Observations and Inferences About the Fixed Cost in Protective Put Strategy

Abstract The Options Based Portfolio Insurance (OBPI) should avoid losses and capture gains, by the cost of a "premium", but in practice the premium cost can be too high to capture gains in the different market conditions. Based on this premise, the objective of this study is to analyze an optimized alternative to Options Based Portfolio Insurance, assuming that the probability of an Extreme Negative Event varies with time, and therefore the insurance floor should not be maintained at any cost. To achieve the objective, data were crossed between the rollover date and a fixed insurance cost on equity, using historical data from 678,546 trades carried out in the US market, which resulted in 3,700 portfolios. Data analysis was performed using descriptive and econometric statistics with simple linear regressions. The results demonstrated that the insured floor varied with the volatility implied in option prices, indicating the possibility of obtaining better return-risk ratio and lower drawdown when compared to the unprotected portfolio, showing a positive correlation between insurance cost and return-risk ratio for low costs and a negative correlation for higher costs. Furthermore, it can be indirectly inferred that the explanation for the results comes from the perception of risk by the market not representing the real risk. As with the stabilization and possible recovery of prices that occur after a significant drop, market confidence is not restored to the same proportion. That's why volatility remains at high levels, consequently, raising the insurance cost, even with lower probabilities of Extreme Negative Events.

Keywords: Options Based Portfolio Insurance; OBPI; Implied Volatility; Dynamic Floor; Fixed Insurance Cost; and Protective Put Strategy.
JEL Code: G10, G11, G14.

1. Introduction

In the first quarter of 2020, you wish you had adopted a portfolio insurance strategy. The idea of portfolio insurance as a financial product dates back to the mid-1970s, when Hayne Leland and Mark Rubinstein were looking for products that could appeal to the financial community. Based on the stock market decline in 1973 and 1974, which directly affected pension funds, Leland imagined that if there had been an insurance option available, pension funds could have continued on the market Leland and Rubinstein (1988).

Leland (1980) proposed the Option-Based Portfolio Insurance (OBPI) strategy in order to protect against losses and capture gains at the cost of a "premium". Initially, Leland (1980) classified investors seeking portfolio insurance, due to risk aversion and optimism, into two groups. The first were investors whose risk tolerance increases with wealth, faster than that of the average investor and who prioritize safety. The second, those whose expectations are more optimistic than the average, as well as institutions with managed portfolios with the expectation of above-average returns.

With investors seeking portfolio insurance, other protection strategies have emerged over the years, one of them is the Stop-Loss order. Although still used today for its simplicity, it presents the Gap [1] issue, due to lack of liquidity at the desired value, leading to possible values below the stipulated portfolio floor (Rubinstein; 1985).

In early 1980s, three US stock exchanges began trading futures contracts on market index, and one of the first studies on the use of futures contracts as portfolio insurance was done by (Figlewski and Kon; 1982). The pioneering spirit of Figlewski and Kon (1982), in relation to the strategy of selling stock index futures contracts as portfolio insurance, was criticized by Bierman Jr (1988). The author stated that this strategy requires the sale of a large amount of futures contracts and this triggers further declines in stock prices.

When the stock prices fall, large funds that insure their portfolios employing the sale of index futures contracts, make use of this tool and sell large amounts of this derivative. Institutions that practice arbitrage in the market then seize the opportunity and buy cheap futures and sell stocks, which further pushes their prices down, leading to a repeating cycle that can bring huge losses.

Also in the 1980s, a new portfolio insurance strategy was proposed: the Constant Proportion Portfolio Insurance (CPPI), a dynamic allocation strategy initially proposed by Perold (1986) and later addressed by Black and Rouhani (1989) and Black and Perold (1992). The strategy basically consists of allocating part of risky assets and the rest into risk-free assets. The percentage allocated to risk assets comes from the product of a coefficient and difference between the total amount and the floor value. The coefficient, in turn, depends on the investor's profile and periodically this portfolio must rebalanced.

One of the problems with implementing a CPPI strategy is that it doesn't immediately offset the risk when markets move in the opposite direction. As what happens in the immediate valuation of a put when the underlying asset decreases in price. In addition to presenting the gap issue, mentioned above, according to the study by Cont and Tankov (2009).

Among so many strategies, what would be the most appropriate way to secure a stock portfolio? European Put options protected from the payment

[1] A gap is a discontinuity of prices in the trading of an asset, usually occurring between the closing of the market and the following opening. For example, when selling interest increases significantly following a news or event, resulting in an opening price well below the previous day's closing price.

of dividends were considered by Rubinstein (1985) to be the ideal portfolio insurance however, unfeasible at the time. Thus, the Short Maturity Index Options Roll was pointed out by the author as the closest feasible strategy to the ideal. Recent studies address portfolio insurance with empirical tests and practical applications associated with theory.

Applying rebalancing every month and every three months Tian (1996) analyzed whether the OBPI strategy worked in practice. Using data from January 1988 to December 1993, he verified that portfolio insurance using S&P 500 put options guaranteed a value greater than the required floor of 90% or 95% in all tested cases.

However, due to high costs, the returns of the insured portfolio were negative, against a return of 13.47% of the uninsured portfolio, contrasting Leland (1980) in its statement that the OBPI would allow the investor, in addition to avoiding losses, to capture the gains at the cost of the "premium" on put option.

Because maintaining portfolio insurance in a full time fixed floor, regardless of its cost, compromises the capture of gains, as seen in Tian (1996), the "premium" on put options can cost even more than the portfolio's return, or the market index's return. In an attempt to improve the portfolio's return with OBPI strategy Zimmermann (1996) proposed the Constant Return Participating (CRP) strategy, which consists of selling calls to cover the high cost of puts, but which still significantly limits the portfolio's return.

It was evidenced in these previous studies that the high cost of fixed floor insurance is financially unfeasible. However, inspired by them, this research aimed to verify if, historically, the fixed insurance cost on equity allows the OBPI to have a return-risk ratio as good or better than the unprotected portfolio, reduce the maximum drawdown, in addition to protecting against the risk of Extreme Negative Events[2]. For this OBPI strategy optimization, the fixed daily percentage of insurance cost on equity is defined at the time of rollover, allowing the percentage of insured capital, represented by the strike-spot ratio, to vary with the implied volatility of the put options.

Two decades after Tian (1996), Welch (2016) also used historical data from S&P 500 put options to suggest that put price data, far below the reference asset price, can be used to measure the time-varying risk of Extreme Negative Events effectively and quantitatively. In empirical tests, the author found that the annual insurance cost on equity reached 6% in 2008, even with a floor of 85%.

[2]Extreme Negative Event is an event that implies very negative returns, with a very small probability of occurrence and, therefore, outside the scope of market expectations, and which has an extreme impact on an investment portfolio.

