MODEL-BASED OPTIMIZATION FOR LEGGED LOCOMOTION: AN OVERVIEW

Yue Hu
Active & Interactive Robotics Lab (A.I.R. Lab)
Department of Mechanical and Mechatronics Engineering, University of Waterloo, Canada
yue.hu@uwaterloo.ca

Jan. 30th 2023, LACIAM, FGV, Rio De Janeiro, Brazil

UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Active & Interactive Robotics Lab

Outline

- Legged robot walking: the problem
- Dynamic models
- Contact models
- Solution methods
- Realizing motion plans

This talk is based o the survey paper: “Optimization-Based Control for Dynamic Legged Robots”, Patrick M. Wensing, Michael Posa, Yue Hu, Adrien Escande, Nicolas Mansard, Andrea Del Prete, submitted to Transaction on Robotics, 2022

Submitted as part of activities of the IEEE-RAS Technical Committee on Model-Based Optimization for Robotics

LEGGED ROBOTS LOCOMOTION

The problem formulation

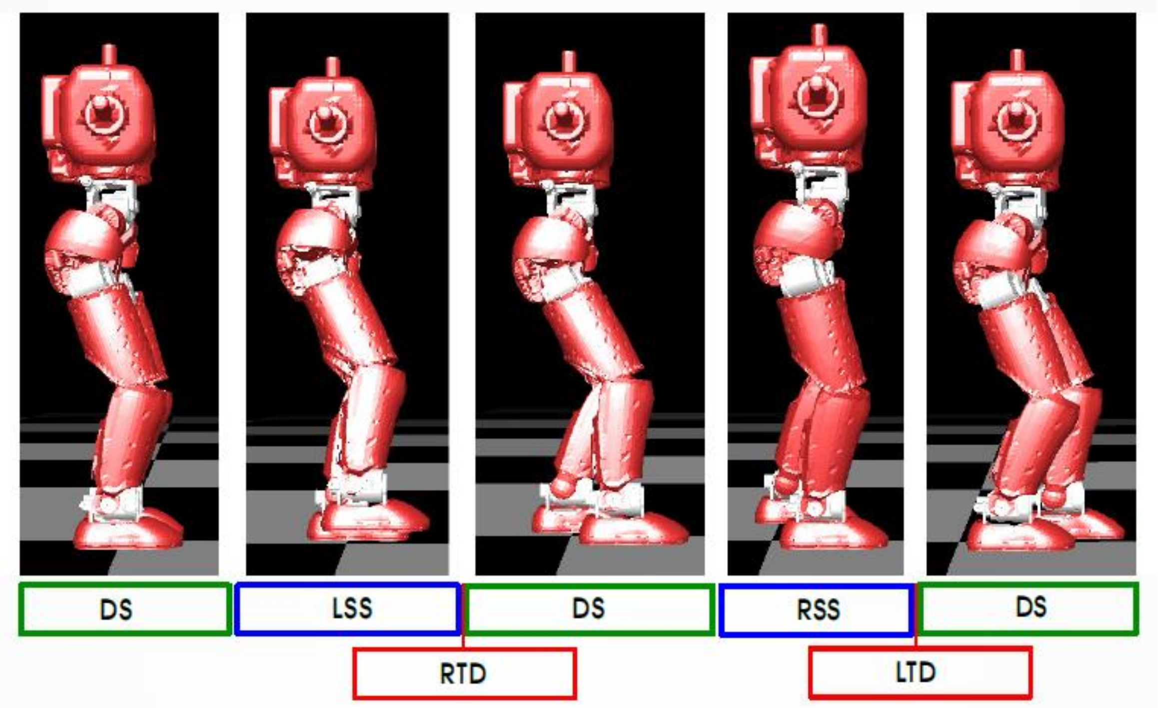
Locomotion is (still) a hard problem for legged robots

Darpa Robotics Challenge 2015

Boston Dynamics 2023

The problem(s) of legged locomotion

- Complex structures:
  - Multiple degrees of freedom
    - ~12 for quadrupeds
    - ~36 for full bipedal humanoids
  - Redundancy
  - Instability – easy to fall



A bit of history

- 1990s:
  - Availability of commercially available solvers and advent of computational power
- 2000s:
  - Application of optimization and optimal control to legged locomotion
- 2015:
  - DARPA Robotics Challenge – push for advances
- 2015 – now:
  - Exploitation of higher complexity models for challenging environments

The problem formulation

$$\underset{x(\cdot),u(\cdot),\lambda(\cdot)}{\text{minimize}} \quad \text{Cost}(x(\cdot), u(\cdot), \lambda(\cdot))$$

