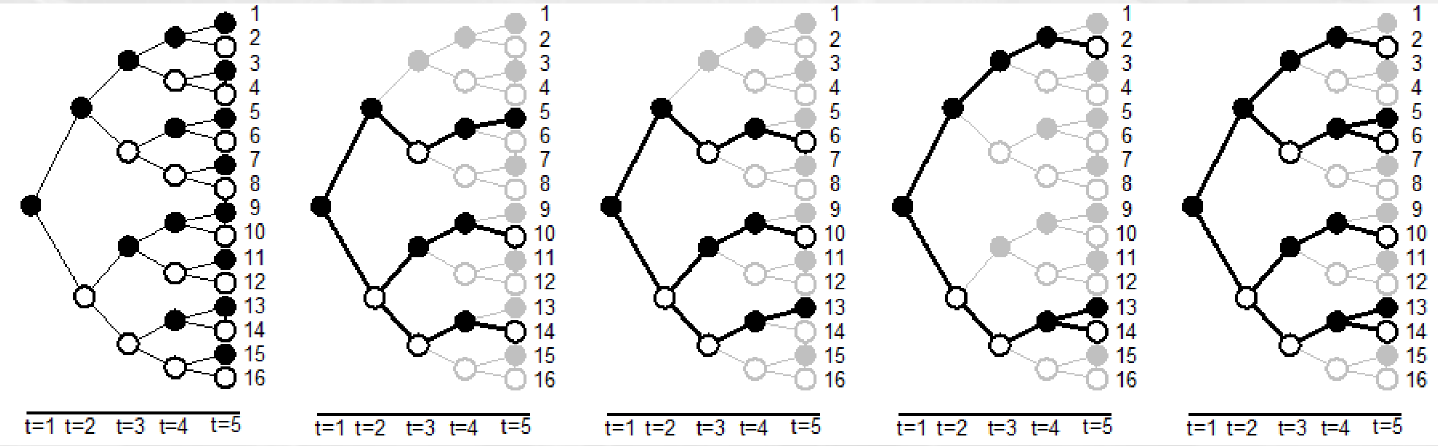
Gerd Infanger [a,1,*], David P. Morton [b,2]

Operations Research Letters 36 (2008) 450–455

On the convergence of stochastic dual dynamic programming and related methods

A.B. Philpott*, Z. Guan

**Iter #1** **Iter #2** **Iter #3** **Iter #4** **Iter #5**



# Literature on SDDP (1/6)

A pesquisa que constrói o futuro

- Variants in the **sampling / cut building procedures**

**1999**

JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 102, No. 3, pp. 497–524, SEPTEMBER 1999

**Convergent Cutting-Plane and Partial-Sampling Algorithm for Multistage Stochastic Linear Programs with Recourse[1]**

Z. L. CHEN[2] AND W. B. POWELL[3]

**2006**

*Algorithmic Operations Research Vol.1 (2006) 20–30*

**The Abridged Nested Decomposition Method for Multistage Stochastic Linear Programs with Relatively Complete Recourse**

Christopher J. Donohue[a] John R. Birge[b]

**2014**

Journal of Applied Operational Research (2014) 6(1), 2–15

ReSa: A method for solving multistage stochastic linear programs

Magnus Hindsberger *

- **Parallelization** strategies

**2003**

IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, VOL. 14, NO. 8, AUGUST 2003 721

Parallel Processing Applied to the Planning of Hydrothermal Systems

Edson Luiz da Silva, *Senior Member, IEEE*, and Erlon Cristian Finardi

**2013**

4888 IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 4, NOVEMBER 2013

An Efficient Parallel Algorithm for Large Scale Hydrothermal System Operation Planning

Roberto J. Pinto, CarmenL. T. Borges, *Senior Member, IEEE*, and Maria E. P. Maceira

**2021**

Electric Power Systems Research 191 (2021) 106907

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

ELSEVIER

Asynchronous parallel stochastic dual dynamic programming applied to hydrothermal generation planning

Felipe D.R. Machado[a], Andre Luiz Diniz[a], Carmen L.T. Borges[a,b], Lilian C. Brandão[a]

**2021**

Computational Management Science
https://doi.org/10.1007/s10287-021-00411-x

**Parallel and distributed computing for stochastic dual dynamic programming**

**D. Ávila[1] · A. Papavasiliou[1] · N. Löhndorf[2]**

# Literature on SDDP (2/6)

## ➢ Convergence Analysis and Stopping Criteria

**1996**

STOPPING RULES FOR A CLASS OF SAMPLING-BASED STOCHASTIC PROGRAMMING ALGORITHMS

DAVID P. MORTON

*The University of Texas at Austin, Austin, Texas*

(Received May 1994; revision received February 1996; accepted November 1996)

**2011**

European Journal of Operational Research 209 (2011) 63–72

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Analysis of stochastic dual dynamic programming method

Alexander Shapiro *

**2018**

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 33, NO. 4, JULY 2018

A Convergence Criterion for Stochastic Dual Dynamic Programming: Application to the Long-Term Operation Planning Problem

Rafael Bruno S. Brandi, André Luís Marques Marcato, *Senior Member, IEEE*, Bruno Henriques Dias, *Senior Member, IEEE*, Tales Pulinho Ramos, and Ivo Chaves da Silva Junior

**2008**

Operations Research Letters 36 (2008) 450–455

On the convergence of stochastic dual dynamic programming and related methods

A.B. Philpott*, Z. Guan

*Department of Engineering Science, The University of Auckland, Private Bag 92019, Auckland, New Zealand*

**2011**

Energy Syst (2011) 2: 1–31
DOI 10.1007/s12667-011-0024-y

**Sampling strategies and stopping criteria for stochastic dual dynamic programming: a case study in long-term hydrothermal scheduling**

**Tito Homem-de-Mello · Vitor L. de Matos · Erlon C. Finardi**

## ➢ Resampling

**2010**

On SDDP algorithm implementation – forward re-sampling

Murilo Pereira Soares and Joari Paulo da Costa

February 22, 2010

**2012**

XII SEPOPE
20 a 23 de Maio 2012
May – 20th to 23rd – 2012
RIO DE JANEIRO (RJ) - BRASIL

XII SIMPÓSIO DE ESPECIALISTAS EM PLANEJAMENTO DA OPERAÇÃO E EXPANSÃO ELÉTRICA

XII SYMPOSIUM OF SPECIALISTS IN ELECTRIC OPERATIONAL AND EXPANSION PLANNING

**Aplicação de Reamostragem de Cenários Hidrológicos na Definição da Estratégia de Operação Energética de Médio Prazo**

D. D. J. PENNA[1], M. E. P. MACEIRA[1,2], J. M. DAMÁZIO[1,2] e A. L. DINIZ[1,2]
[1]Centro de Pesquisas de Energia Elétrica
[2]Universidade Estadual do Rio de Janeiro
Rio de Janeiro, Brasil

# Literature on SDDP (3/6)

Cepel A pesquisa que constrói o futuro

- **Accelerating techniques**
  - ✓ Number or forward samples per iteration
  - ✓ Cut elimination / cut selection

**2011**

17th Power Systems Computation Conference — Stockholm Sweden - August 22-26, 2011

SOLVING LONG-TERM HYDROTHERMAL SCHEDULING PROBLEMS

Vitor L. de Matos, Universidade Federal de Santa Catarina, Florianópolis, Brazil, vitor@labplan.ufsc.br
Andrew B. Philpott, University of Auckland, Auckland, New Zealand, a.philpott@auckland.ac.nz
Erlon C. Finardi, Universidade Federal de Santa Catarina, Florianópolis, Brazil, erlon@labplan.ufsc.br
Ziming Guan, University of Auckland, Auckland, New Zealand, z.guan@auckland.ac.nz

**2015**

Journal of Computational and Applied Mathematics 290 (2015) 196–208

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Improving the performance of Stochastic Dual Dynamic Programming

Vitor L. de Matos[a,*], Andy B. Philpott[b], Erlon C. Finardi[c]

**2011**

Report for technical cooperation between Georgia Institute of Technology and ONS – Operador Nacional do Sistema Elétrico

Alexander Shapiro and Wajdi Tekaya
School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332-0205, USA

Joari Paulo da Costa and Murilo Pereira Soares
ONS - Operador Nacional do Sistema Elétrico, Rua da Quitanda, 196, Centro, Rio de Janeiro, RJ, 20091-005, Brasil

**2012**

Operations Research 61(6):1435-1449.

