Meta-heuristic Optimisation Applied to Astrophysical Blazar Modelling

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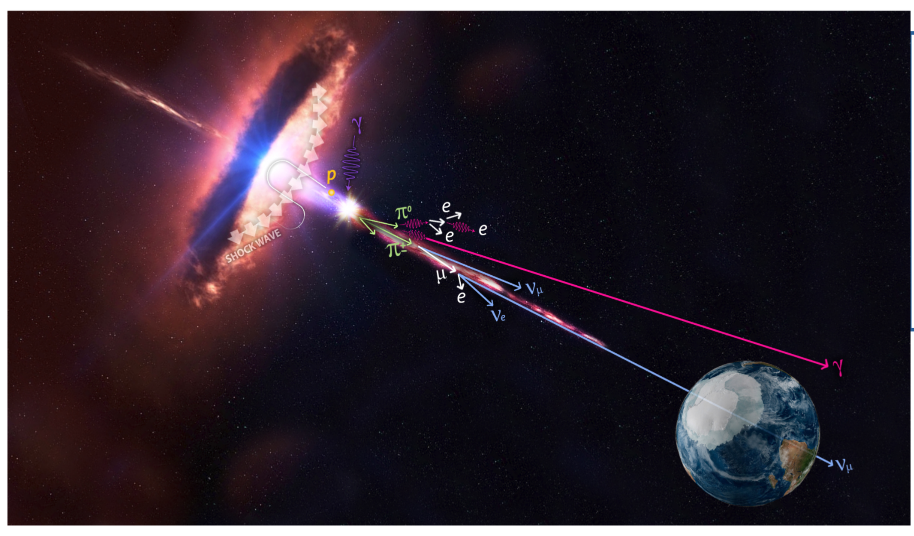
Introduction: In high-energy astrophysics, fitting experimental data through mathematical models is a widely used procedure to test scientific hypotheses. Here we focus on the fitting of radiative models to data sets observed from blazars, which are extragalactic high energy sources being currently measured through multiple ground-based as well as onboard observatories [8]. State-of-the-art models employed to interpret such astrophysical measures are highly non-linear, making systematic data fitting challenging.

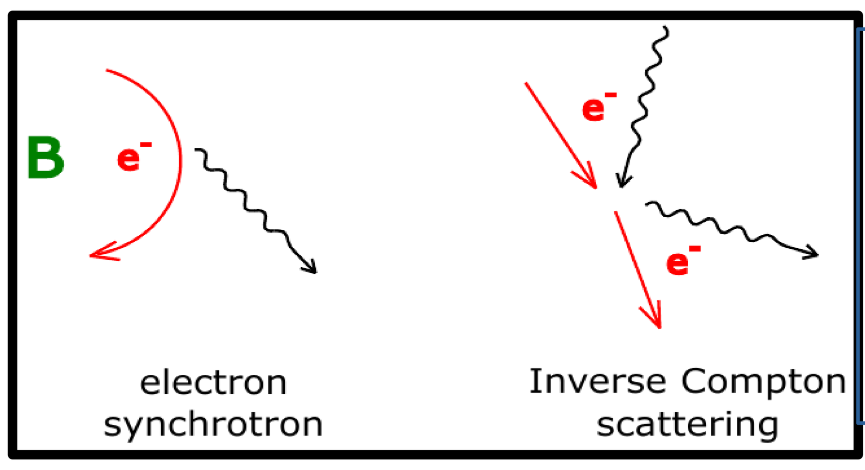
The problem: The fitting of data through a physical model can be addressed as an optimisation problem, where particular solutions of a mathematical model are evaluated through an objective or merit function. The derivation of a single blazar emission solution typically requires specifying about ten free parameters (depending on the adopted physical model). With a such number of free parameters, an exhaustive deterministic exploration of the model parametric space can be computationally prohibitive when searching for acceptable fits to a particular data set. In such situations, stochastic approaches like those adopted by meta-heuristic algorithms (or simply meta-heuristics) can be suitable alternatives to perform efficient optimisation (see [1] and reference therein). However, it not established a general criterion for choosing the most efficient meta-heuristic for a particular problem.