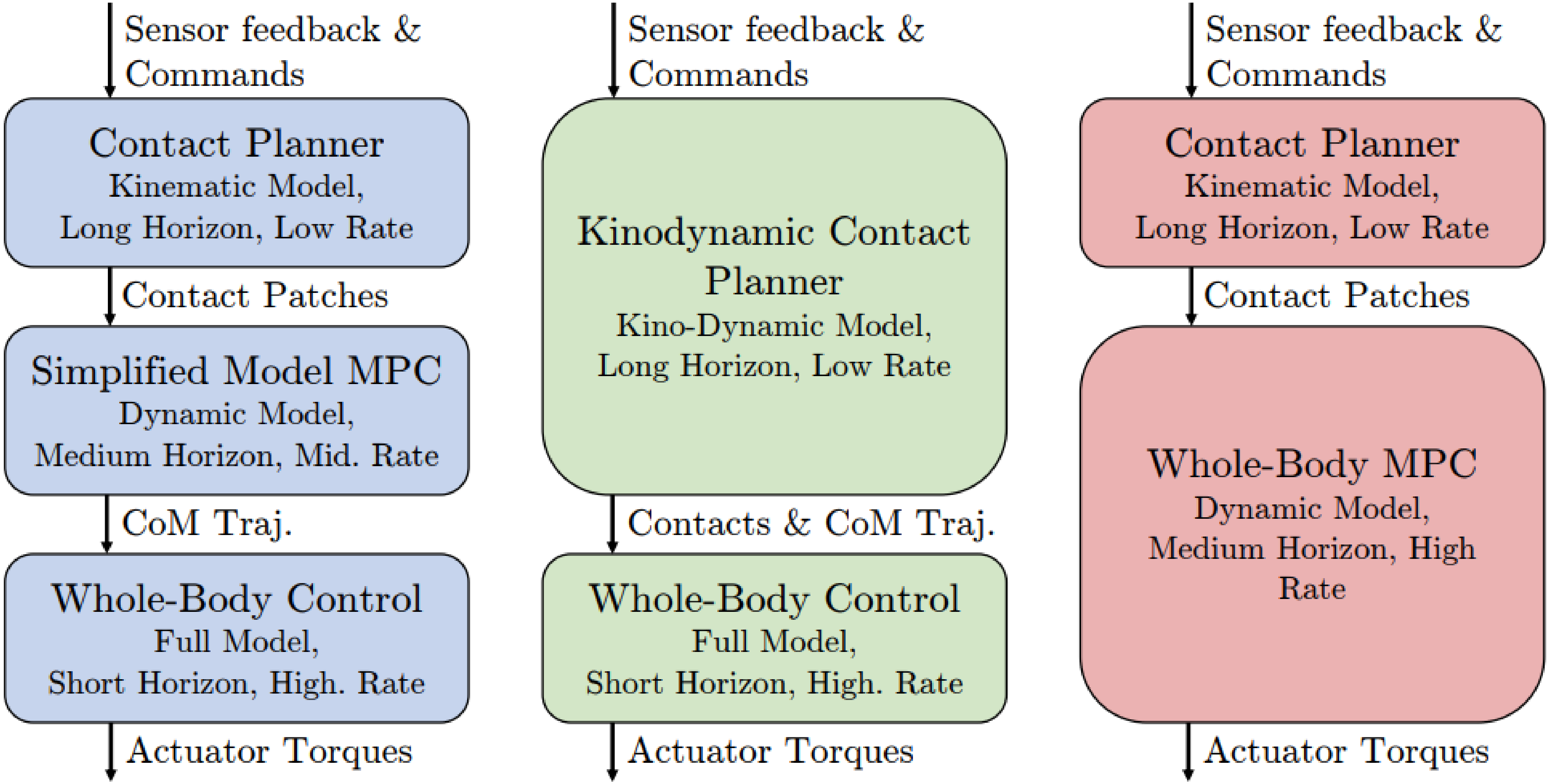
subject to $\boxed{M(q)\,\dot{\nu} + C(q,\nu)\,\nu + \tau_g(q) = S^T\tau + J(q)^T\lambda}$ Nonlinear body dynamics

Non penetration, friction, etc $\boxed{\text{ContactConstraints}(x(t), \lambda(t), u(t), \text{Env})}$

Joint position, velocity limits, etc $\boxed{\text{KinematicsConstraints}(x(t))}$