Worst-Case-Expectation Approach to Optimization Under Uncertainty

Alexander Shapiro, Wajdi Tekaya, Murilo Pereira Soares, Joari Paulo da Costa

**2017**

DUAL DYNAMIC PROGRAMMING WITH CUT SELECTION: CONVERGENCE PROOF AND NUMERICAL EXPERIMENTS

VINCENT GUIGUES
Fundação Getulio Vargas, School of applied mathematics

**2020**

LII Simpósio Brasileiro de Pesquisa Operacional
João Pessoa-PB, 3 a 5 de novembro de 2020

ESTRATÉGIA DE SELEÇÃO DE CORTES DE BENDERS PARA REDUÇÃO DO TEMPO COMPUTACIONAL DA PROGRAMAÇÃO DINÂMICA DUAL ESTOCÁSTICA

André L. Diniz [(1), (2)] - diniz@cepel.br
Maria Elvira P. Maceira [(1), (2)] - elvira@cepel.br / melvira@ime.uerj.br
Roberto J. Pinto [(1)] - rjpinto@cepel.br
Débora D. J. Penna [(1)] - debora@cepel.br
Cristiane B. C. Oliveira [(1)] - criscruz@cepel.br
Cesar L. V. Vasconcellos [(1)] - cesarluis@cepel.br

- **Assessment of Policy Quality** (lower/Upper bounds)

**1999**

Operations Research Letters 24 (1999) 47–56

Monte Carlo bounding techniques for determining solution quality in stochastic programs

Wai-Kei Mak[a], David P. Morton[b], R. Kevin Wood[c,*]

**2006**

Math. Program., Ser. B 108, 495–514 (2006)
Digital Object Identifier (DOI) 10.1007/s10107-006-0720-x

Güzin Bayraksan · David P. Morton

Assessing solution quality in stochastic programs

**2017**

Ann Oper Res
DOI 10.1007/s10479-016-2107-6

Assessing policy quality in a multistage stochastic program for long-term hydrothermal scheduling

Vitor L. de Matos[1] · David P. Morton[2] · Erlon C. Finardi[3]

# Literature on SDDP (4/6)

Cepel | A pesquisa que constrói o futuro

➢ Modeling of **higher order time-dependencies** on **random** variables

**1996 & 2013 – Cut Sharing**

Mathematical Programming 75 (1996) 241–256

Cut sharing for multistage stochastic linear programs with interstage dependency

Gerd Infanger [a,1,*], David P. Morton [b,2]

Operations Research Letters 41 (2013) 311–316

Contents lists available at SciVerse ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

ELSEVIER

Sharing cuts under aggregated forecasts when decomposing multi-stage stochastic programs

Anderson Rodrigo de Queiroz [a,b], David P. Morton [b,*]

**1993 – Par(p)**

Nº Técnico 237/93

TÍTULO:

Programação Dinâmica Dual Estocástica Aplicada ao Planejamento da Operação Energética de Sistemas Hidrotérmicos com Representação do Processo Estocástico de Afluências por Modelos Auto-regressivos Periódicos.

AUTORES:

Maria Elvira Piñeiro Maceira

**2022 – Par(p)-A**

Periodic Time Series Model with Annual Component Applied to Operation Planning of Hydrothermal Systems

F. Treistman, Civil Engineer Program, COPPE/UFRJ, Rio de Janeiro, Brazil

M.E.P. Maceira, Energy Optimization and Environment Department, CEPEL

J.M. Damázio, Energy Optimization and Environment Department, CEPEL

C.B. Cruz, Energy Optimization and Environment Department, CEPEL

978-1-7281-2822-1/20/$31.00 ©2020 IEEE PMAPS 2020

**2022 – diferente inflow regimes**

Water Resources Research 10.1029/2021WR030582

Optimizing the Management of Multireservoir Systems Under Shifting Flow Regimes

Vahid Espanmanesh[1] and Amaury Tilmant[1]

[1]Department of Civil Engineering and Water Engineering, University of Laval, Quebec City, QC, Canada

➢ Modeling of **higher order time-dependencies** on **decision** variables

**2012 - Anticipated Dispatch of thermal plants**

July 21, 2012 11:11 9in x 6in Applications in Finance, Energy, Planning and Logistics b1392-ch16

Chapter 16

Multi-Lag Benders Decomposition for Power Generation Planning with Nonanticipativity Constraints on the Dispatch of LNG Thermal Plants

Andre L. Diniz* and Maria E. P. Maceira†

**2012 - Time-Window emission constraints**

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 27, NO. 1, FEBRUARY 2012

Stochastic Hydro-Thermal Scheduling Under $CO_2$ Emissions Constraints

Steffen Rebennack, *Member, IEEE*, Bruno Flach, *Member, IEEE*, Mario V. F. Pereira, *Fellow, IEEE*, and Panos M. Pardalos

# Literature on SDDP (5/6)

Cepel A pesquisa que constrói o futuro

## ➢ Risk Averse Approaches

### CVaR

2011

European Journal of Operational Research 209 (2011) 63–72

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Analysis of stochastic dual dynamic programming method

Alexander Shapiro *

2011

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Dynamic sampling algorithms for multi-stage stochastic programs with risk aversion ☆

A.B. Philpott [a,*], V.L. de Matos [b]

2013

European Journal of Operational Research 224 (2013) 375–391

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Risk neutral and risk averse Stochastic Dual Dynamic Programming method

Alexander Shapiro [a,*], Wajdi Tekaya [a], Joari Paulo da Costa [b], Murilo Pereira Soares [b]

2015

Electrical Power and Energy Systems 72 (2015) 126–135

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

ELSEVIER

Application of CVaR risk aversion approach in the expansion and operation planning and for setting the spot price in the Brazilian hydrothermal interconnected system

M.E.P. Maceira [a,b,*], L.G.B. Marzano [a], D.D.J. Penna [a], A.L. Diniz [a,b], T.C. Justino [a]

2015

Math. Program., Ser. A (2015) 152:275–300
DOI 10.1007/s10107-014-0787-8

FULL LENGTH PAPER

**Evaluating policies in risk-averse multi-stage stochastic programming**

**Václav Kozmík · David P. Morton**

### Risk Averse Surface (rule curves)

2017

XXIV SNPTEE
SEMINÁRIO NACIONAL DE PRODUÇÃO E TRANSMISSÃO DE ENERGIA ELÉTRICA
22 a 25 de outubro de 2017
Curitiba - PR

XXIV SNPTEE
Seminário Nacional de Produção e Transmissão de Energia Elétrica

**Avaliação do Uso de Restrições Probabilísticas para a Superfície de Aversão a Risco no Problema de Planejamento de Médio Prazo da Operação Hidrotérmica**

L.F. RODRIGUES(*)[1,3], A.L. DINIZ[1,2], R.B. PRADA[3]

[1]CEPEL – Centro de Pesquisa de Energia Elétrica [2]UERJ -Universidade do Estado do Rio de Janeiro
[3]PUC-RIO -Pontifícia Universidade Católica do Rio de Janeiro-Departamento de Engenharia Elétrica

2020

Annals of Operations Research

S.I. : STOCHASTIC OPTIMIZATION:THEORY&APPLICATIONS IN MEMORY OF M.BERTOCCHI

**A combined SDDP/Benders decomposition approach with a risk-averse surface concept for reservoir operation in long term power generation planning**

Andre Luiz Diniz[1,2] · Maria Elvira P. Maceira[1,2] · Cesar Luis V. Vasconcellos[1] · Debora Dias J. Penna[1]

### Robust Optimization
**(demand uncertainty)**

2012

Operations Research 61(6):1435-1449.

Worst-Case-Expectation Approach to Optimization Under Uncertainty

Alexander Shapiro, Wajdi Tekaya, Murilo Pereira Soares, Joari Paulo da Costa

# Literature on SDDP (6/6)

- **Combination of decomposition approaches** to model different types of uncertainties

**1999 – SDP / SDDP**

A. Gjelsvik, M. M. Belsnes, and A. Haugstad, "An algorithm for stochastic medium-term hydrothermal scheduling under spot price uncertainty," in *Proc. 13th Power Syst. Comput. Conf.*, Trondheim, Norway, 1999, pp. 1079–1085.

**2016 – Sampling X scenario tree**

Math. Program., Ser. A (2016) 156:343–389
DOI 10.1007/s10107-015-0884-3

CrossMark

FULL LENGTH PAPER

**Combining sampling-based and scenario-based nested Benders decomposition methods: application to stochastic dual dynamic programming**

**Steffen Rebennack**

- SDDP with **Reinforcement Learning / AI**

**2021**

Batch Learning in Stochastic Dual Dynamic Programming

Daniel Ávila*, Anthony Papavasiliou
*Center of Operations Research and Econometrics, Université Catholique de Louvain, 1348 Louvain-La-Neuve, Belgium*
Nils Löhndorf
*Luxembourg Centre for Logistics and Supply Chain Management, University of Luxembourg, 1511 Luxembourg, Luxembourg*

*Preprint submitted to European Journal of Operational Research* *May 17, 2021*

**2021**

NEURAL STOCHASTIC DUAL DYNAMIC PROGRAMMING

A PREPRINT

**Hanjun Dai*, Yuan Xue*, Zia Syed, Dale Schuurmans, Bo Dai**
Google
{hadai, yuanxue, zsyed, schuurmans, bodai}@google.com

# SDDP with Nonconvex Constraints (1/3)

**2016**

- Nonconvex state-dependent irrigation constraints
- **Convex relaxation** in the backward pass (Loose cuts)

Energy Procedia 00 (2016) 000–000

5th International Workshop on Hydro Scheduling in Competitive Electricity Markets

Optimizing hydrothermal scheduling with non-convex irrigation constraints: Case on the Chilean electricity system

Eduardo Pereira-Bonvallet[a,*], Sebastian Püschel-Løvengreen[a], Marcelo Matus[a], Rodrigo Moreno[a,b]

[a]Energy Centre, Electrical Engineering Department, University of Chile, Av. Tupper 2007, Santiago, Chile.
[b]Electrical and Electronic Engineering Department, South Kensington Campus, Imperial College London, UK.