Also dealing with the variation of risk over time, the Seo and Wachter (2019) model suggests links between the probability of rare economic events, option prices, and equity risk premiums. It further suggests that option prices reflect the risk of extreme negative events across the economy and that this risk varies over time.

Following this reasoning, the logic behind this strategy is to allow the auto-adjustment of the insured percentage. The strike of the put options rises as the implied volatility decreases, that is, when the option prices are lower the floor will be closer to 100% of the value portfolio, and vice versa. This way, as the percentage amount to be spent on insurance increases, as long as there is sufficient liquidity in the options market, it is expected to observe the return-risk ratio of the portfolio rise to a peak before it declines.

2. Literature Review

The use of put options as portfolio insurance is based on studies carried out by Leland (1980); Rubinstein (1985); Chidambaran and Figlewski (1995); Tian (1996); Welch (2016); Bharadwaj and Wiggins (2003); Fung et al. (2004) that addressed the results and impact of targeted decisions on portfolios. According to Leland (1980), the author of the idea for options-based portfolio insurance (OBPI) strategy, a put option on a reference portfolio, such as a market index, is equivalent to a portfolio insurance, which allows the investor to avoid losses, but capture gains by paying the "premium" of the Put option.

Analyzing several methods of portfolio insurance, Rubinstein (1985) states that European put options protected from the payment of dividends are the ideal portfolio insurance. However, in the absence of liquidity in options with long maturities, especially over three months (Tian; 1996; Bharadwaj and Wiggins; 2003; Fung et al.; 2004), the rolling short maturity index options is the best strategy.

By observing the behavior of option returns over time, Baird (1992) and Tannous and Lee-Sing (2008) state that the decay rate increases with the proximity of expiration. Due to this phenomenon, Chidambaran and Figlewski (1995) suggests the rollover of puts one month before expiration. However, for out-the-money options there is an initial increase in returns before decaying to the maturity (Tannous and Lee-Sing; 2008). Some recent studies, mentioned below, address portfolio insurance with empirical tests and practical applications associated with theory.

Tian (1996) tested the use of market index options for hedging, using 6-year historical data from January 1988. The number of contracts was determined using the 0.5 β; 1 and 1.5 as an adjustment, while the amount spent on insurance is a consequence of the first two. The floors were set at 90% and 95% of the portfolio's value, obtaining annualized average returns, with dividends reinvested, of -0.89% and -0.7% respectively, much lower than the reference portfolio, 13.47% in the same period.

Also using historical data from S&P 500 put options, in a more recent analysis, Welch (2016) tested the strategy with a floor set at 85% of the portfolio value, with contracts sufficient to cover 100% of the portfolio's assets. The study made it clear that the average cost on equity to protect from the risk of extreme events was 1% to 2% per year in the period observed, but that this varied over time according to market uncertainty, reaching 6% in 2008 and 0.5% in 2015. Israelov (2018) used Monte Carlo simulations with the floor fixed at 95% and recurring put purchases to reach the conclusion similar to Tian (1996) and like Welch (2016), that this is an ineffective strategy in reducing the drawdown and the volatility per unit of expected return.

Like Welch (2016), Lee et al. (2010) also used the floor far below the value of the insured portfolio, but instead of using historical data from the S&P500, the author performed 1,500 stock price simulations. The study was carried out in two insurance strategies: the CPPI with a multiple of 1.5 and an initial floor of 80%; and the synthetic put with annual volatility fixed at 20% and an initial floor of 85%. The objective was to compare the return and the standard deviation between the fixed and dynamic floor portfolios, the latter being readjusted at each variation of 5%. The conclusion was that the dynamic floor works better both in protecting from downside and in generating the Sharpe ratio in the long term, but with even worse returns than those of fixed floor insurance.

Opposing Lee et al. (2010), that fixed the annual volatility at 20% for the synthetic put strategy, Seo and Wachter (2019) proposed that option prices and stock risk premiums are related to the probability of rare economic events and this is not constant, it varies over time. Wachter (2013) goes further, showing that the price/dividend or price/earnings ratio, as in Shiller (1989) Chap. 26, is inversely proportional to the probability of Extreme Negative Events.

Corroborating the idea that higher stock prices imply greater probabilities of Extreme Negative Events, especially when rising prices occur for a longer or faster period than dividend or earnings growth can keep up. In this study, the percentage of insured capital varied with the implied volatility of the put options, seeking to capture the variation in risk perception by the market. Therefore, in the optimization proposed by this study, instead of fixing the floor at 95%, 90% or 85%, as Tian (1996) or Welch (2016), the maximum daily insurance cost on equity was tested in all possibilities with positive return and beyond, different of Lee et al. (2010) that fixed the floor adjustment to every 5% changes, the floor was readjusted with each rollover.

It is important to remember that the optimization of an investment strategy is substantially affected by investor's profile. Following a portfolio insurance strategy means that the investor has to give up part of the profits in times of rising markets, when compared to an investor with less risk aversion Berkelaar et al. (2004).

3. Data and Methodology

The performance of the insurance strategy was analyzed in a fictitious index fund, which replicates the SPY - ETF, which follows the S&P 500 - with β equal to 1. The performance of the insured portfolio was evaluated compared to the uninsured portfolio. For simplicity, dividends flows were ignored.

Data on SPY options traded on the CBOE were acquired from the website https://www.barchart.com/. The entries contain all daily data that were possible to acquire and which comprises the closing prices of trades between January 04, 2016 and November 19, 2021, and contains the closing price, expiry date, exercise and closing price of the option. Note that it was not possible to acquire intraday data or transaction volume data, this led to results being obtained from closing price data.

3.1 Parameters

To support this study, there are some parameters to be defined in a portfolio insurance based on market index options. If we consider that a portfolio can be the union of two other portfolios, one insured and the other not, it is possible to treat the insured portion as a full portfolio. For portfolios that are not perfectly represented by the marked index, insuring put options on that index will provide a cross-hedge (Figlewski and Kon; 1982), which can be adjusted by the β between the portfolio to be insured and the index, in order to determine the number of contracts in the OBPI strategy, as done by Tian (1996).

Once the β adjustment is determined and disregarding the uninsured portion of the portfolio, if any, the amount spent on insurance depends directly on the minimum amount accepted by the investor, that is, the insured floor. Once the insurance cost on equity or floor is determined, the other will be a consequence. Last but not least, to determine the rollover period, necessary for long-term insurances, because as pointed out by Tian (1996); Bharadwaj and Wiggins (2003) and Fung et al. (2004), the lack of supply and liquidity as factors that restrict the use of index options for portfolio insurance, especially for options with a maturity of more than three months.