Motor torque limits, etc $\boxed{\text{InputConstraints}(u(t), x(t))}$

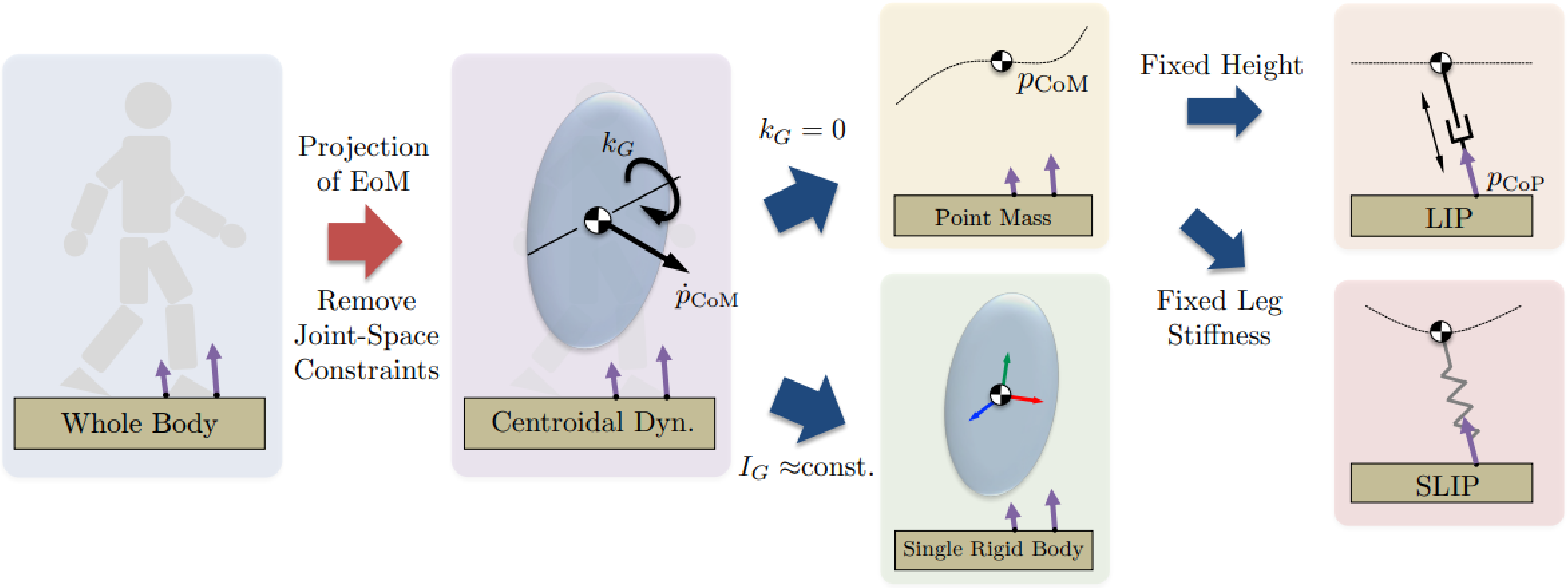
Initial state, field-of-view, etc $\boxed{\text{TaskConstraints}(x(t), u(t), \lambda(t))}$ $\forall t\,.$



Possible workflows

DYNAMIC MODELS

Model complexities



UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Dynamic models for legged robots

- General dynamic model of a floating base system:

$$\begin{bmatrix} M_{bb} & M_{bj} \\ M_{jb} & M_{jj} \end{bmatrix} \begin{bmatrix} \dot{\nu}_b \\ \dot{\nu}_j \end{bmatrix} + C(q, \nu)\nu + \tau_g(q) = \begin{bmatrix} 0 \\ \tau \end{bmatrix} + J(q)^T \lambda$$

- Simplified model with centroidal dynamics:
  - Centroidal momentum $h_G = (k_G,\ l_G)$

$$\dot{h}_G = \begin{bmatrix} \dot{k}_G \\ \dot{l}_G \end{bmatrix} = \begin{bmatrix} 0 \\ -M a_g \end{bmatrix} + \sum_{i=1}^{n_c} \begin{bmatrix} (p_i - p_{\mathrm{CoM}}) \times \lambda_i \\ \lambda_i \end{bmatrix}$$

- Tradeoffs between flexibility, accuracy, and computational efficiency needed

Dynamic models for legged robots

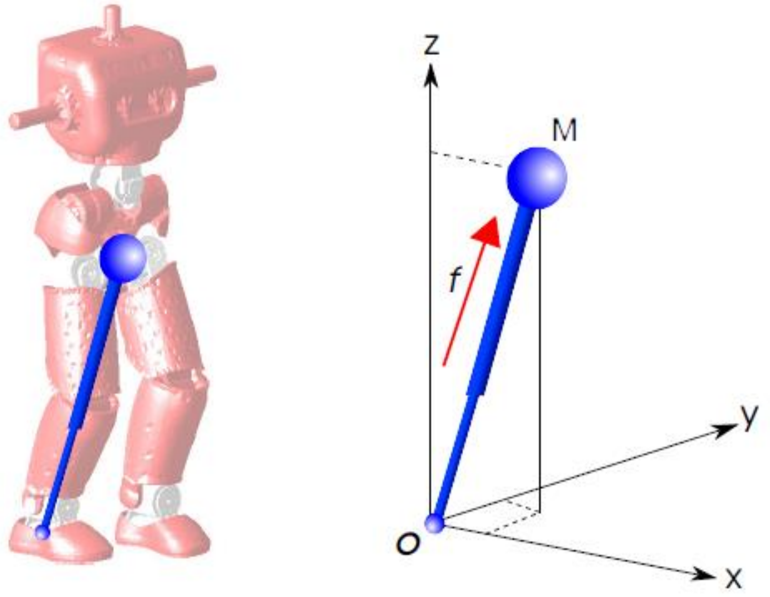
$$\dot{h}_G = \begin{bmatrix}\dot{k}_G \\ \dot{l}_G\end{bmatrix} = \begin{bmatrix}0 \\ -Ma_g\end{bmatrix} + \sum_{i=1}^{n_c}\begin{bmatrix}(p_i - p_{\mathrm{CoM}}) \times \lambda_i \\ \lambda_i\end{bmatrix}$$