**2012**

- Nonconvex model of the Hydro production function with **McCormick envelopes**
- solve MILP in forward passes and **Lagrangian relaxation of subproblems in backward passes**, yileding tighter cuts

European Journal of Operational Research 218 (2012) 687–697

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Stochastic dual dynamic programming applied to nonconvex hydrothermal models

Santiago Cerisola *, Jesus M. Latorre, Andres Ramos

Instituto de Investigación Tecnológica, ICAI, Universidad Pontificia Comillas, Alberto Aguilera 23, 28015 Madrid, Spain

**2016**

- Strategic Bidding Problem
- DP equations are considered in the LR procedure

European Journal of Operational Research 00 (2016) 1–30

Dynamic Convexification within Nested Benders Decomposition using Lagrangian Relaxation: An Application to the Strategic Bidding Problem

Gregory Steeger, Steffen Rebennack[a]

Colorado School of Mines, Division of Economics and Business, Golden, CO 80401

**2020**

- **Step Function** model for the FCF
- **MILP subproblems are solved** in forward/backard passes

MIDAS: A mixed integer dynamic approximation scheme

A. B. Philpott, F. Wahid & J. F. Bonnans

Mathematical Programming **181**, 19–50 (2020)

# SDDP with Nonconvex Constraints (2/3)

## SDDiP – Stochastic Dual Dynamic Integer Programming

2019

Math. Program., Ser. A

FULL LENGTH PAPER

**Stochastic dual dynamic integer programming**

Jikai Zou[1] · Shabbir Ahmed[1] · Xu Andy Sun[1]

- Modeling on nonconvexities in an explicit way with a MILP problem
- Binary expansion to derive 0-1 state variables

**yields convex FCFs in the $\{0,1\}^2$ space**

- 3 types of cuts with increasing degree of exacteness:
  - ✓ Traditional Benders cuts
  - ✓ Lagrangian cuts
  - ✓ Strenghtened Benders cuts

**Applications so far (small systems)**

2019

IEEE TRANSACTIONS ON SUSTAINABLE ENERGY, VOL. 10, NO. 1, JANUARY 2019 481

Nonconvex Medium-Term Hydropower Scheduling by Stochastic Dual Dynamic Integer Programming

Martin N. Hjelmeland, *Student Member, IEEE*, Jikai Zou, Arild Helseth, *Member, IEEE*, and Shabbir Ahmed, *Senior Member, IEEE*

2019

IEEE TRANSACTIONS ON POWER SYSTEMS, SEPTEMBER 2018

Multistage Stochastic Unit Commitment Using Stochastic Dual Dynamic Integer Programming

Jikai Zou, Shabbir Ahmed, *Senior Member*, and Andy Sun, *Senior Member*

2020

Nonconvex Environmental Constraints in Hydropower Scheduling

Arild Helseth, Birger Mo, Hans Olaf Hågenvik
SINTEF Energy Research
Trondheim, Norway
arild.helseth@sintef.no

# SDDP with Nonconvex Constraints (3/3)

Cepel A pesquisa que constrói o futuro

## Stochastic Lipschitz Dynamic Programming

- If the FCFs are known to be Lipschitz-continuous, build lower approximations by Lipschitz continuous cuts

Stochastic Lipschitz dynamic programming

Shabbir Ahmed, Filipe Goulart Cabral & Bernardo Freitas Paulo da Costa

*Mathematical Programming* **191**, 755–793 (2022)

- When the Lipschitz continuity hipothesis may not hold, applies a Lipschitz continuous regularized FCF

Zhang, Shixuan and Sun, Xu A. 2022. "Stochastic dual dynamic programming for multistage stochastic mixed-integer nonlinear optimization."
https://doi.org/10.1007/s10107-022-01875-8
Springer Berlin Heidelberg

# OPERATION POLICY BY SDDP FOR MID/LONG TERM PLANNING IN BRAZIL

Newave Planejameto da Operação

Cepel A pesquisa que constrói o futuro

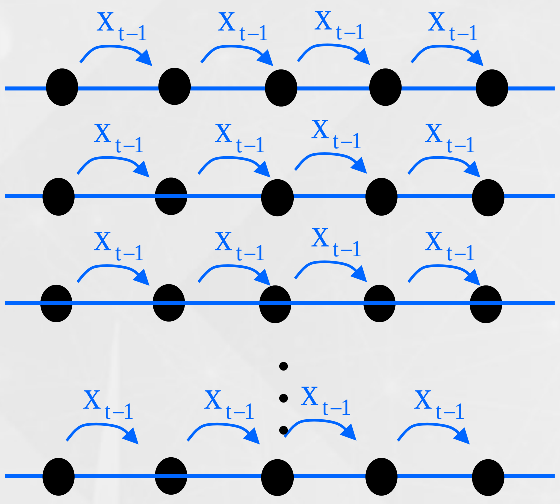
## FORWARD PASS

*Scenario 1*

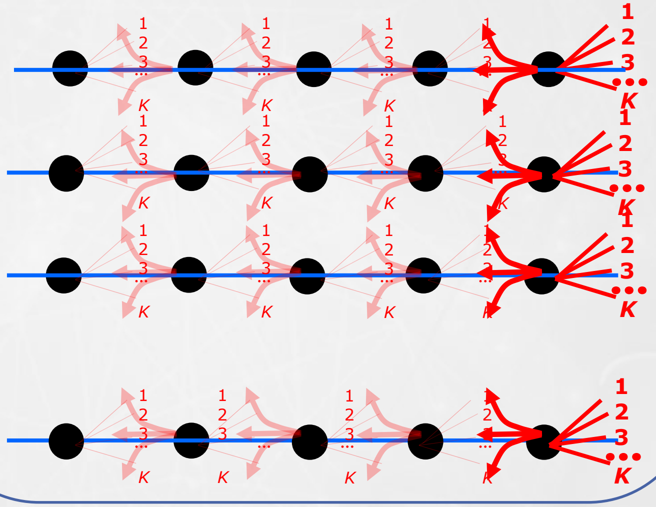
*Scenario 2*

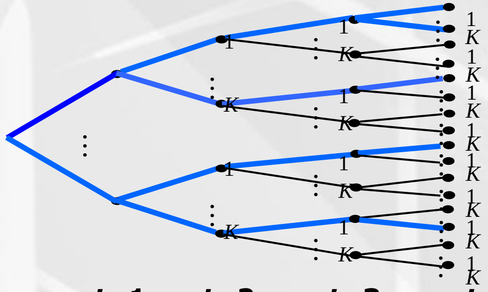
*Scenario 3*

*Scenario S << $K^T$*




## BACKWARD PASS




**Benders cuts:**

$$\varphi_t(x_{t-1}) \ge \sum_{\omega=1,\dots K} p_\omega \left[ z_{t,\omega^*} + \left\langle \frac{\partial z_{t,\omega^*}}{\partial x_{t-1}}(\hat{x}_{t-1,s^*}), x_{t-1} - \hat{x}_{t-1,s^*} \right\rangle \right]$$

**OUTPUT:**

**Optimal Operation Policy**

- Time dependency on random variables (Par(P)-A)
- Time-dependency on decision variables (LNG plants, rule curves)
- Forward resampling
- Accelerating techniques (cut selection, warm starts)
- Parallel processing
- Stopping criteria: lower bound stability
- Assessment of solution quality with 2,000 scenarios

**Nonlinear / Nonconvex terms**

# Nonlinear/Nonconvex constraints in the NEWAVE model

Cepel A pesquisa que constrói o futuro

## Previous Assessments

- Performs na assessment of nonconvex terms related to the modeling of energy inflows
- Proposes **linear approximations instead of quadratic functions** in  the modeling of nonlinear terms related to state variables (in the modeling of equivalent reservoirs)

**2012**

Electrical Power and Energy Systems 43 (2012) 1443–1452
Contents lists available at SciVerse ScienceDirect
Electrical Power and Energy Systems
journal homepage: www.elsevier.com/locate/ijepes
ELSEVIER

A computational study of a stochastic optimization model for long term hydrothermal scheduling

Vitor Luiz de Matos *, Erlon Cristian Finardi

- Application of **dynamic piecewise linear models** for convex thermal Generation costs and convexified non-conconcave Hydro production function

XXII SNPTEE
Seminário Nacional de Produção e Transmissão de Energia Elétrica
XXII SNPTEE
SEMINÁRIO NACIONAL
DE PRODUÇÃO E
TRANSMISSÃO DE
ENERGIA ELÉTRICA
BR/GOP/18
13 a 16 de Outubro de 2013
Brasília - DF

ESTRATÉGIA DE PDDE LINEAR POR PARTES DINÂMICA APLICADA A PROBLEMAS DE PLANEJAMENTO HIDROTÉRMICO ESTOCÁSTICO

Renato Neves Cabral(*) CEPEL
Michel Igor Ennes COPPE/UFRJ
André Luiz Diniz CEPEL/UERJ

# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear functions of decisions variables $x$

- Since the problems should be convex, we must have:
  - ✓ equations ?

# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear functions of decisions variables $x$

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  - ✓ equations => must have a linear formulation

# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear functions of decisions variables $x$

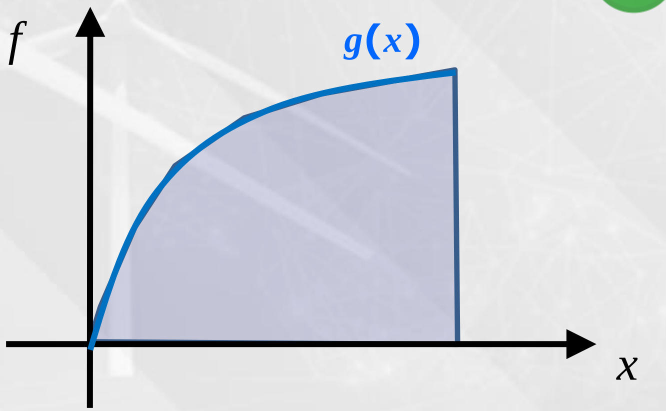
- Since the problems should be convex, we must have:
  - ✓ equations => must have a linear formulation
  - ✓ inequalities ?