Studies such as Rubinstein (1985); Bookstaber and Langsam (1988); Choie and Novomestky (1989); Chidambaran and Figlewski (1995) and Tannous and Lee-Sing (2008) indicate the rollover of short-term put options as an alternative to long-term insurance. In order to assess the return-risk ratio, the mean return to standard deviation ratio, which will be called (δ), can be used as a representative measure of the Sharpe index Bansal et al. (2004); Lee et al. (2010). For comparison purposes, in this analysis, δ **Ratio** stands for the ratio between the Insured Portfolio δ and the Uninsured Portfolio δ will be used.

3.2 Methods

Data analysis was performed using descriptive and econometric statistics with simple linear regressions. Previously, the fixed floor portfolio insurance strategies have already been tested with simulations as in Lee et al. (2010); Pézier and Scheller (2013); Israelov (2018) and with back-tests as in Tian (1996); Welch (2016). Just as the dynamic floor has already been tested with simulations on CPPI and Synthetic Put strategies by Lee et al. (2010) and in this study back-tests of historical data were used to test the dynamic floor in OBPI with daily fixed cost.

Some considerations had to be made when choosing the method. Although the use of historical prices to calculate the returns of a strategy does not guarantee the determination of a pattern essentially representative of the market (Chidambaran and Figlewski; 1995). The simulations have certain limitations that, according to the objective of this research, make their application unfeasible, such as:

Simulations with Brownian geometric processes, as realized by Pézier and Scheller (2013), are not suitable for the purpose of this study because they use constant volatility. Just as it is not appropriate to use the Monte Carlo method applied by Boyle (1977) to derive the value of options, as it requires the assumption of risk neutrality, assuming that the returns of the underlying stock follow a log-normal distribution.

Tannous and Lee-Sing (2008) used Merton (1976) model in the simulations of asset returns over time, to analyze the expected time value decay of options. Merton (1976) model unites the geometric Brownian motion of "normal" vibrations and "abnormal" vibrations, which come from a process that produces change only when important news arrives. However, even this mixed model uses the instantaneous standard deviation of the asset's return conditioned to the non-occurrence of jumps, assuming a normal variation in volatility.

More recently, to treat portfolio optimizations with floor constraints, Sekine (2012) employs a long-term risk-sensitive criterion, introducing examples of solutions in two models, the first one having a constant market price of risk and the second in a "linear-quadratic" structure model. However, the author himself states that scale-invariance is considered a disadvantage of the long-term risk-sensitive criterion and that an alternative criterion would be desirable. Thus, since the core of the fixed cost on equity in a protective Put strategy is the auto-adjustment of the guaranteed floor, based on the implied volatility of the Put options.

The current lack of a "good enough" method, particularly to simulate the variation of implied volatility, evidenced at risk of Extreme Negative Events, restricts its application. Thus, the method chosen as the most suitable for this research, in order to represent the real world, was the statistical study of the observable past, as used by Tian (1996) when back-testing historical data.

Using the last six years of available data, to investigate the optimization of the results in relation to the uncovered portfolio, the daily fixed percentage of insurance cost on equity at the time of rollover was defined, in order to allow the percentage of insured capital, represented by the strike-spot ratio, to vary with the implied volatility of the Put options.

3.3 Backtesting

Historical data on the closing prices of 678,546 trades were used, crossing the data between the rollover date and the fixed percentage of insurance cost on equity, obtaining the daily returns of 3,700 portfolios at the end of the tested period. The criteria adopted due to price and trading gaps are described in Table 1.

For comparison purposes, additional tests were carried out with the fixed floor strategy, obtaining the daily returns of 7,000 portfolios at the end of the tested period. The criteria adopted due to price and trading gaps are described in Table 2:

4. Results

Assuming that the probability of an Extreme Negative Event varies with time, it is expected that it is possible to optimize the return-risk ratio from the variation of the percentage value of the portfolio to be spent on insurance. This causes the insured floor to vary with the volatility implied in the prices of the options, in order to enable a lower drawdown and better returns than the unprotected portfolio.

In Figure 1 the blue line represents the Maximum Daily Insurance Cost on Equity (x) versus the δ **Ratio** (y), and the red line its simple linear regression,

Table 1

Backtesting Criteria in Fixed Cost Strategy	
1.	The purchased Put was the one in which the ratio between the price and the total amount invested, adjusted for interest compounded by the daily rate, considering the business days until expiration, was closest to and below the maximum daily cost, with variations of 0.0005% between 0.0005% and 0.0500%.
2.	If no options are available below the maximum daily budget, the cheapest available has been purchased.
3.	Every N days before expiry, the option was rolled to the next expiry.
4.	If there was no negotiation on the day scheduled for the purchase or sale, the purchase or sale was considered to be the next day that that Put was traded.
5.	To avoid periods without coverage, when necessary the sale day was delayed to the next purchase day or later.
6.	To prevent the portfolio from being uncovered for some period, tests were started with purchases from 35 business days before expiration.
7.	To calculate the portfolio, the days that the purchased Put was not traded remained as the last recorded value.
8.	If the Put has remained in the portfolio until the expiration date and this Put has not been traded on that day, its intrinsic value was computed.
9.	When a Put was purchased, the value was taken from the portfolio.
10.	When a Put was sold, the value was added to the portfolio.
11.	If, due to lack of negotiation, the sale occurred after the purchase of the following maturity, the sale amount remained in cash until the next Put was purchased.
12.	Only Puts with a strike less than or equal to the closing of the underlying asset could be purchased.
13.	To avoid distortions, purchases with up to 71 business days to maturity were tested, due to the options expiring on April 18, 2019 having only started on January 7, 2019, 71 business days before expiration.

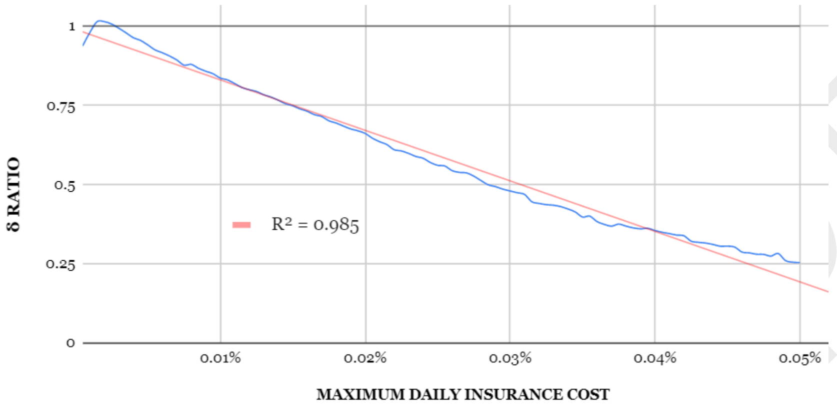
Table 2

Backtesting Criteria in Fixed Floor Strategy	
1'.	The purchased Put was the one in which the floor was immediately below the tested floor, with variations of 1% between 1% and 100%.
2'.	If there are no options available immediately below the tested floor, the one with floor immediately above the tested floor was purchased.
3'. to 13'.	The same of 3. to 13. in Fixed Cost Strategy.