- Simplifications of the centroidal dynamics
  - Removing orientation dynamics
  - CoM height fixed -> h
  - Assumption that the Center of Pressure (CoP) has zero value on the z direction

$$\ddot{c}_{x,y} = \omega^2(c_{x,y} - p_{x,y})$$

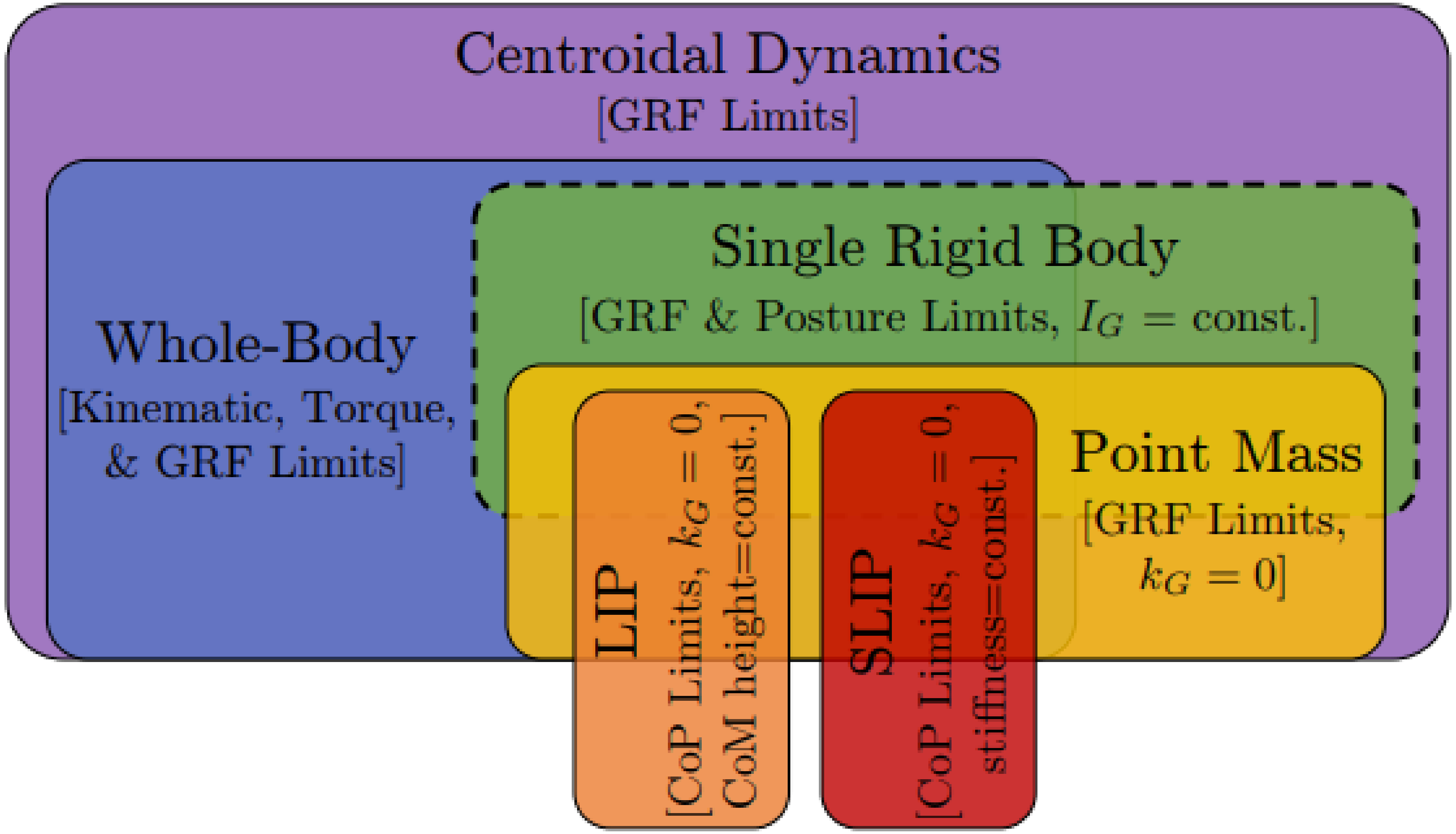
$$\omega = \sqrt{g/h}$$

  - This is linear -> convex optimization



Linear Inverted Pendulum (LIP)