# Requirement for application of nonlinear constraints

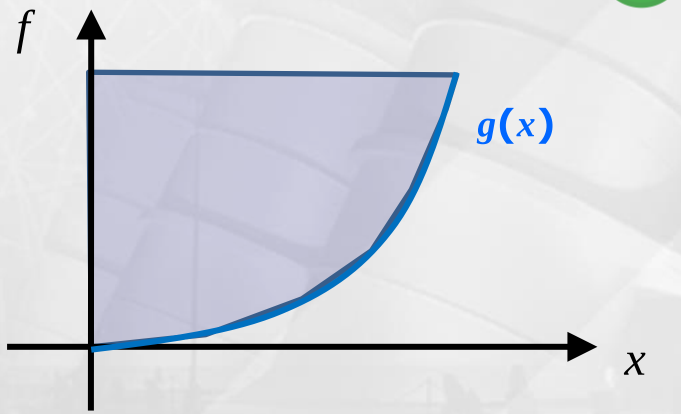
## 1st case: explicit nonlinear functions of decisions variables $x$

- Since the problems should be convex, we must have:
  - ✓ equations => must have a linear formulation
  - ✓ inequalities => must satisfy:

$f \leq g(x)$, w/ $g$ concave



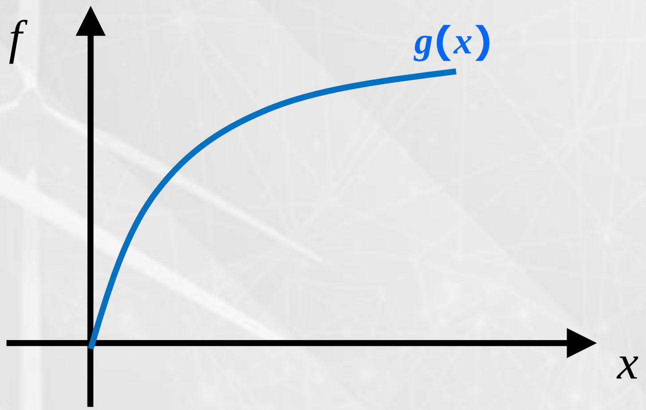

$f \geq g(x)$, with $g$ convex

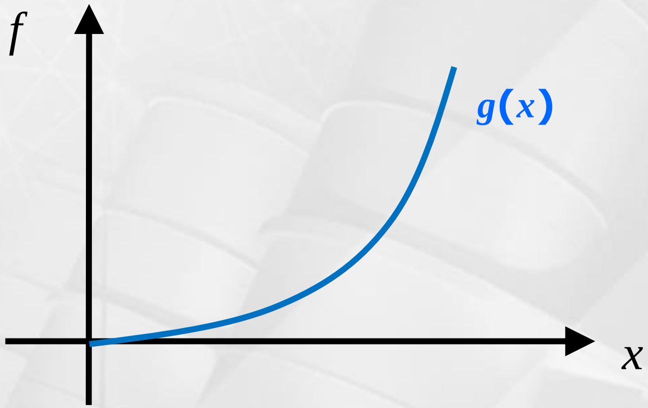


# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **nonlinear (concave/convex) equations**?

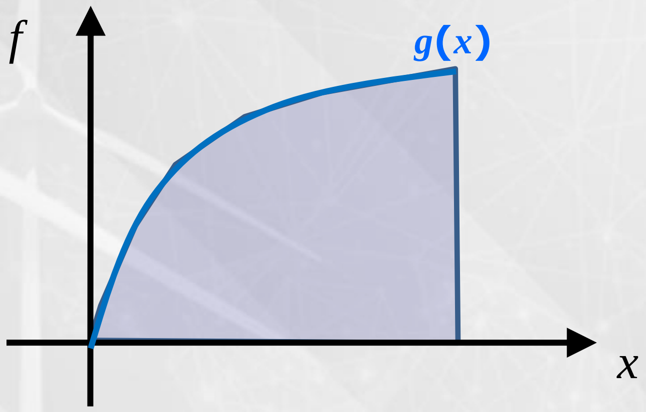


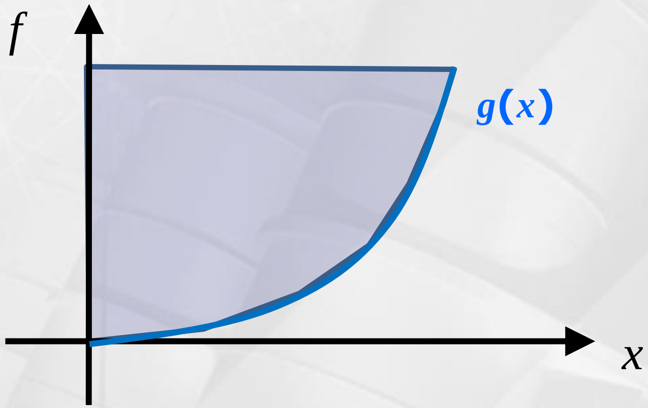



# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **nonlinear (concave/convex) equations**?
  - ✓ 1st step: expand the feasible region by transforming the equation into an inequality relation

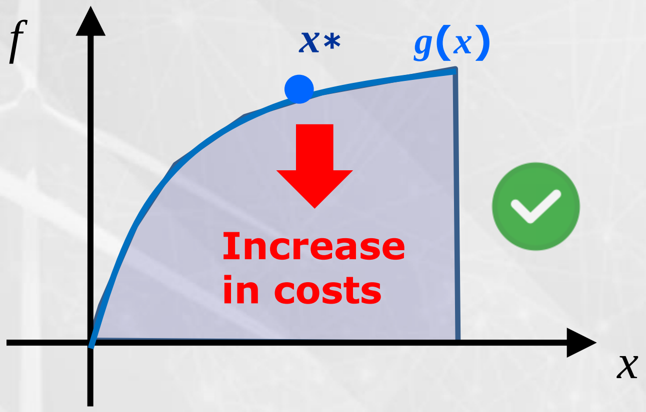


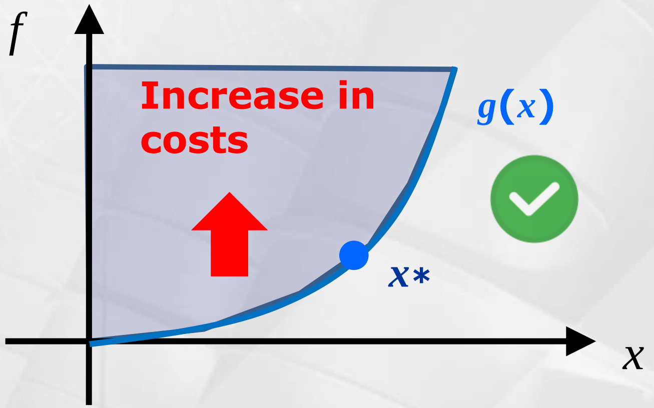



# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **nonlinear (concave/convex) equations**?
  - ✓ 1st step: expand the feasible region by transforming the equation into an inequality relation
  - ✓ 2nd step: check whether the optimal solution will tend to lie in the boundary of the augmented feasible region




(example for cost minimization problems)

# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear function of decisions variables $x$

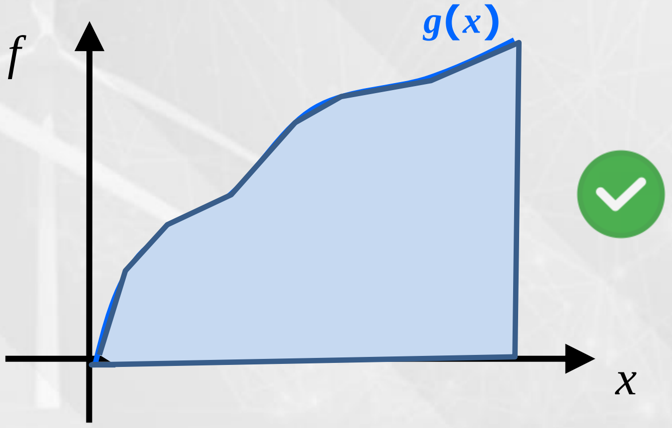
- What if we have **non concave/non convex inequalities**?

# Requirement for application of nonlinear constraints

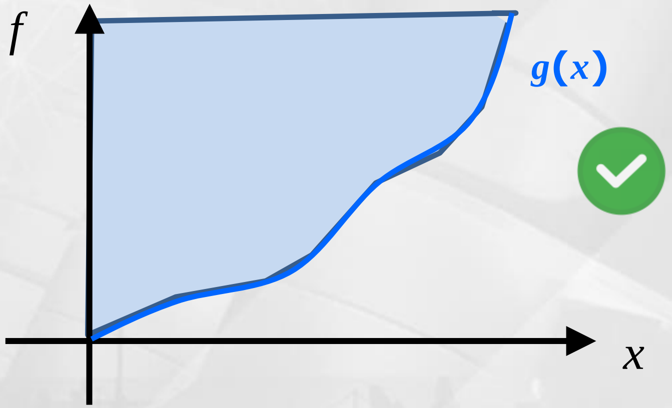
## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **non concave/non convex inequalities**?
  - ✓ 1st step: check the conditions:

$f \leq g(x)$, w/ $g$ nearly concave




$f \geq g(x)$, with $g$ nearly convex

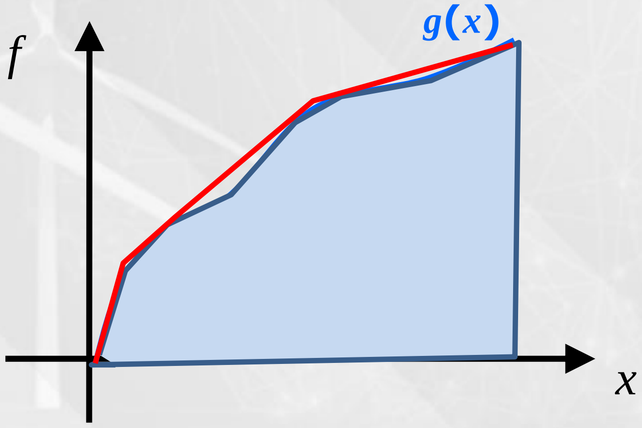


# Requirement for application of nonlinear constraints

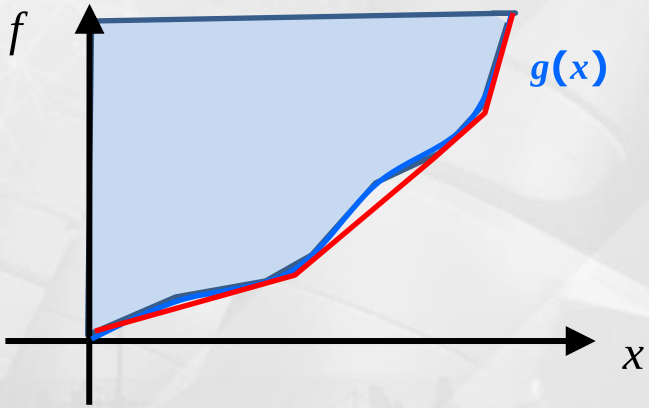
## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **non concave/non convex inequalities**?
  - ✓ 1st step: check the conditions:
  - ✓ 2nd step: apply a convexification procedure to yield outer concave/convex approximations

$f \leq g(x)$, w/ $g$ nearly concave




$f \geq g(x)$, with $g$ nearly convex



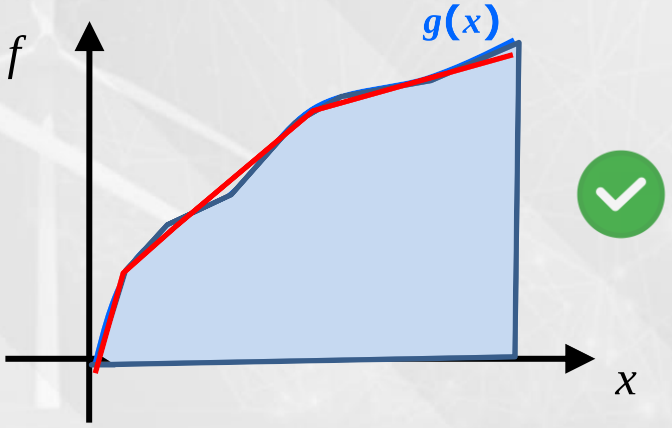
# Requirement for application of nonlinear constraints

Cepel A pesquisa que constrói o futuro

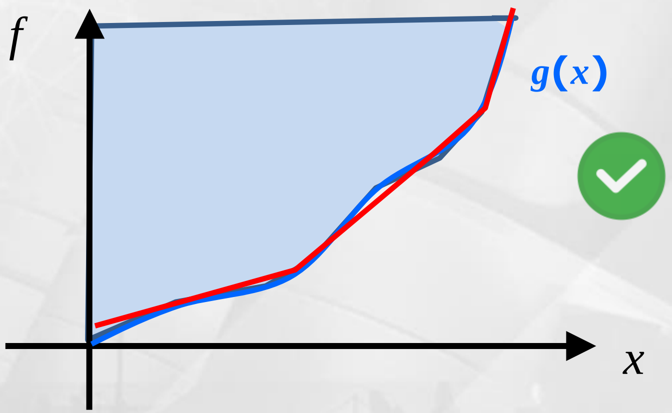
## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **non concave/non convex inequalities**?
  - ✓ 1st step: check the conditions:
  - ✓ 2nd step: apply a convexification procedure to yield outer concave/convex approximations
  - ✓ 3rd step: apply a regression coefficient to decrease errors between both functions

$f \leq g(x)$, w/ $g$ nearly concave



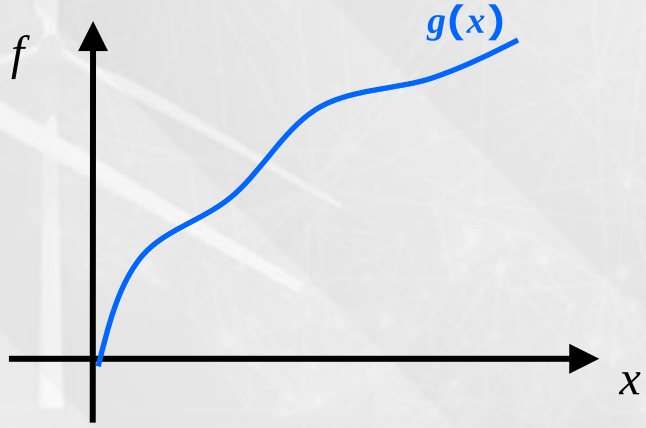

$f \geq g(x)$, with $g$ nearly convex

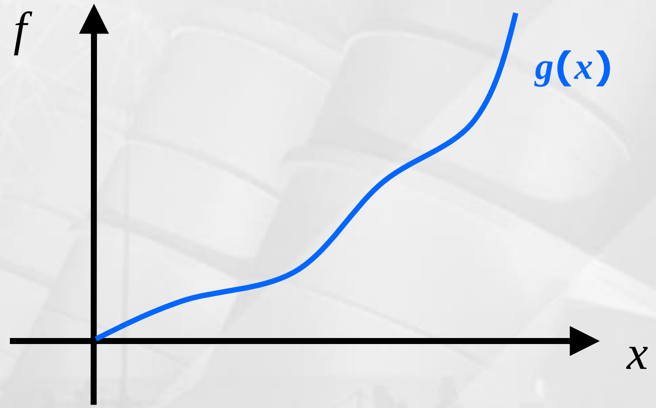


# Requirement for application of nonlinear constraints

## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **non concave/non convex equations**?

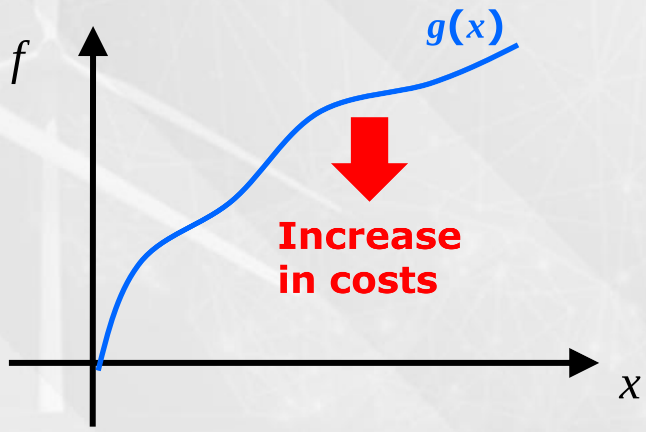


# Requirement for application of nonlinear constraints

## $1^{st}$ case: explicit nonlinear function of decisions variables $x$

- What if we have **non concave/non convex equations**?
  - $1^{st}$ step: check whether the optimal solution will tend to lie in the boundary of the (to be augmented) feasible region

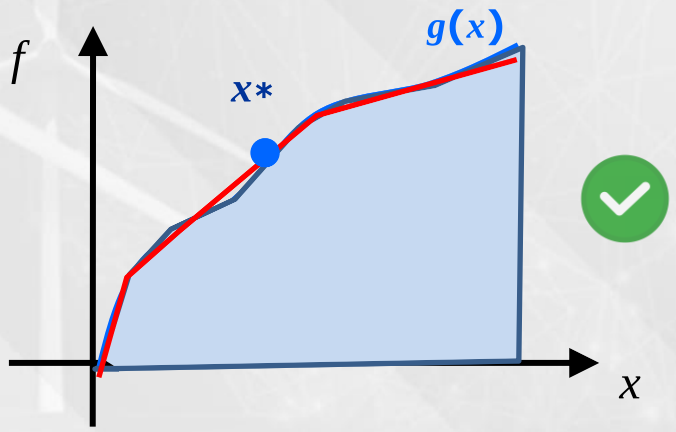


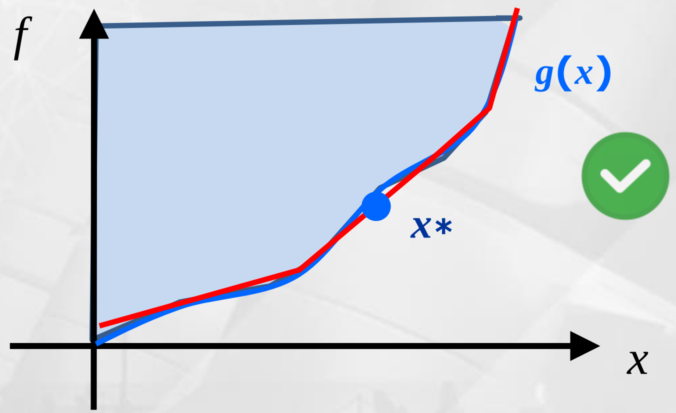

# Requirement for application of nonlinear constraints

Cepel A pesquisa que constrói o futuro

## 1st case: explicit nonlinear function of decisions variables $x$

- What if we have **non concave/non convex equations**?
  - 1st step: check whether the optimal solution will tend to lie in the boundary of the (to be augmented) feasible region
  - 2nd step: apply the convexification/regression procedure previously described



# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) \leq a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

