Valuation of a Financial Claim Contingent on the Outcome of a Quantum Measurement

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We consider a rational agent who at time 0 enters into a financial contract for which the payout is determined by a quantum measurement at some time $T > 0$. The state of the quantum system is given by a known density matrix $\hat{p}$. How much will the agent be willing to pay at time 0 to enter into such a contract? In the case of a finite dimensional Hilbert space, each such claim is represented by an observable $\hat{X}_T$ where the eigenvalues of $\hat{X}_T$ determine the amount paid if the corresponding outcome is obtained in the measurement. We prove, under reasonable axioms, that there exists a pricing state $\hat{q}$ which is equivalent to the physical state $\hat{p}$ such that the pricing function Π_{0T} takes the form $\Pi_{0T}(\hat{X}_T) = P_{0T}\,\mathrm{tr}(\hat{q}\hat{X}_T)$ for any claim $\hat{X}_T$, where P_{0T} is the one-period discount factor. By "equivalent" we mean that $\hat{p}$ and $\hat{q}$ share the same null space: thus, for any $|\xi\rangle \in \mathcal{H}$ one has $\langle\bar{\xi}|\hat{p}|\xi\rangle = 0$ if and only if $\langle\bar{\xi}|\hat{q}|\xi\rangle = 0$. We introduce a class of optimization problems and solve for the optimal contract payout structure for a claim based on a given measurement. Then we consider the implications of the Kochen-Specker theorem in such a setting and we look at the problem of forming portfolios of such contracts. Based on work carried out in collaboration with Leandro Sànchez-Betancourt. The paper is available at ArXiv: 2305.10239.

Key words: Quantum mechanics, quantum measurement, contingent claims, discount bonds, absence of arbitrage, rate of return, density matrices, Gleason's theorem, Kochen-Specker theorem.