Dynamic models for legged robots



UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Whole-body vs Centroidal

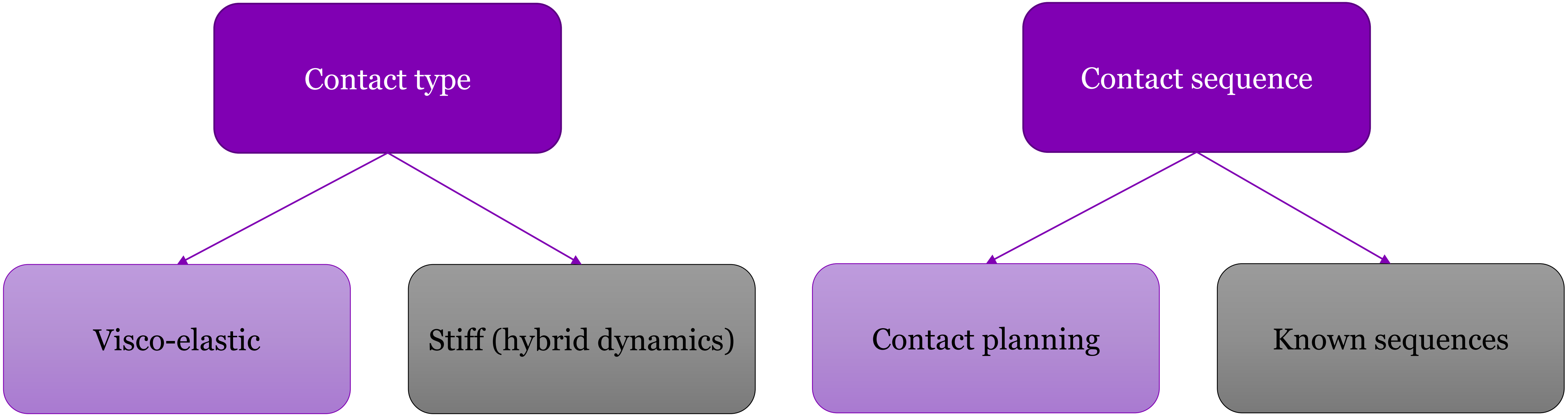
Koch, Kai Henning, et al. "Optimization based exploitation of the ankle elasticity of HRP-2 for overstepping large obstacles." *2014 IEEE-RAS International Conference on Humanoid Robots*. IEEE, 2014.

Kudruss, Manuel, et al. "Optimal control for whole-body motion generation using center-of-mass dynamics for predefined multi-contact configurations." *2015 IEEE-RAS 15th International Conference on Humanoid Robots (Humanoids)*. IEEE, 2015.