$y = -1594\ x + 0.99$, $R^2 = 0.985$. The Maximum Daily Insurance Cost varies between 0.005% and 0.05% with purchases between 35 and 52 days to expiration. The mean standard deviation of the δ **Ratio**, represented by blue line, is 0.1343. A strong negative correlation between these variables is evidenced, -0.9926, when observing the entire daily cost window, up to 0.05%.

For comparison purposes, in Figure 2, on the primary axis is represented The Average Insurance Cost in the Fixed Floor Strategy. Here, the blue

Figure 1
δ Ratio Vs Maximum Daily Cost in Fixed Cost Strategy



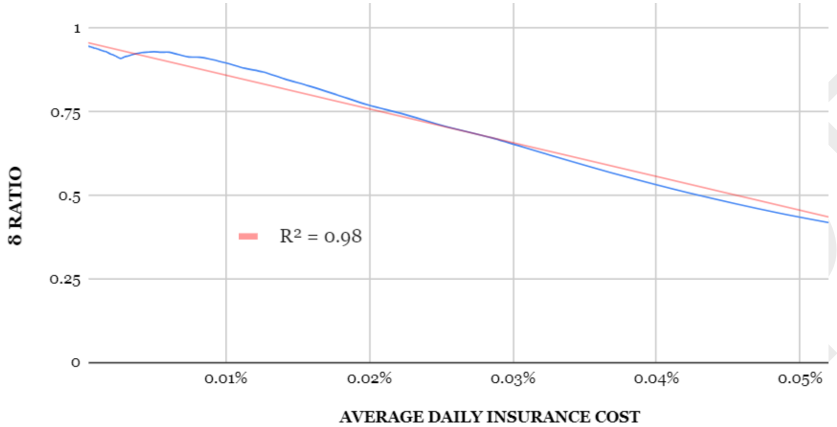
line represents the Average Insurance Cost on Equity (x) versus the δ **Ratio** (y), and the red line its simple linear regression, y = -1010 x + 0.961, R^2 = 0.980. With purchases between the same 35 and 52 days to expiration, an even stronger negative correlation, -0.9899, between cost and δ **Ratio** is evidenced. The mean standard deviation of the δ **Ratio**, represented by blue line, is 0.0921. However, the slope of the simple linear regression, $\beta 1$ = -1011 with R^2 = 0.980, is almost two-thirds that presented in the Fixed Cost Strategy, $\beta 1$ = -1594 with R^2 = 0.985. That is, in the Fixed Cost Strategy, the increase in cost influences the δ **Ratio** much more negatively than in the Fixed Floor Strategy.

Figure 3 is a extract from Figure 1, in wish the Maximum Daily Insurance Cost varies only until 0.0020% with purchases between 35 and 52 days to expiration. Here, the blue line represents the Maximum Daily Insurance Cost on Equity (x) versus the δ **Ratio** (y), and the red line its simple linear regression, y = 5302 x + 0.92, R^2 = 0.879. The mean standard deviation of the δ **Ratio**, represented by blue line, is 0.0967. It is possible to observe the strong positive correlation, 0.9375, between these variables in this data window.

From Figures 1 and 3 it can be observed that there was a positive correlation between cost and δ **Ratio** for lower costs and a negative correlation for higher costs. That is, the initial increase in the cost of insurance improves the risk-return ratio, but beyond a certain cost range, the opposite is true.

In Figure 4 the cost varies between 0.005% and 0.05% with purchases between 53 and 71 days to expiration. Here, the blue line represents the Maximum Daily Insurance Cost on Equity (x) versus the δ **Ratio** (y), and the

Figure 2
δ Ratio Vs Average Daily Cost in Fixed Floor Strategy



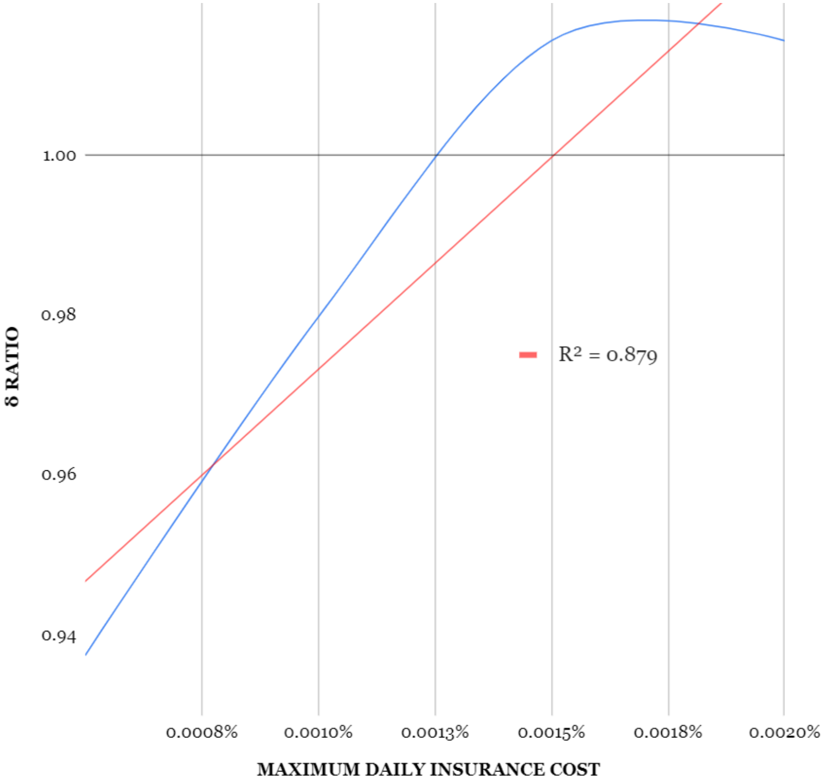
red line its simple linear regression, y = -1848 x + 0.96, $R^2 = 0.978$.

A possible explanation to almost constant declination on δ **Ratio** is the lack of supply and liquidity. Factors that restrict the use of index options for portfolio insurance, especially for options with a maturity of more than three months, as pointed out by Tian (1996); Bharadwaj and Wiggins (2003) and Fung et al. (2004), as a determinant factor to the rollover period at long-term insurances. Adopting this assumption as true, to avoid distortions, let's observe the data with purchases between 35 and 52 business days to maturity in Figure 5.