What we did: In this work, we study for the first time the performance of selected meta-heuristics in fitting blazar emission data. We consider different types of swarm optimization and genetic algorithms. For building the optimization objective function, we employ a χ2 criterion and consider a synchrotron self-Compton physical scenario. The best performance was obtained with the DE algorithm, as shown by the convergence curves of our study. Further details about the present work will be reported in a forthcoming paper [9]

The Physical Scenario



Blazars are astrophysical sources of radiation, which emission is produced in relativistic and collimated ejections of plasma pointing towards our line of sight.



We consider a physical model where the blazar emission is produced by electrons spiralling magnetic fields (synchrotron radiation), and up-scattering low energy photons (Inverse Compton), a.k.a. synchrotron self-Compton (SSC) [8].

The Objective Function

$$\chi^2 \equiv \sum_{i=1}^{N}\left(\frac{y_i - y(x_i; c_1, .., c_k)}{\sigma_i}\right)^2$$

The objective function of our optimisation problem. This function measures the goodness of the radiative models in fitting observed data points of radiation flux represented by the y_i points. The radiative model y is the SSC model and is evaluated at specific energies of photons, represented by the x_i points, and depends on 9 free parameters (k=9), representing physical quantities of the blazar emission model, namely

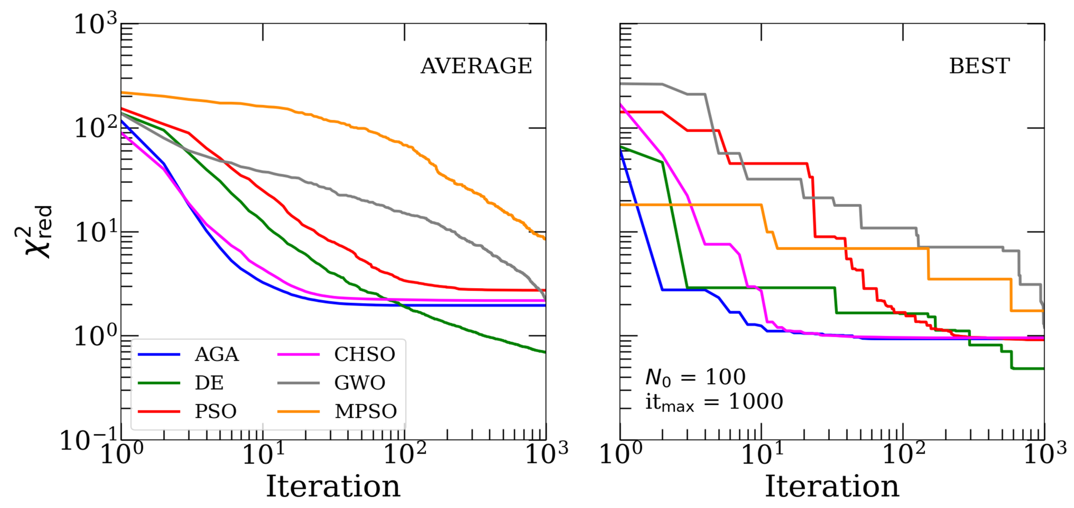
$E_{e,min}$: minimum energy of electrons
$E_{e,b}$: energy break of the electron population
$E_{e,max}$: maximum energy of electrons
α: spectral index of the electron population
α_b: break spectral index of the electron population
L_e: power of the electrons in the black hole frame.
r_b: size of the emission region
B: average magnetic field within the emission region in the jet co-moving frame
Γ_j: bulk Lorentz factor of the emission region