**1st situation:**

# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

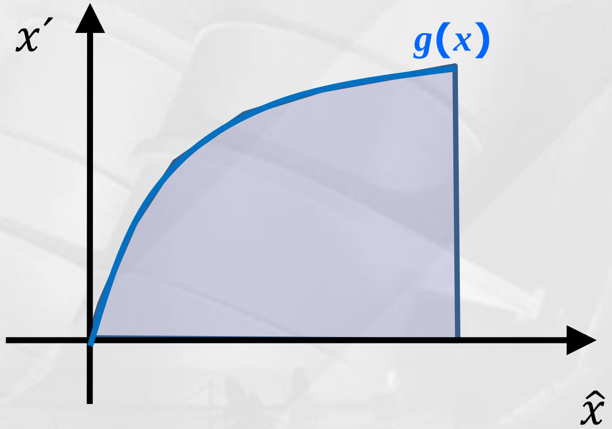
$$\begin{cases} Ax = \; b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) \leq a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

$$\begin{cases} Ax + x' = \; b \\ x' \leq a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

**1st situation:**

**1st condition:**
$g(\hat{x})$ **must be concave (a<0)**



# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

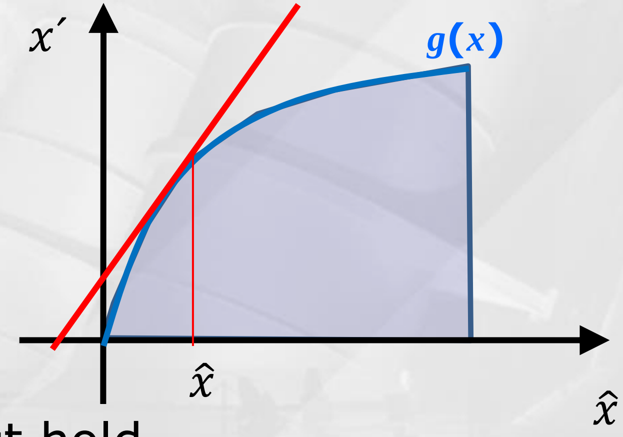
$$\begin{cases} Ax = b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) \leq a\hat{x}^2 + b\hat{x} + c, \end{cases} \quad \Longleftrightarrow \quad \begin{cases} Ax + x' = b \\ x' \leq a\hat{x}^2 + b\hat{x} + c, \quad \leftarrow \lambda^* \end{cases}$$

$$x' = g(\hat{x})$$

**1st situation:**

=> **Multiplier $\lambda^*$ has to taken into account** in the computation of the derivative of the state variable $\hat{x}$ **, multiplied by the derivative of the function $g(\hat{x})$ w.r.t. to $\hat{x}$**

=> **The effect of the nonlinear constraint will eventually impact the shape of the cost to go function**




The same requirement regarding the sign of the objective function inside the feasible region must hold

# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) = a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

**2nd situation:**

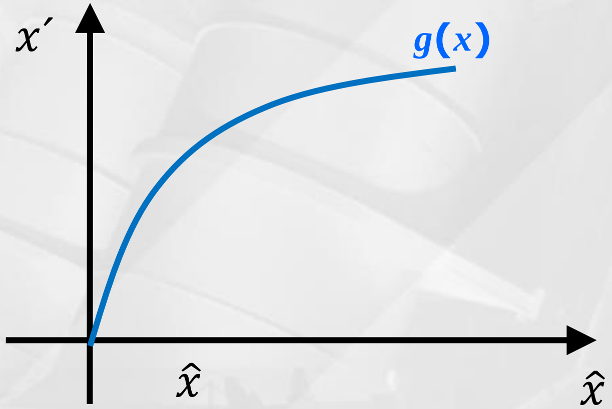
# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = \ b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) = a\hat{x}^2 + b\hat{x} + c, \end{cases} \quad \Longleftrightarrow \quad \begin{cases} Ax + x' = \ b \\ x' = a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

**2nd situation:**



# Requirement for application of nonlinear constraints

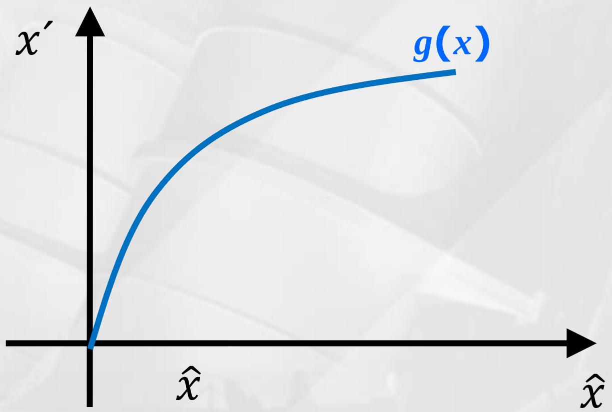
## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = \ b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) = a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

$$\begin{cases} Ax + x' = \ b \\ x' = a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

**2nd situation:**




In order to augment the feasible region, the same requirement regarding the "direction" of the objective function into the feasible region must hold

# Requirement for application of nonlinear constraints

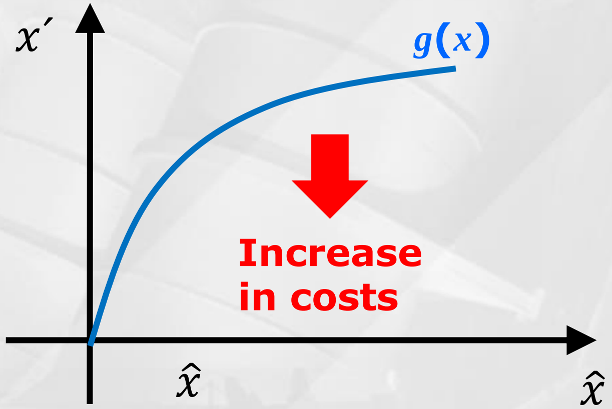
## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = b - g(\hat{x}), & \text{with} \\ g(\hat{x}) = a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

$$\begin{cases} Ax + x' = b \\ x' = a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

**2nd situation:**




In order to augment the feasible region, the same requirement regarding the “direction” of the objective function into the feasible region must hold

# Requirement for application of nonlinear constraints

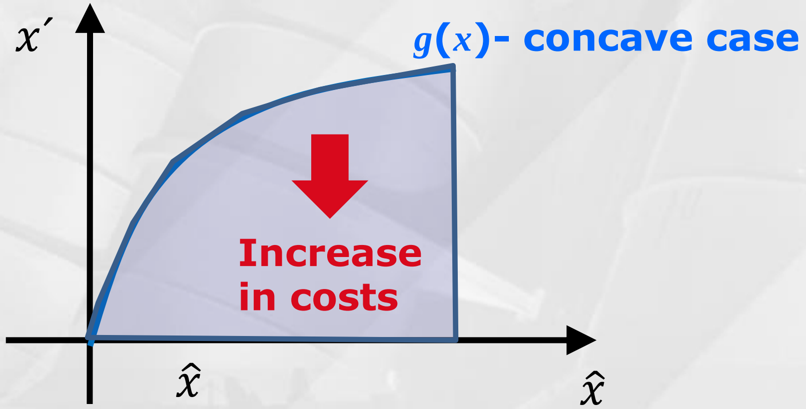
## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) \leq a\hat{x}^2 + b\hat{x} + c, \end{cases} \iff \begin{cases} Ax + x' = b \\ x' \leq a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

**2nd situation:**

In order to augment the feasible region, the same requirement regarding the "direction" of the objective function into the feasible region must hold



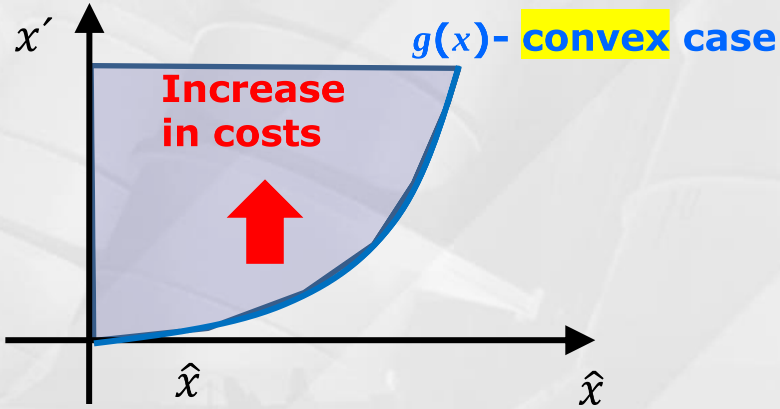
# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = b - g(\hat{x}), & \text{with} \\ g(\hat{x}) \boxed{\geq} a\hat{x}^2 + b\hat{x} + c, \end{cases} \quad \Longleftrightarrow \quad \begin{cases} Ax + x' = b \\ x' \geq a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

**2nd situation:**




In order to augment the feasible region, the same requirement regarding the "direction" of the objective function into the feasible region must hold

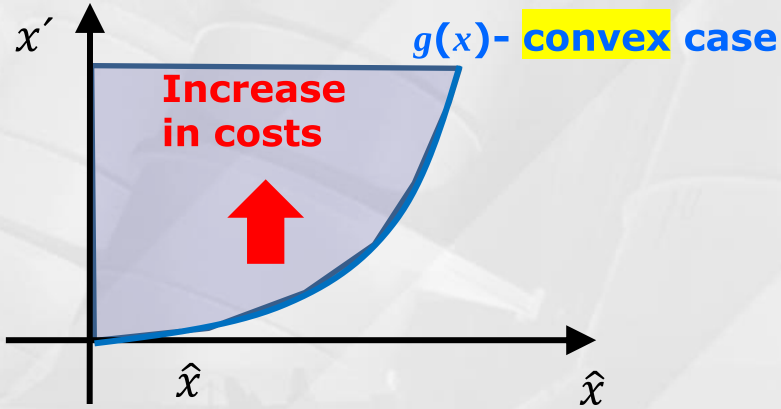
# Requirement for application of nonlinear constraints