In Figure 5, comparing the behavior of δ **Ratio** in both strategies, fixed floor (red line) and fixed cost (blue line), at the first, it almost only decreases as the insurance floor increases, it is also noted that the rate of decline increases considerably after the 79% floor. At the second, before it declines from the 71.56% floor, the δ **Ratio** rises as the insurance floor increases. The lines start almost together at the floor of 60% and 59.82% respectively, that is the lowest average floor available in the fixed cost strategy, with a maximum daily cost of 0.0005%. The lines intersect when the floor is at 85.45%, from which point the fixed floor strategy is more worthwhile than the fixed cost strategy,although from that point on it is also not advantageous in relation to the uninsured portfolio. This finding is in line with the study by Welch (2016) which analyzes the fixed cost at 85% and refers to the cost of insurance with this floor as a quantitative indicator of the risk of Extreme Negative Events.

Welch (2016) suggests that the average cost of protection, with an 85%

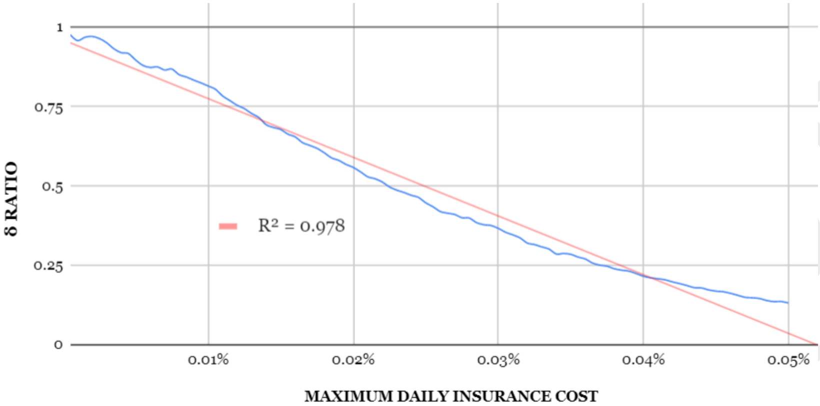
Figure 3
Maximum Daily Cost until 0.0035% in Fixed Cost Strategy



fixed floor, using S&P 500 Put options as portfolio insurance, for hedging the risk of Extreme Negative Events was 1% (0.0039% per day) to 2% (0.0079% per day) per year in the period observed. It also shown that the cost of protection varied over time according to market uncertainty, reaching 6% (0.0231% per day) in 2008 and 0.5% (0.0020% per day) in 2015.

At the cross section shown in Figure 6, the mean standard deviation of the Average δ **Ratio**, represented by blue line, is 0.2532. In this, the average δ **Ratio** grows as the time to maturity, on the rollover date, increases, reaching a maximum value within 45 business days of maturity. Its possible to observe that the Average δ **Ratio** drops considerably from this point forward, considering that a month usually has 21 business days, the maximum result occurred approximately two months to maturity, with rollover at the end of

Figure 4
Purchases from 53 Business days in Fixed Cost Strategy



the first month.

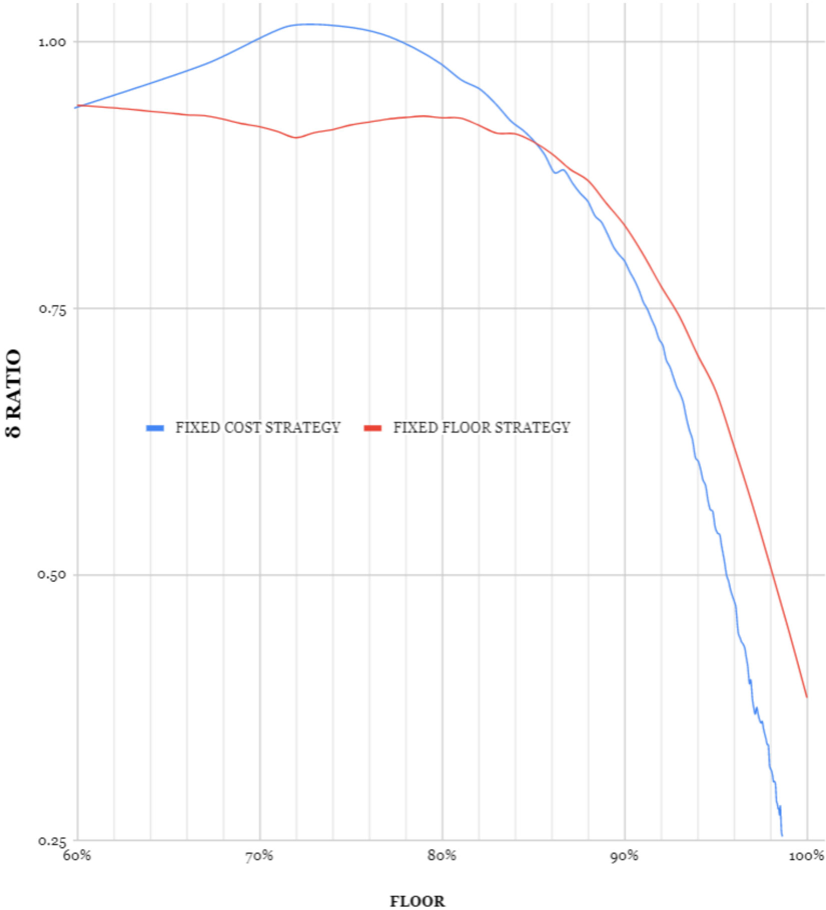
This corroborates the findings of Chidambaran and Figlewski (1995) who suggest buying protective Puts two months before expiration and rolling one month before expiration, to avoid the sharp loss in the last month to expiration, when the fastest decline occurs. Since, according to Baird (1992) and Tannous and Lee-Sing (2008), the decay rate increases over time. As the average return in Figure 6 covers all the results obtained, including: at and out-the-money options, similar to Tannous and Lee-Sing (2008) it is also inferred that there is an initial increase on returns before it declines.

Table 3 contains the extract of all δ **Ratios** for 3,700, obtained from crossing the Maximum Daily Cost with the number of Business Days to Expiration in the Rollover in the Fixed Cost Strategy. Values in black show insured portfolios that had better return-risk ratios than uninsured portfolios.

From now on, the analysis will be based on the best relative result, δ **Ratio** of 1.2511, the yellow one in Table 3 - Maximum Daily Cost of 0.0020% and Rolls at 46 Business Day. This result showed a return-risk ratio 25.11% higher than the uninsured portfolio. The Average Daily Return (ADR) of the Insured Portfolio with the Fixed Cost Strategy was 0.07%, while that of the uninsured portfolio was 0.06%, in the period between February 9, 2016 and February 19 November 2021, referring to the first Put purchase and the last Put sale respectively.