Meta-heuristics

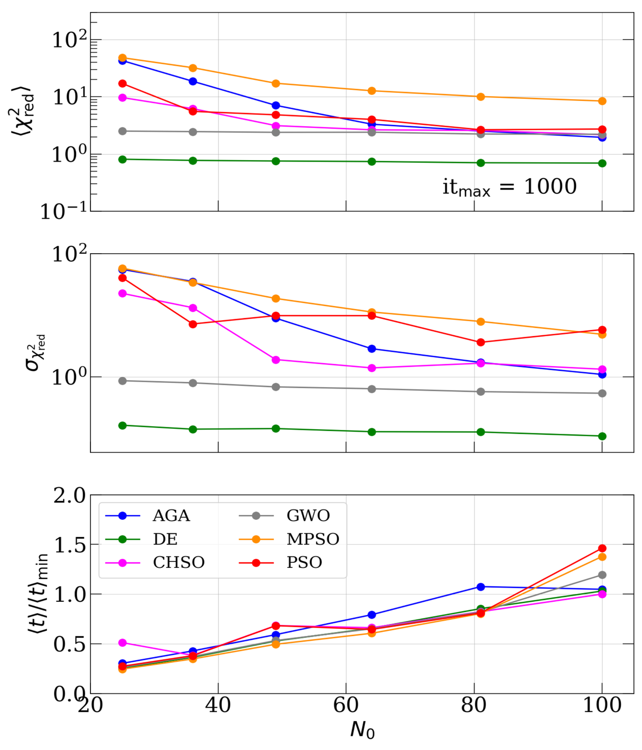
- We compare the performance of selected metaheuristic methods in optimizing the χ2 function described above, namely:

- asexual genetic algorithm (AGA) [1]
- differential evolution (DE) [2]
- particle swarm optimisation (PSO) [3]
- chicken swarm optimisation (CHSO) [4]
- grey wolf optimisation (GWO) [5]
- mean particle swarm optimisation (MPSO) [6].

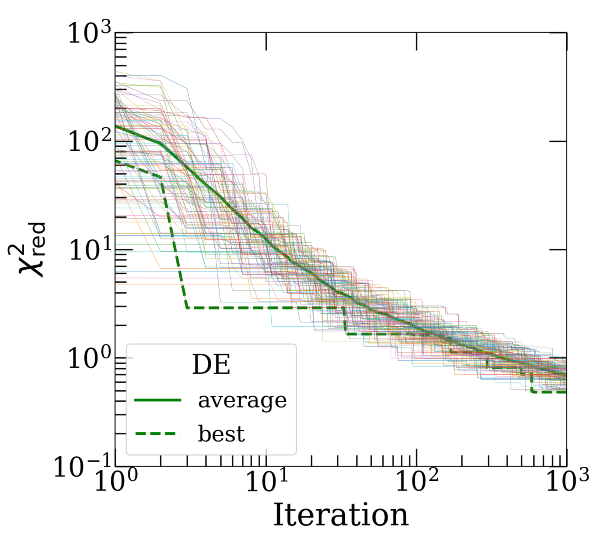
Results



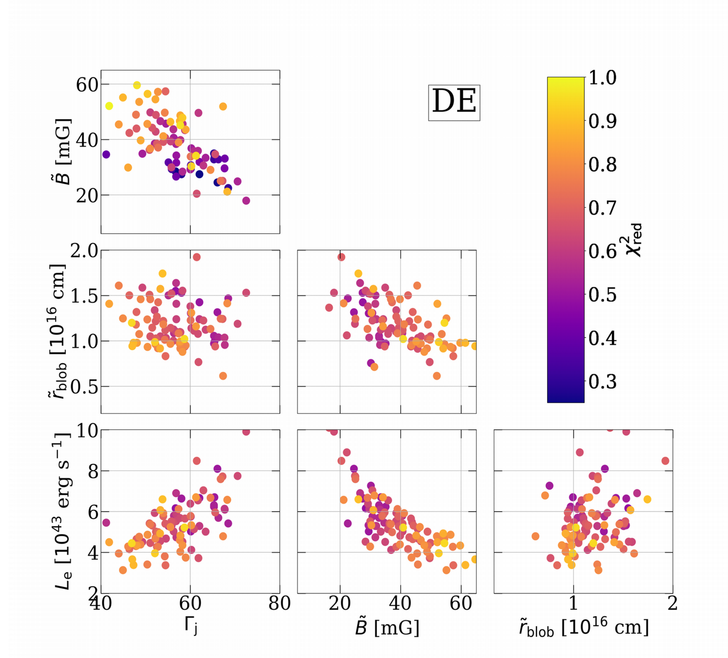
Convergence curves of the optimisation performed by the metaheuristics methods. Left: average curves of 100 runs carried out by each method. Right: The best convergence curve of the same runs.



