Periodic variants of stochastic dual dynamic programming in energy operation planning

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Abstract

The energy planning problem can be modeled as a multistage stochastic program, where at each stage (at each month of the year) an agent needs to choose a generation quantity from thermal and hydro such that minimize a performance index (operation costs) subject to the energy balance equation and the reservoir dynamics. A characteristic of the uncertainties in the problem of interest is their periodic behavior (typically the inflows to the reservoirs follow the annual rainfall regime). The stochastic dual dynamic programmming (SDDP) [3, 4, 6] is a method for solving the energy planning problem, where the "curse of dimensionality" of dynamic programming is avoided. In order to reduce the computational effort for solving the energy planning problem, in this work we study and analize multistage stochastic optimization methods that explore periodic structures. To achieve this, we analize the proposals [5] and consequently model and simulate the energy planning problem as a linear policy graph [2, 1].

The energy operation planning problem

For the Brazilian interconnected power system it is usual to consider four energy reservoirs, one in each one of the four interconnected main regions SE, S, N, and NE. The purpose of hydrothermal system operation planning is to define an operation strategy, for each stage of the planning period, given the system state at the beginning of the stage, produces generation targets for each plant. The objective is to minimize the expected value of the total cost along the planning period [6].

Modeling

Let $N = \{1, 2, 3, 4\}$ be the set of reservoirs (SE, S, N, and NE) for the Brazilian interconnected power system. For each system $i \in N$, we denoted by x_t^i the energy stored in the reservoir at time t. The system's dynamics is given by

$$x_{t+1}^i = x_t^i + a_{t+1}^i - h_{t+1}^i - s_{t+1}^i, \tag{1}$$

where a_{t+1}^i is the random water inflow in the reservoir, h_{t+1}^i is the water flowing through a turbine between time t and $t+1$ in order to produce electricity, and s_{t+1}^i is a spillage as a recourse variable to avoid reservoir overflow. At system i, the energy balance equation is

$$h_t^i + \sum_{k \in NT_i} g_t^{ik} + \sum_{j=1}^{5} f_t^{ji} - \sum_{j=1}^{5} f_t^{ij} + \sum_{j=1}^{4} r_t^{ij} = d_t^i \tag{2}$$

$$\sum_{j=1}^{4} f_t^{j5} = \sum_{j=1}^{4} f_t^{5j}, \tag{3}$$

where g_t^{ik} is the production of the thermal plant k (with unit cost c_t^{ik}), f_t^{ji} is the energy inflow from $j \in N$ or a trans-shipment station ($j = 5$). If the demand d_t^i is not satisfied, system i borrows r_t^{ij} units of energy from the deficit account in j and a total cust δ_t^i will be incurred. The objective is to minimize

$$\sum_{t=1}^{T} \left[\sum_{i=1}^{4} \left(\sum_{k \in NT_i} c_t^{ik} g_t^{ik} + \delta_t^i \sum_{j=1}^{4} r_t^{ij} \right) \right], \tag{4}$$

subject to (1), (2), (3), where T is the study horizont.

Results

We use the open-source stochastic programming solver SDDP.jl [2], that uses the policy graph formulation of a multistage stochastic program [1], for simulating the energy operation planning problem. The data is from [6]. The experiments were performed on a computer with Intel(R) Core(TM) i9-12900K, operational system Ubuntu.

Iteration	**Time** (h)	**Lower Bound**	**Confidence Interval**
100	$0{,}22 \times 10^{-1}$	$300{,}782 \times 10^6$	$(348{,}216 \pm 5{,}347) \times 10^6$
200	$0{,}45 \times 10^{-1}$	$307{,}355 \times 10^6$	$(344{,}202 \pm 5{,}534) \times 10^6$
500	0,12	$313{,}155 \times 10^6$	$(334{,}996 \pm 4{,}955) \times 10^6$

Table 1: Study horizont $T = 120$

Table 1 shows the lower bound and confidence interval, for the upper bound, of the optimal policy's quality found when the study horizont is $T = 120$. The plots, in Figures 1, 2, 3, 4, displays ribbons of the 0-100, 10-90, and 25-75 percentiles. The dark, solid line in the middle is the median (i.e. 50'th percentile).

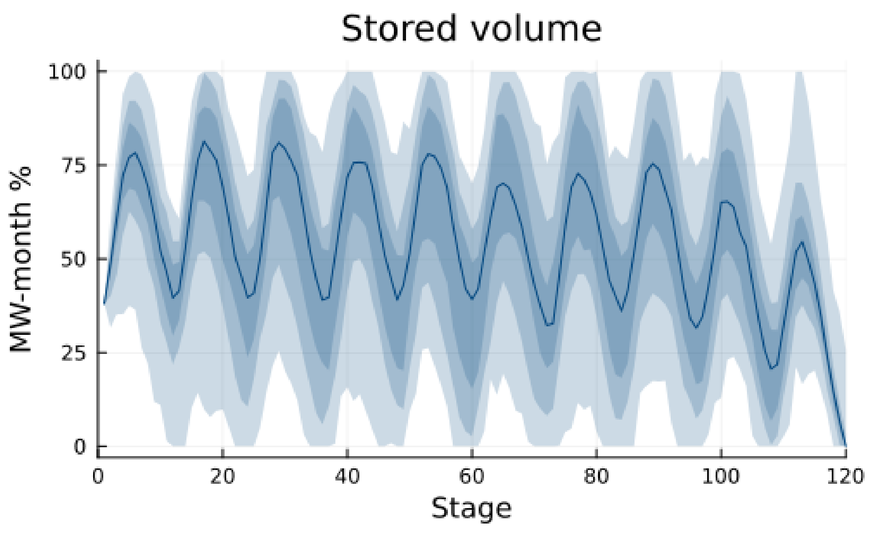

Figure 1: SE system percentiles of stored volumes for each stage

Figure 1 shows the water store volume for each month of the year. We can observe the periodic behavior (with period 12).