In Figure 7, the blue line represents the return on the insured portfolio, and the red line that of the uninsured SPY in a logarithmic scale. The yellow line represents the floor, that is, the highest strike value of the Puts in the

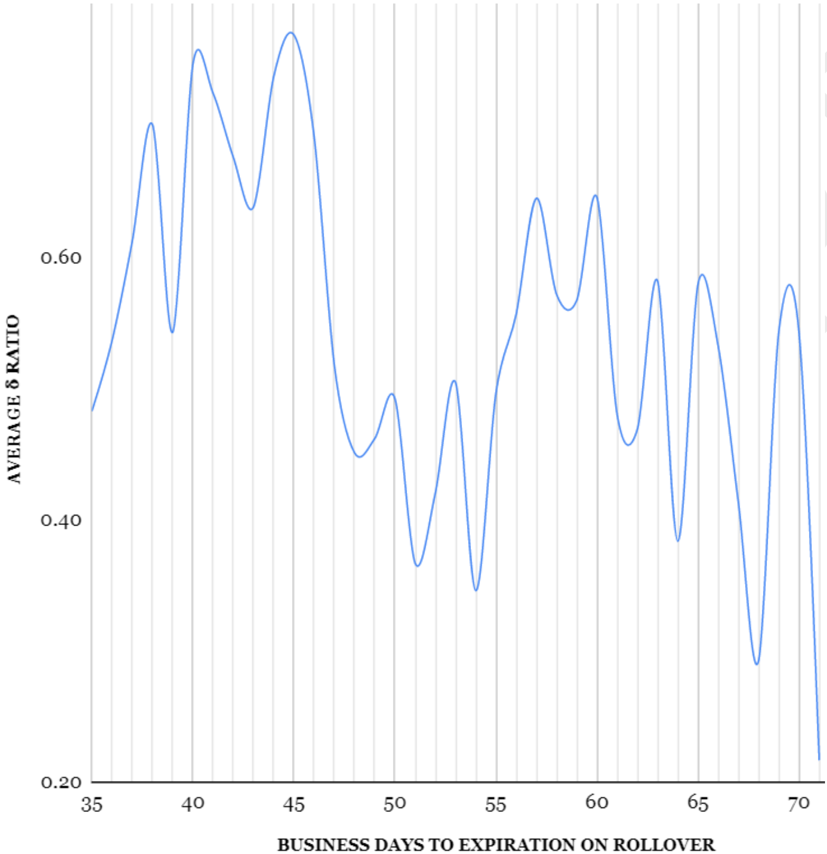
Figure 5
δ Ratio Vs Floor in Fixed Cost and Fixed Floor Strateg



portfolio each day.

Similar to figure 7, in figure 8, the blue line represents the return on the insured portfolio, the red line that of the uninsured SPY in a logarithmic scale, and the yellow line represents the floor, however, only the Extreme Negative Event is observed here. In fact, in the first quarter of 2020, you wish you had adopted a portfolio insurance strategy. On the day after February 7, 2020, as a result of the news published at the time, the price of SPX put options began to overvalue, reflecting the implied volatility priced by the market. The

Figure 6
Average δ Ratio Vs Business Days to Expiration on Rollover in Fixed Cost Strategy



fund took place on March 23, 2020, when the market started its recovery. In this period, the insured portfolio varied -7.95% while the SPY -32.89%, demonstrating that the dynamic floor of the fixed cost strategy can work well in protecting against Extreme Negative Events.

As shown in Figure 9, the insured portfolio had a lower maximum Drawdown than the uninsured portfolio. Here, the blue line represents the Drawdown from the Insured Portfolio, which presented a maximum value of -19.15%, and the red line represents the uninsured SPY, which presented a maximum value of -34.1%.

Table 3
Extract of all δ Ratio from 3,700 portfolios

	δ Ratio in Fixed Cost Strategy													
	Business Days to Expiration on Rollover													
Maximum Daily Cost	35	36	37	...	43	44	45	46	47	...	69	70	71
0.0005%	0.7719	0.8282	0.9391	...	0.9236	0.9643	1.0067	1.0901	0.9224	...	1.1359	1.1366	0.7883
0.0010%	0.7927	0.8483	0.9584	...	0.9850	1.0501	1.0931	1.1645	0.9830	...	1.1204	1.1246	0.7750
0.0015%	0.8260	0.8788	0.9874	...	1.0172	1.1115	1.1576	1.2111	1.0071	...	1.1345	1.1351	0.7609
0.0020%	0.8243	0.8778	0.9842	...	1.0246	1.1281	1.1751	**1.2511**	1.0098	...	1.1263	1.1294	0.7583
0.0025%	0.8218	0.8739	0.9808	...	1.0048	1.1233	1.1686	1.2453	0.9967	...	1.1086	1.1211	0.7512
0.0030%	0.8154	0.8660	0.9748	...	1.0038	1.1100	1.1558	1.2292	0.9723	...	1.0892	1.1232	0.7348
0.0035%	0.8019	0.8500	0.9597	...	0.9945	1.0946	1.1413	1.2130	0.9446	...	1.0560	1.0950	0.7187
0.0040%	0.7897	0.8381	0.9439	...	0.9780	1.0856	1.1284	1.1942	0.9266	...	1.0280	1.0844	0.7114
0.0045%	0.7776	0.8250	0.9370	...	0.9673	1.0668	1.1137	1.1859	1.0175	...	1.0873	1.0681	0.6980
0.0050%	0.7646	0.8101	0.9309	...	0.9670	1.0554	1.0976	1.0513	1.0063	...	1.0062	1.0572	0.6773
0.0055%	0.7498	0.7944	0.9156	...	0.9499	1.0418	1.0845	1.0386	0.9925	...	0.9639	1.0299	0.6662
0.0060%	0.7410	0.7857	0.9162	...	0.9376	1.0267	1.0700	1.0377	0.9859	...	0.9373	1.0308	0.6594
0.0065%	0.7337	0.7773	0.9042	...	0.9267	1.0146	1.0598	1.0511	0.9796	...	0.9177	1.0186	0.6439
0.0070%	0.7233	0.7684	0.8934	...	0.9105	1.0006	1.0405	1.0370	0.9749	...	0.9134	1.0076	0.6248
0.0075%	0.7094	0.7520	0.8770	...	0.9050	0.9909	1.0331	1.0250	0.8238	...	0.9704	0.9978	0.6042
0.0080%	0.7016	0.7418	0.8652	...	0.9018	0.9949	1.0368	1.1194	0.8247	...	0.9580	1.0019	0.5925
0.0085%	0.6947	0.7321	0.8562	...	0.8885	0.9879	1.0248	1.0175	0.8247	...	0.9413	0.9838	0.5617
0.0090%	0.6821	0.7207	0.8476	...	0.8771	0.9788	1.0188	1.0077	0.9566	...	0.9167	0.9502	0.5367
0.0095%	0.6735	0.7156	0.8378	...	0.8701	0.9731	1.0160	0.9946	0.9361	...	0.9644	0.9443	0.5561
0.0100%	0.6601	0.7021	0.8199	...	0.8572	0.9639	1.0050	0.9927	0.7951	...	0.9555	0.9468	0.5652
...	...	...	...	...	...	...	...	...	...	...	...	...	...
0.0485%	0.2541	0.3213	0.3453	...	0.3896	0.4415	0.4693	0.2985	0.1398	...	0.1224	0.1279	-0.1459
0.0490%	0.2424	0.3213	0.3383	...	0.3659	0.4315	0.4674	0.2855	0.1341	...	0.1200	0.1248	-0.1491
0.0495%	0.2395	0.3016	0.3401	...	0.3568	0.4236	0.4638	0.2826	0.1331	...	0.1186	0.1248	-0.1515
0.0500%	0.2406	0.3047	0.3290	...	0.3646	0.4218	0.4601	0.2831	0.1472	...	0.1011	0.1182	-0.1684