## 2nd case: nonlinear function of state variables of a given subproblem

$$\begin{cases} Ax = b - g(\hat{x}), \quad \text{with} \\ g(\hat{x}) \geq a\hat{x}^2 + b\hat{x} + c, \end{cases} \iff \begin{cases} Ax + x' = b \\ x' \geq a\hat{x}^2 + b\hat{x} + c, \end{cases}$$

$$x' = g(\hat{x})$$

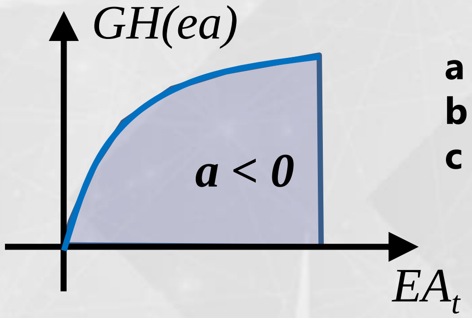
**2nd situation:**




- In order to augment the feasible region, the same requirement regarding the "direction" of the objective function into the feasible region must hold
- Convexification procedures may be necessary

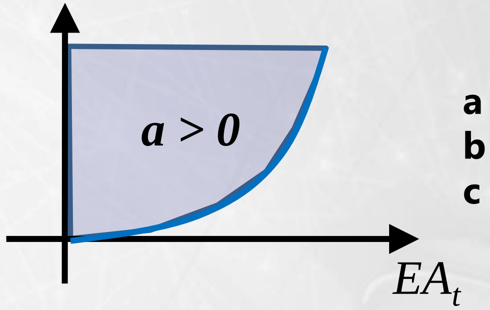
# Nonlinear expressions for application of nonlinear constraints

**Generation as a function of storage**



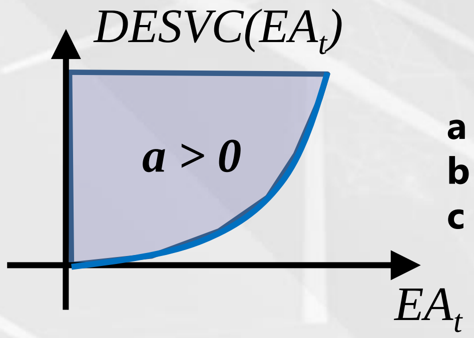

**a = -0.111736E-11**
**b = 0.767911E-06**
**c = 0.920171E+00**

Evaporation as a function of storage



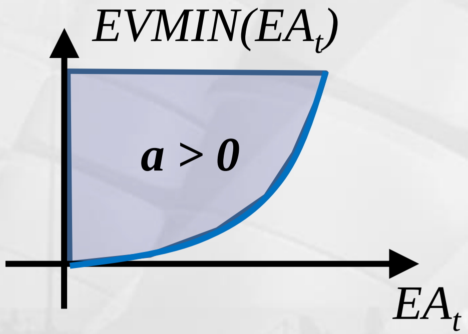

**a = -0.5025E-09**
**b = 0.7149E-03**
**c = 0.1004E+03**

Water intakes as a function of storage




**a = 0.260846E-09**
**b = -0.19384E-03**
**c = -0.38308E+03**

Minimum outflow as a function of storage

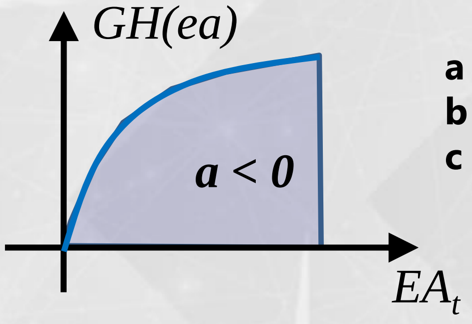



**a = -0.789412E-08**
**b =0.534699E-02**
**c = 0.629055E+04**

# Nonlinear expressions in the SDDP-based NEWAVE model

Cepel A pesquisa que constrói o futuro

## Generation as a function of storage

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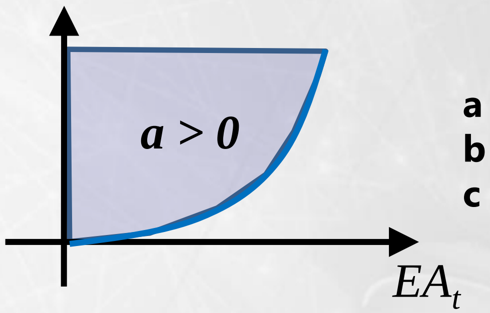


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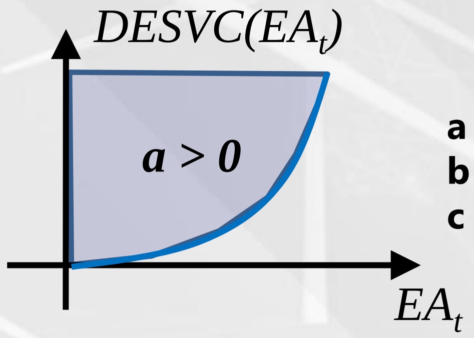


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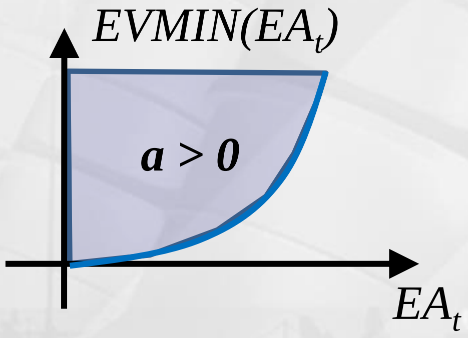


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# Nonlinear expressions in the SDDP-based NEWAVE model

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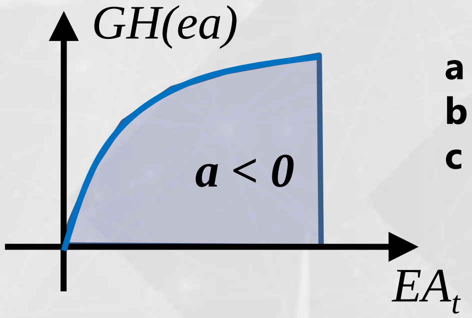


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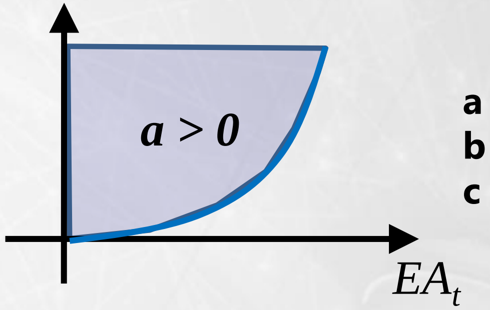


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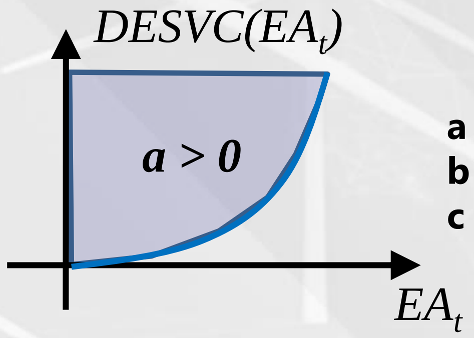


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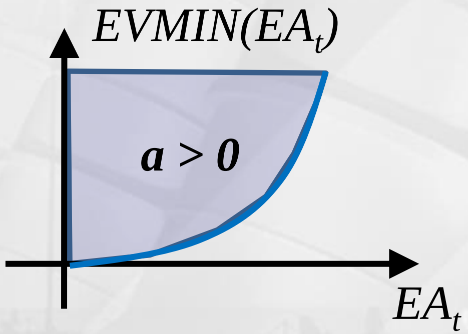


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# Nonlinear expressions in the SDDP-based NEWAVE model

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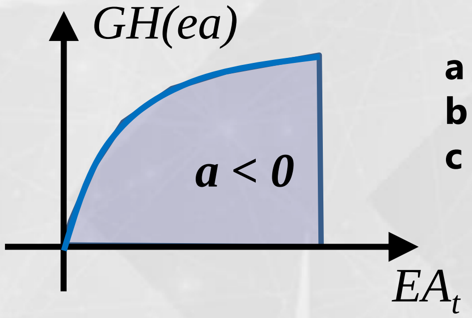


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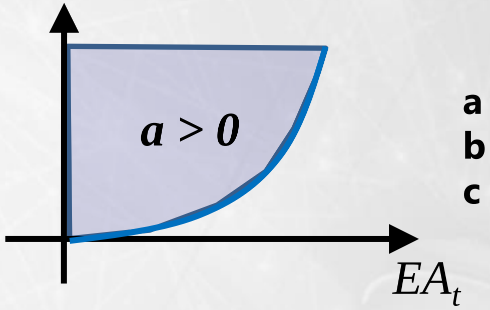


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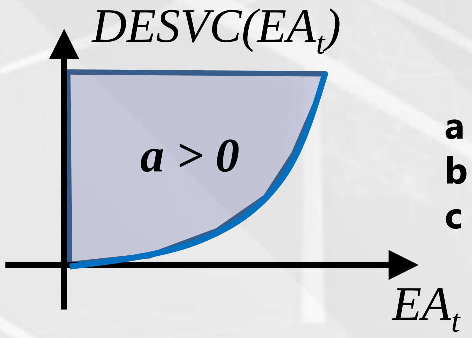