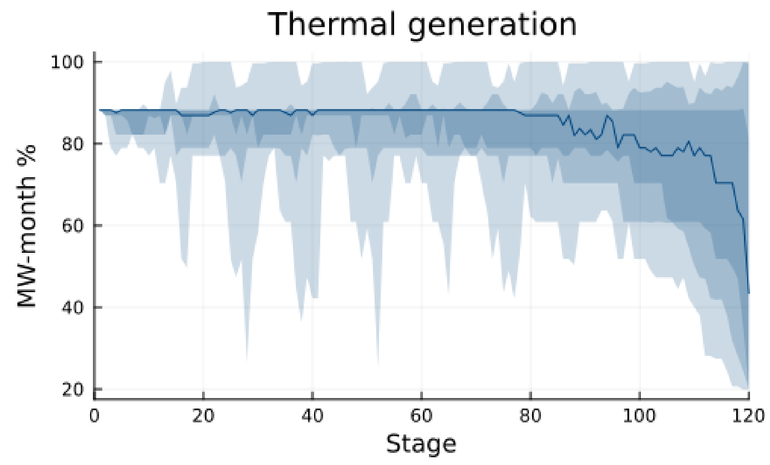

Figure 2: SE system percentiles of thermal geration for each stage

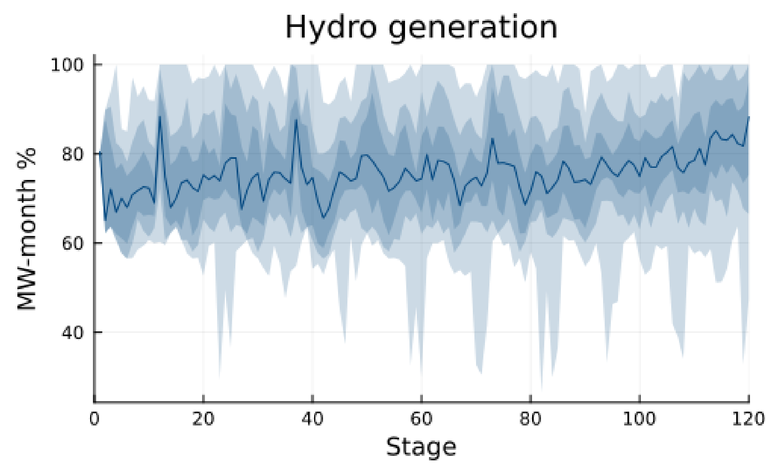

Figure 3: SE system percentiles of hydro generation for each stage

Figures 2, 3 show the thermal and hydro generation, respectively, in order to meet a time-variant deterministic demand.

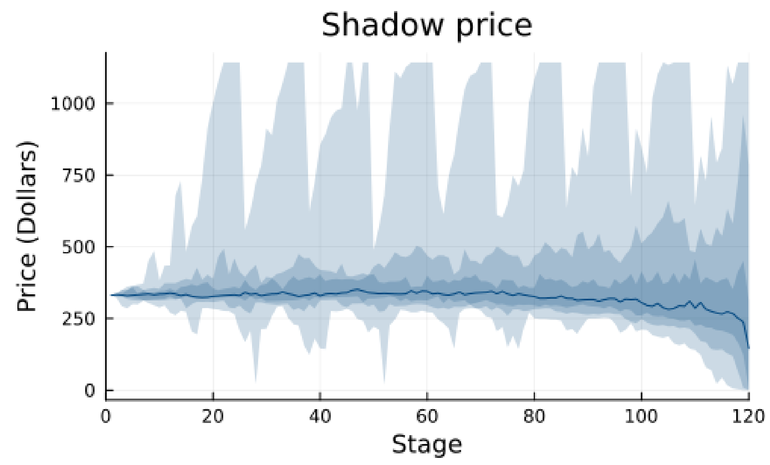

Figure 4: SE system percentiles of cost of each additional unit of demand

Figure 4 shows the dual of the demand constraint that corresponds to the price we should charge for electricity, sice it represents the cost of each additional unit of demand.

In periodic variants of SDDP, the periodical SDDP solves a much smaller problem per iteration (ten times smaller in this case sinse there are 10 periods, with period 12, for $T = 120$). Therefore, it is interesting to make periodic calculations. The performance comparison of periodical and non-periodical SDDP variants is work in progress.

Forthcoming Research

- Explore periodical structures when incorporating environmental constraints to take into account the multiple uses of water.

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