CONTACT MODELS

Contact types and sequences

$$\lambda = F_{\text{contact}}(\phi, J\nu)$$

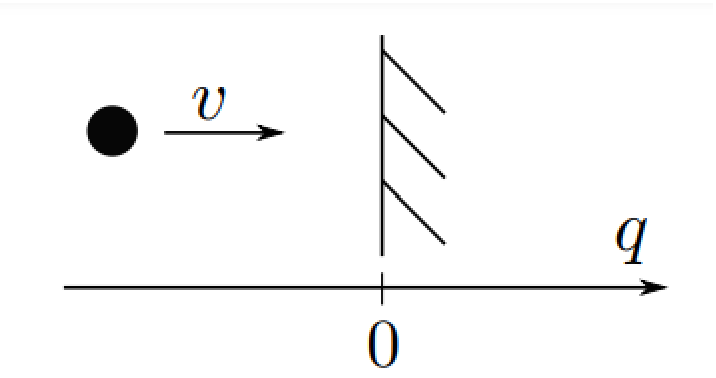


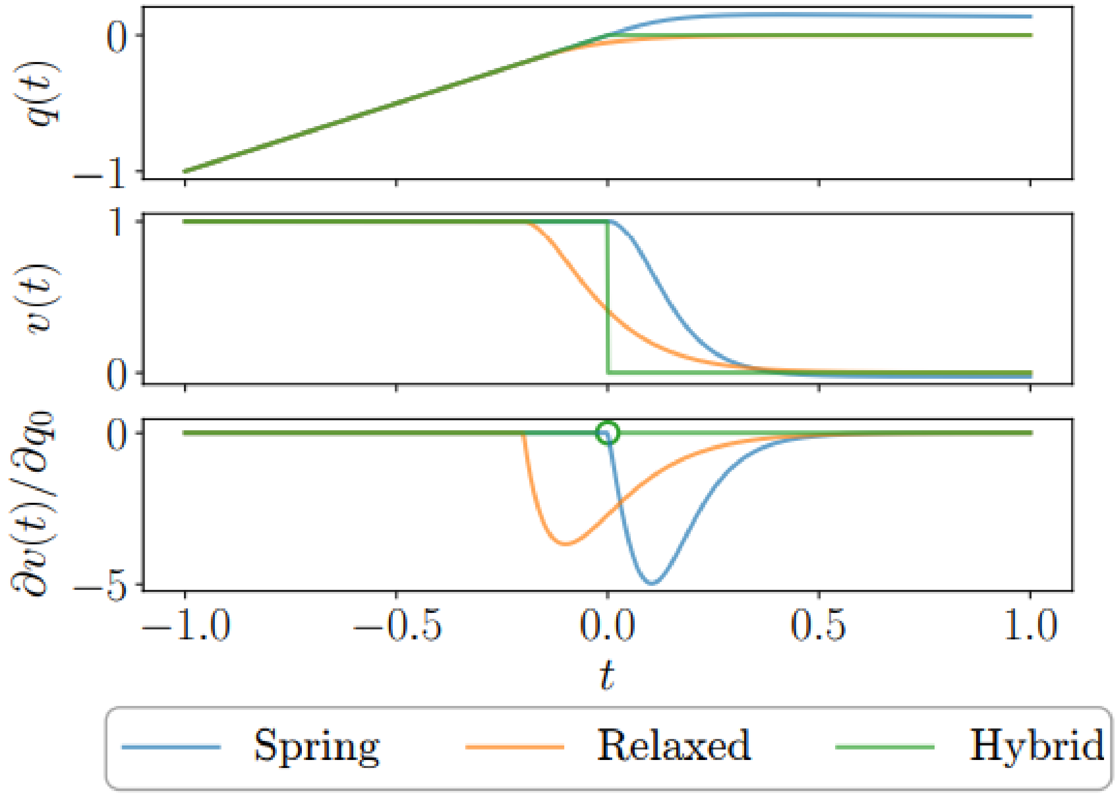
Impacts: $x^{+} = R(x^{-})$

Non-penetration: $0 \leq \lambda_n \perp \phi(q) \geq 0$

Contact modeling

- Example: single mass contact





- Well-conditioned if contact modes known
- Need relaxation if unknown

Contact planning

- One of the most challenging problems for legged robots
- Known sequences: solve the hybrid optimal control problem with respect to the transcribed decision variables

Hybrid sequence optimization:

- Jointly optimize over the hybrid sequence and the robot motion
- Mixed-integer formulation or bilevel optimization
- Sampling based methods (e.g. RRT)

Contact implicit optimization:

- Variation on hybrid optimization
- Contact dynamics: complementarity formulations or smooth approximations
- Poor numerical conditioning
- Can require high quality initial guesses

SOLUTION METHODS

Transcription of the problem

- General formulation:

$$\begin{aligned} \underset{x(\cdot),u(\cdot),p(\cdot)}{\text{minimize}} \quad & \int_{t_0}^{t_f} \ell(x(t),u(t),p(t))dt + L(x(t_f)) \\ \text{subject to} \quad & \dot{x}(t) = f(x(t),u(t),p(t)) \\ & 0 = g(x(t),u(t),p(t)) \\ & \qquad\qquad \forall t \in [t_0, t_f]\,, \end{aligned}$$

- Multiphase formulation:

$$\begin{aligned} \underset{\substack{x(\cdot),u(\cdot),p(\cdot),\\ s_1,\dots,s_{n_{\text{ph}}}}}{\text{minimize}} \quad & \sum_{j=1}^{n_{\text{ph}}} \left[\int_{s_{j-1}}^{s_j} \ell_j(x(t),u(t),\lambda(t))dt + L_j(x(s_j)) \right] \\ \text{subject to} \quad & \dot{x}(t) = f_j(x(t),u(t),\lambda(t)) \\ & g_j(x(t),u(t),\lambda(t)) = 0 \\ & h_j(x(s_j^-)) = 0 \\ & x(s_j^+) = R_j(x(s_j^-)) \\ & \ \forall t \in [s_{j-1}, s_j],\ j = 1, \dots, n_{\text{ph}} \\ & s_0 = t_0, s_{n_{\text{ph}}} = t_f\,, \end{aligned}$$

Direct methods are preferred in the legged community

Solution methods

Direct multiple-shooting

- Discretized grid "shooting nodes"
- State trajectories are obtained via forward integration
- Time horizon parametrized with a series of Initial Value Problems (IVP)
- Continuity constraints
- Often used with Hybrid Dynamics with predefines contact sequences

Direct collocation

- Discretized intervals are called "finite elements"
- States approximated by polynomials
- Continuity constraints
- Collocation constraints at collocation points
- Often used with whole-body models and complementary constraints for contacts