In Figure 10, the yellow line represents the daily Floor Variation in relation to 100% of SPY, the black line represents the cumulative average, which fluctuates around 75% of the total value of the portfolio. The sharp rise in the floor at March 23, 2020 is due to the drop in the SPY price, during an Extreme Negative Event, not a Puts at or in-the-money rollover. The insured floor is directly proportional to the cost and this, in turn, is directly proportional to the implied volatility in Put prices. By establishing the maximum cost, the insured floor followed the variation of market volatility, as proposed, adjusting to the market's perception of risk.

In figure 10, it is observed that months before the drop in SPY price, the floor was around 80%, and soon after the beginning of the recovery, the floor was around 60% for months, demonstrating that the perception of risk by the market do not represent the real risk of an Extreme Negative Event and that market confidence was not restored to the same proportion as prices recovered.

At the top of Figure 11, the blue line represents the return of the SPY in a

Table 4
Descriptive statistics of the effective returns of the selected Insured Portfolio and SPY

Statistics of Returns of Portfolios: February 9, 2016 to November 19, 2021		
Coefficients	Insured Portfolio	SPY
Accumulated Returns (%)	196.26%	152.87%
ADR - Average Daily Return (%)	0.07%	0.06%
STD - Standard Deviation (%)	1.06%	1.13%
δ (ADR / STD)	0.0660	0.0531
Maximum drawdown (%)	-19.15%	-34.10%
Average Floor (%)	74.30%	0.00%
δ Ratio	1.2511	1.0000
Business Days to Expiration on Rollover	46	-
Maximum Daily Insurance Cost (%)	0.0020%	0.0000%
Equivalent Monthly Cost (%)	0.04%	0.00%
Equivalent Annual Cost (%)	0.51%	0.00%

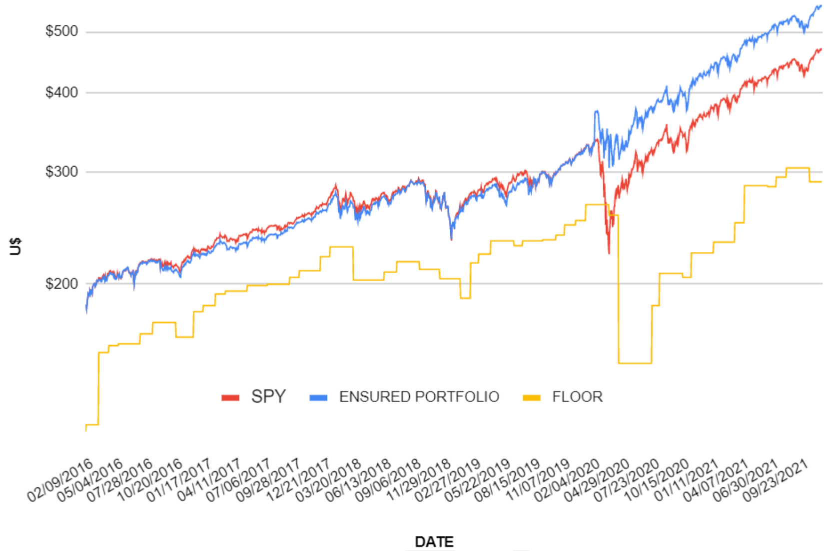
logarithmic scale. At the bottom, the red line represents the drawdown of the SPY, and the yellow line the daily variation of the cushion, which in turn, is the complement of the floor in relation to the portfolio.

The cushion variation was used instead of the floor variation in order to better illustrate graphically. For the same reason, the graph is sectioned in 4: section 1 from October 24, 2019 to February 19, 2020; section 2 from February 19, 2020 to March 23, 2020; section 3 from March 23, 2020 to August 24, 2020; e section 4 from August 24, 2020 to September 07, 2021. These periods were chosen because they more effectively reflect what is wanted to present.

Analyzing Figure 11, we have: in section 1, during the upward movement of SPY, the cushion remains around -20%. In section 2, the drop in SPY price, also presented in the detachment of the drawdown line, makes the portfolio Puts go into the money, as expected. In section 3, prices rise again until the drawdown returns to zero, however, the cushion remains around -50%. In section 4, despite the rising line in the SPY price, since the beginning of section 3, and the drawdown having returned to zero, with prices at levels above the maximum prior to the fall in section 2, the cushion remains around of -40%.

It is possible to infer that high price movements lead to an increase in market confidence, reducing fluctuations (section 1). Wachter (2013) shows that the price/dividend or price/earnings ratio, as in Shiller (1989) Chap. 26, is inversely proportional to the probability of Extreme Negative Events. Cor-

Figure 7
Returns and Floor.



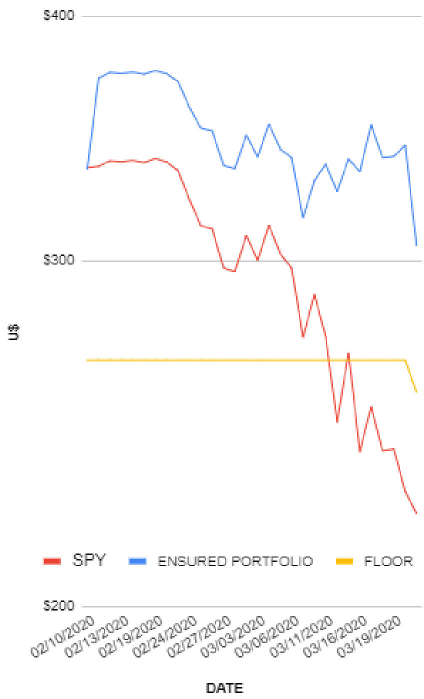
roborating the idea that higher stock prices imply greater probabilities of Extreme Negative Events, especially when rising prices occur for a longer or faster period than dividend or earnings growth can keep up.