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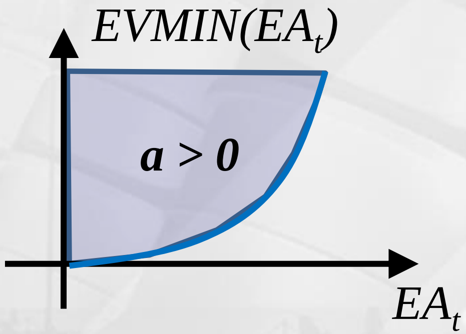


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# Nonlinear expressions in the SDDP-based NEWAVE model

Cepel A pesquisa que constrói o futuro

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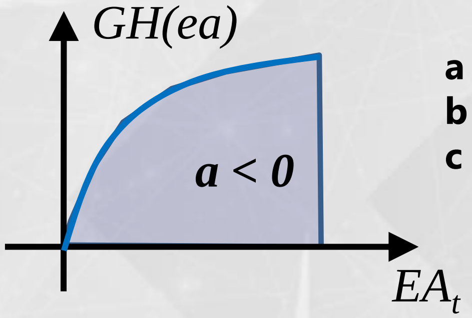


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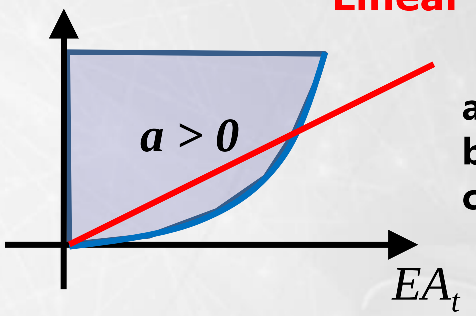
## Evaporation as a function of storage

**Linear approximation**

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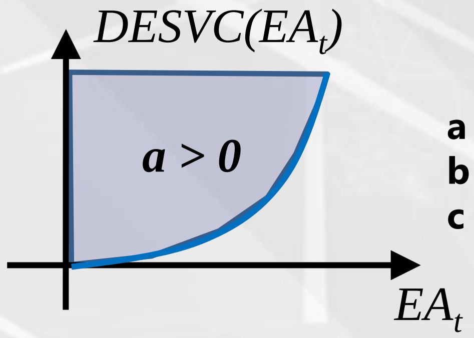


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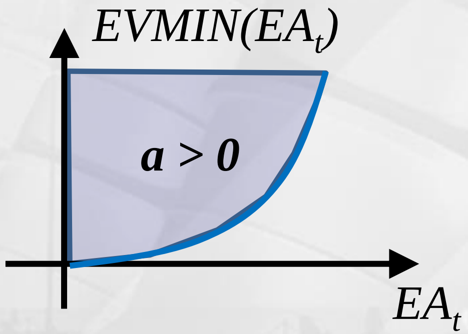


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# Nonlinear expressions in the SDDP-based NEWAVE model

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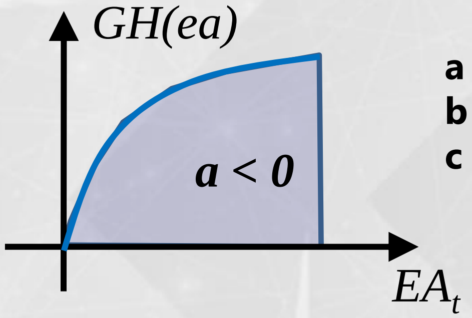


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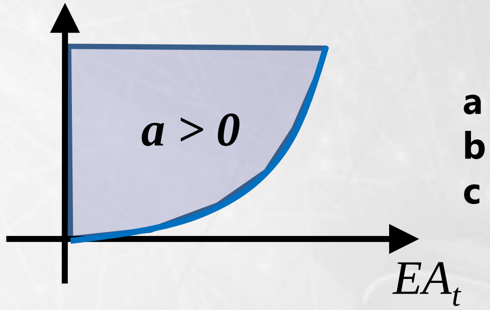


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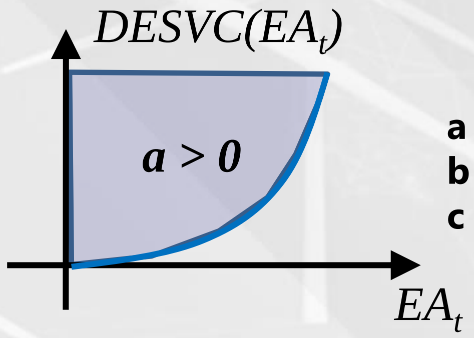


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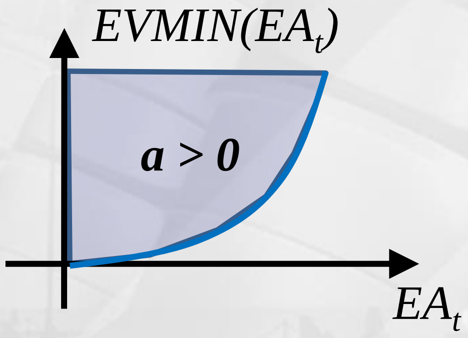


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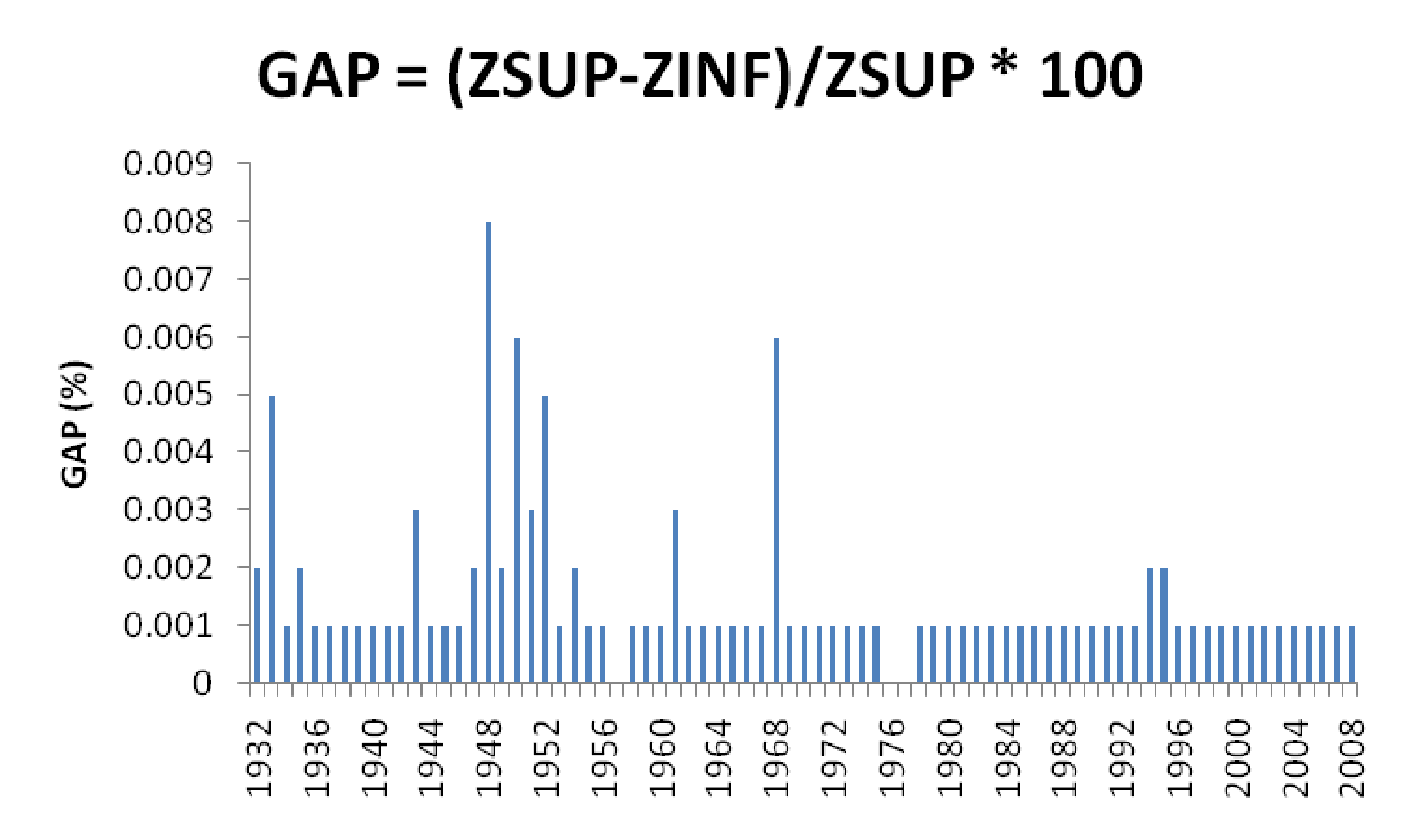
# Convergence check: deterministic cases



# CONCLUSIONS

**Linear (affine) approximation should be employed only if, after having checked all these conditions, one of them fail**

# A Survey on SDDP algorithms

STOCHASTIC DUAL DYNAMIC PROGRAMMING AND ITS VARIANTS

CHRISTIAN FÜLLNER* AND STEFFEN REBENNACK*

**Abstract.** We provide a tutorial-type review on stochastic dual dynamic programming (SDDP), as one of the state-of-the-art solution methods for multistage stochastic programs. Since introduced about 30 years ago for solving large-scale multistage stochastic linear programming problems in a hydrothermal context, SDDP has been applied to practical problems from several fields and is enriched by various improvements and enhancements to broader problem classes. We begin with a detailed introduction to SDDP, with special focus on its motivation and required assumptions. Then, we present and discuss in depth the existing enhancements as well as current research trends, allowing for an alleviation of those assumptions.

https://optimization-online.org/2021/01/8217/

# Thank you!

diniz@cepel.br