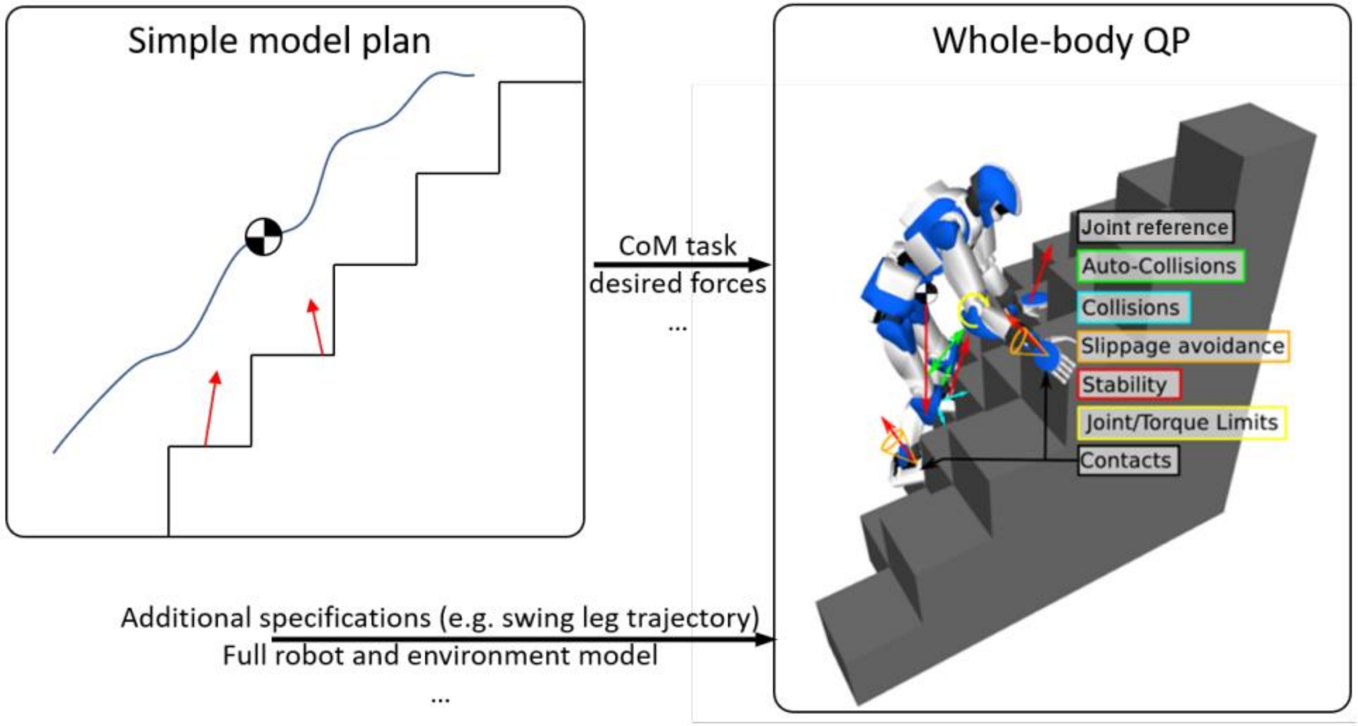
Differential Dynamic Programming

- Expresses the control inputs as linear functions of the state
- State computed by forward integration
- Requires second derivative of dynamics
- Extended to take into account constraints
- Increasingly popular for legged locomotion

REALIZING MOTION PLANS

Reactive control

- Realizing motions planned with the OCP methods, often a separate reactive control is needed to realize the plan
- The time interval in the OCP is reduced to a single instant, the OCP becomes a problem of instantaneous control
- Quadratic Programming is a popular technique



Instantaneous control

- The goal is to drive the error e_i (e.g. tracking of CoM) to zero
  - Regulation of inequality tasks
    - E.g. joint velocity limits
  - Regulation of equality tasks
    - E.g. maintain contacts
- Assumption that during the (sufficiently small) control interval Δt:
  - The state is known and constant
  - The contact is given and fixed

Quadratic Programming approach

- Example of QP for LIP:
  - Tracking reference trajectory
  - Minimize torques

$$
\begin{aligned}
\underset{\dot{\nu},\tau,\lambda}{\text{minimize}}\ & w_1\|J_G\dot{\nu} + \dot{J}_G\nu - (\ddot{p}_G^d + \ddot{e}_G^d)\|^2 + w_2\|\tau\|^2 \\
\text{subject to}\ & M\dot{\nu} + C\nu + \tau_g = S^T\tau + J_c^T\lambda \quad \text{(dynamics)} \\
& J_c\dot{\nu} = a_c \quad \text{(fixed contacts)} \\
& C\lambda \leq 0 \quad \text{(friction cones)} \\
& \ldots \text{(other top-priority constraints)} \ldots
\end{aligned}
$$

- Reactive control complements predictive control

Outlook

- Main trends:
  - Increase in complexity of dynamic models thanks to faster solvers and numerical methods
  - Increase interest and adoption of contact-implicit formulations to avoid predefined sequences
  - Increase in interest for DDP, but convergence is still challenging for complex problems
- Existing problems:
  - Theoretical gap on stability and feasibility of nonlinear problems
  - Complexity of the problem remains very large
  - Bridging with Reinforcement Learning

Paper preprint:

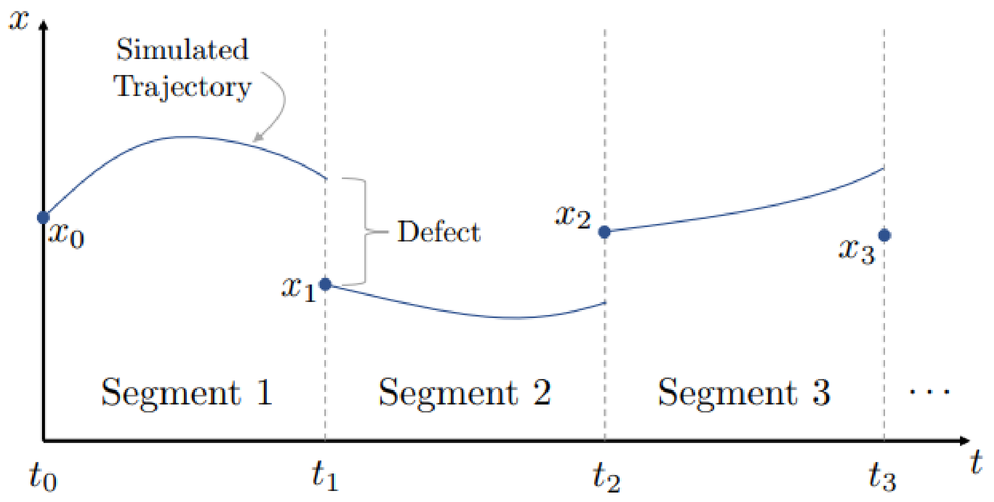
UNIVERSITY OF WATERLOO

FACULTY OF ENGINEERING

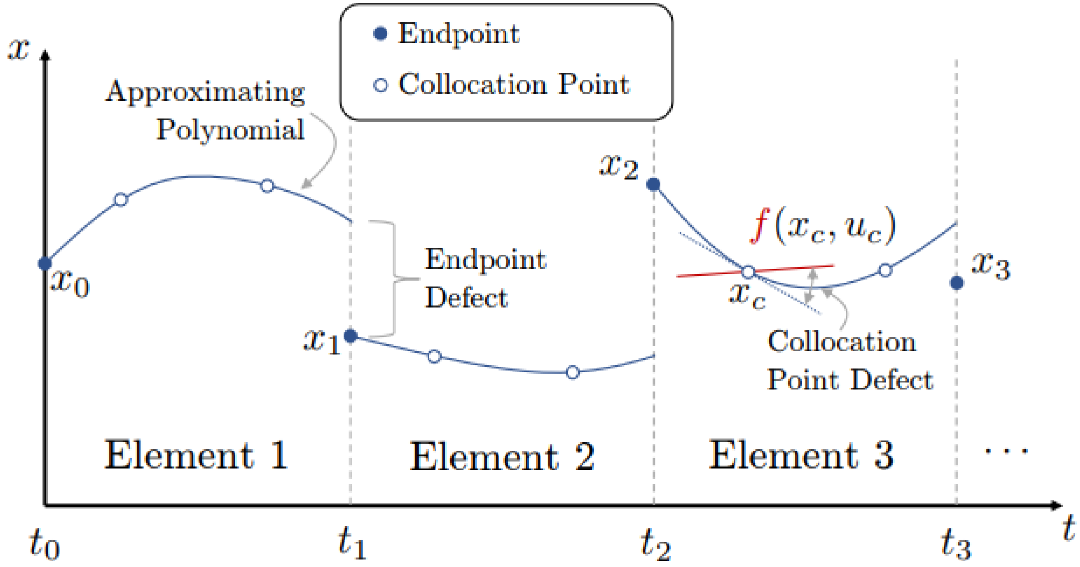
Active & Interactive Robotics Lab (A.I.R. Lab)
Department of Mechanical and Mechatronics Engineering
yue.hu@uwaterloo.ca

Direct multiple-shooting and collocation

		Supporting Theory	Computation
Global		Hamilton-Jacobi-Bellman (HJB)	Numerical PDE Solvers or Dynamic Programming
Local	***Direct*** *(discretize then optimize)*	Numerical Integration	**Direct Shooting/Collocation & NLP**
	Indirect *(optimize then discretize)*	Pontryagin Maximum Principle (PMP)	Indirect Shooting/Collocation & Root Find



Direct multiple-shooting



Direct collocation

QP formulation

Differentiating the error e twice and assuming $\dot{q} = \nu$

$$\ddot{e}_i(q, \nu, \dot{\nu}, t) = J_i(q)\dot{\nu} + \dot{J}_i(q, \nu)\nu + a_i(q, \nu, t) \qquad a_i = \frac{\partial^2 e_i}{\partial t^2}$$

To regulate e to a desired value

$$\ddot{e}_i = \ddot{e}_i^d$$

Equality linear to $\dot{\nu}$

$$J_i(q)\dot{\nu} \geqq \ddot{e}_i^d - \dot{J}_i(q, \nu)\nu - a_i(q, t)$$

If e depends on ν, a single differentiation leads to

$$A_i(q, \nu)\dot{\nu} \geqq b_i(q, \nu, t)$$

$$\frac{\partial e_i}{\partial \nu}\dot{\nu} \geqq \dot{e}_i^d - J_i(q, \nu)\nu - \frac{\partial e_i}{\partial t}(q, \nu, t)$$