Modeling and control of malaria dynamics in fish farming regions

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Overview

In this work we propose a model that represents the relation between fish ponds, the mosquito population and the transmission of malaria. It has been observed that in the Amazonic region of Acre, in the North of Brazil, fish farming is correlated to the transmission of malaria when carried out in artificial ponds that become breeding sites. Evidence has been found indicating that cleaning the vegetation from the edges of the crop tanks helps to control the size of the mosquito population.

We use our model to determine the effective contribution of the fish tanks to the epidemic. The model consists of a nonlinear system of ordinary differential equations with jumps at the cleaning time, which act as *impulsive controls*. We study the asymptotic behaviour of the system in function of the intensity and periodicity of the cleaning, and the value of the parameters. In particular, we state sufficient conditions under which the mosquito population is exterminated or prevails, and under which the malaria is eradicated or becomes endemic. We prove our conditions by applying results for cooperative systems with concave nonlinearities.

Numerical simulations of our model have shown that an increase in cleaning frequency can reduce the mosquito population, which in turn reduces incidence of malaria. Our results suggests that incentives for fish pond maintenance are potential public health measures in the state of Acre.

Model

The following system describes the joint dynamics of malaria, aquatic-stage and adult mosquitoes, and vegetation:

$$\dot V = r(1-V),$$

$$V(n\tau^+) = V(n\tau^-) - \gamma(n\tau)V(n\tau^-), \quad \text{for } n \in \mathbb{N},$$

$$\dot L_p = \alpha\frac{K_p(V)}{K_w(V)+K_p(V)}(M_I+M_S)\left(1-\frac{L_p}{K_p(V)}\right) - (\nu+\mu_L+\mu_p(1-V))L_p,$$

$$\dot L_w = \alpha\frac{K_w(V)}{K_w(V)+K_p(V)}(M_I+M_S)\left(1-\frac{L_w}{K_w(V)}\right) - (\nu+\mu_L)L_w,$$

$$\dot M_S = \nu(L_w+L_p) - \beta_{hv} I M_S - \mu_M M_S,$$

$$\dot M_I = \beta_{hv} I M_S - \mu_M M_I,$$

$$\dot S = -\beta_{vh} S M_I + \kappa I,$$

$$\dot I = \beta_{vh} S M_I - \kappa I.$$

Asymptotic Behaviour

Limit Behaviour of Vegetation: The trajectories of vegetation $V(\cdot)$ satisfy

$$V(t) \to 1 - \frac{\gamma^* e^{-r(t-n\tau)}}{1-(1-\gamma^*)e^{-r\tau}}, \quad \text{for } t \in [n\tau,(n+1)\tau),$$

Limit Behaviour of Mosquito Population: The following assertions hold.

(i) If $\mathcal{N} \max_{t\in[0,\tau)} \frac{\tilde K_w}{\tilde K_w + \tilde K_p} + \mathcal{N}_p(\tau^-) \max_{t\in[0,\tau)} \frac{\tilde K_p}{\tilde K_w+\tilde K_p} \le 1$ *is satisfied, then the trajectories of the mosquito and larvae population asymptotically approach the origin for any feasible initial condition*

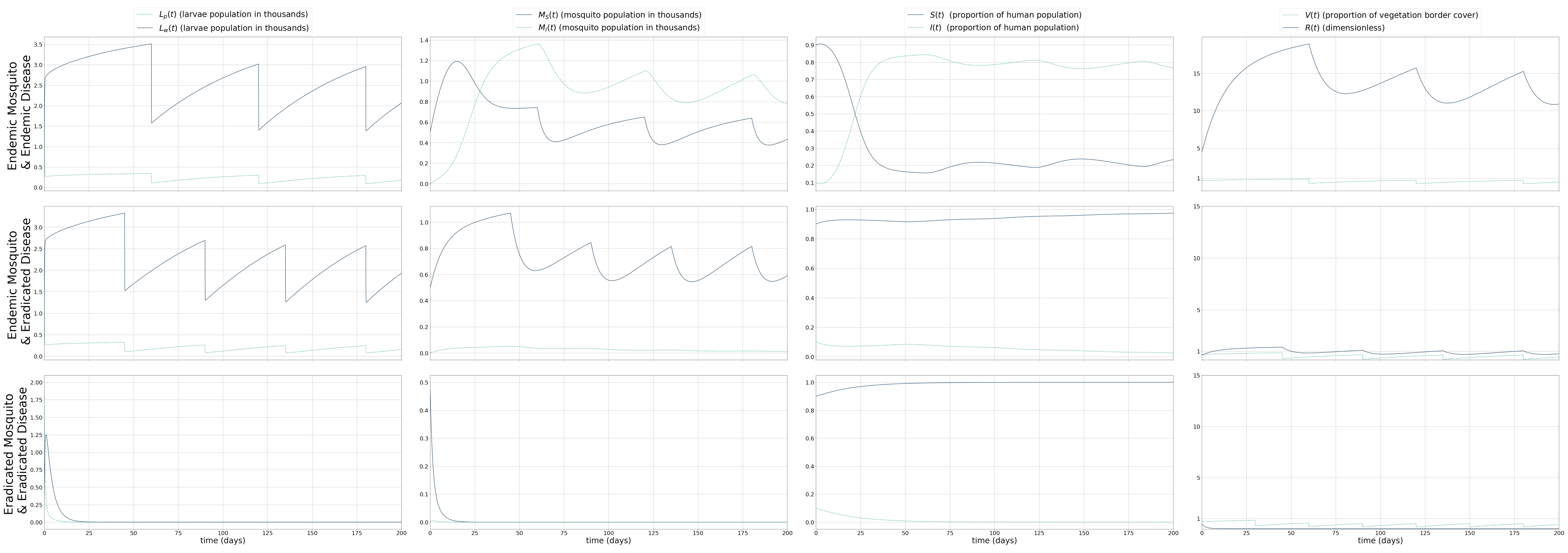
(ii) *If* $\frac{\min_{[0,\tau)}\left(\frac{\tilde K_w(t)}{\tilde K_w(t)+\tilde K_p(t)}\right)}{\mathcal{N}^{-1} + \frac{\tilde K_w(\tau^-)+\tilde K_p(\tau^-)}{\tilde K_w(0)+\tilde K_p(0)}} + \frac{\min_{[0,\tau)} \frac{\tilde K_p(t)}{\tilde K_w(t)+\tilde K_p(t)}}{\mathcal{N}_p(0)^{-1} + \frac{\tilde K_w(\tau^-)+\tilde K_p(\tau^-)}{\tilde K_w(0)+\tilde K_p(0)}} > 1$ *is satisfied, then for all feasible initial conditions, trajectories of the mosquito and larvae population are bounded above and below by asymptotically periodic solutions.*

Limit Behaviour of Disease: The following assertions hold.

(i) If $\frac{\beta_{vh}\beta_{hv}\max_{t\in[0,\tau)} M_{\mathrm{per}}(t)}{\kappa\mu_M} \le 1$, then the trajectories of (M_I, I) verify $\lim_{t\to+\infty}(M_I,I)(t) = 0$ for all feasible initial conditions.

(ii) If $\frac{\beta_{vh}\beta_{hv}\min_{t\in[0,\tau)} M_{\mathrm{per}}(t)}{\kappa\mu_M} > 1$, then there exists a strictly positive periodic solution for (M_I, I), which attracts all feasible initial conditions.

Scenario Simulation



Acknowledgements and References

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Play with the model

An interactive simulation of the model is available in Google Colaboratory through the QR Code.