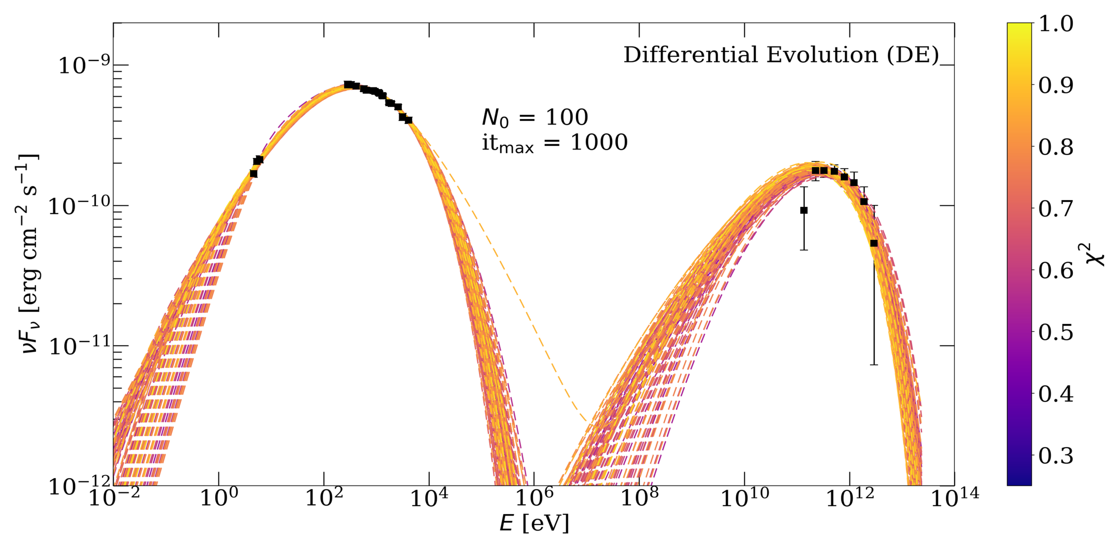
Performance of the metaheuristic methods as a function of the initial individuals and after 1000 iterations. Upper: final mean value of the objective function. Middle: standard deviation of the achieved χ2 values. Lower: Normalised time of computation.



Convergence curves of the 100 runs performed by the DE algorithm (the one showing the best performance in our study). The average curve is this highlighted with the thick curve, whereas the run with the best performance is plotted with the dashed line.



2D cuts of the parametric space where the DE method found the best solutions. The axes in the plots represent selected physical quantities of the blazar radiative model.



Radiation curves as a function of photon energy given by the blazar SSC model and optimised by the DE method. Data points are observations of the astrophysical source Markarian 421 [7].

Conclusions

- **The DE algorithm outperforms all the metaheuristics considered in the present study in minimizing the objective function and takes, in general, the lowest computation time.**
- **The optimization obtained by the DE algorithm exhibits almost no sensitivity to the number of initial individuals N_0.**
- **The AGA algorithm displays the second-best performance, especially when increasing the number of initial individuals N_0.**

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