While moments of uncertainty lead to an increase in the selling force, that pressures the market, leading to the devaluation of assets (section 2). Consequently, the perception of risk becomes accentuated, increasing volatility, and with it, the price of insurance (section 3). It is also observed that the perception of risk by the market remains even after the stabilization of prices, which occurs soon after the fall and that slowly decreases again, with the subsequent increase in prices (section 4).

The risk aversion has a substantial effect on the optimization of an investment strategy. To maintain wealth above a reference point in the planning horizon, an investor, that the utility function curve denotes risk aversion, must follow a portfolio insurance strategy. Therefore, this investor needs to give up part of the profits in times of bull market, when compared to an investor without risk aversion (Berkelaar et al.; 2004).

And this is verified in the back-tests carried out here, corroborating the idea that the relative profit of the strategy proposed in this study does not necessarily show a distortion in prices, but rather shows that it is possible for

Figure 8
Extreme Negative Event.



the loss-averse investor to adjust his utility function to the implied volatility of the Put option prices.

As seen earlier, in general terms, the proposed model self-adjusts as prices vary. It makes the insurance closer to the total value of the portfolio as it increases in value, and moves away as the quotation decreases, keeping the cost fixed in percentage terms on the portfolio's value. At the same time, the market adjustment ensures 100% of the portfolio is covered.

5. Conclusions

This study sought to analyze an optimized alternative to Option-Based Portfolio Insurance, assuming that in practice, the premium cost can be too high to capture gains in different market conditions (Tian; 1996); the cost of

Figure 9
Drawdown.

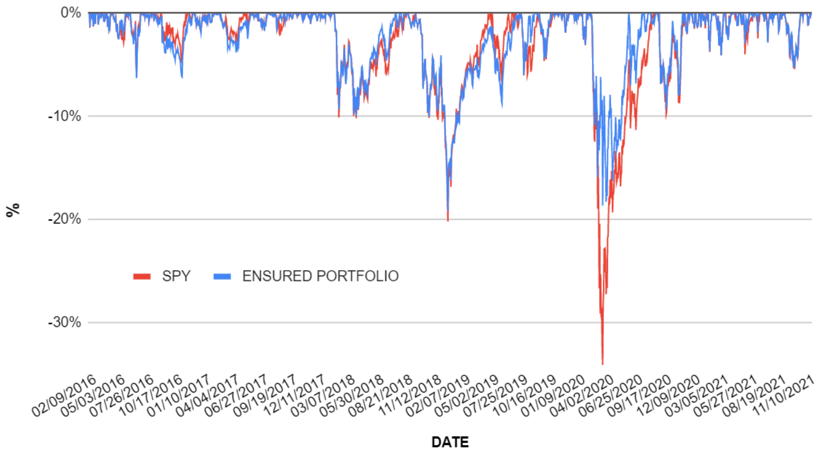
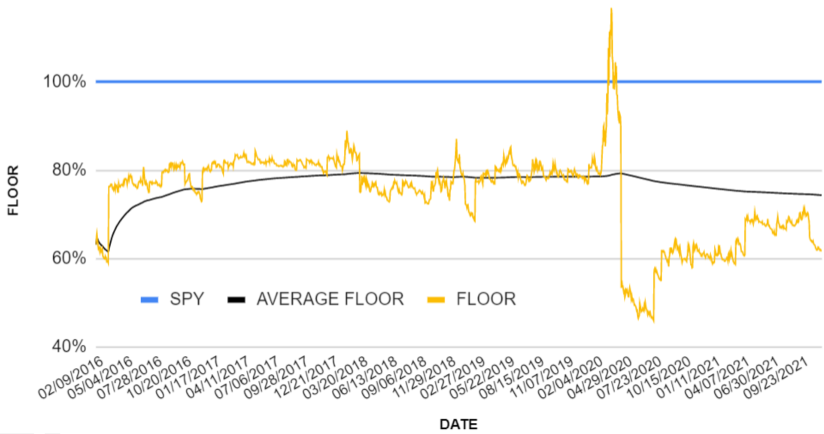
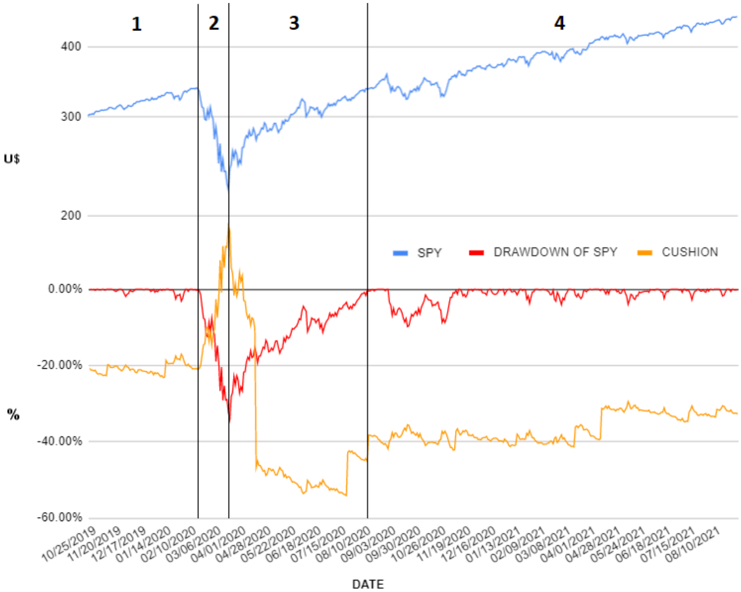


Figure 10
Floor Variation.



protection varies over time according to market uncertainty (Welch; 2016); the probability of an Extreme Negative Event varies with time (Wachter; 2013); price/earnings is inversely proportional to the probability of extreme negative events (Wachter; 2013); and the cost of insurance is directly proportional to volatility and volatility increases during negative events and not

Figure 11
Returns, Cushion Variation and Drawdown.



before them and the insurance floor should not be maintained at any cost.

Using 3,700 portfolios from the intersection of historical data on the closing price of 678,546 trades, the possibility of portfolio insurance was tested in the period between April 1, 2016 and August 28, 2021, using the closing price, expiration date, strike and closing price of the option.

Analyzing the data through descriptive and econometric statistics, with simple linear regressions, it was observed that:

1 - The portfolio insured with the fixed cost strategy had a higher return-risk ratio than the portfolio insured with a fixed floor and the portfolio without insurance;

2 - The insured portfolio had a lower maximum drawdown than the uninsured portfolio;

3 - OBPI optimization with a fixed floor protects the portfolio against extreme negative events;

4 - Lower costs have a positive correlation with the return-risk ratio, which is not observed with higher costs;

5 - Buying protective Puts approximately two months before expiration and rolling over approximately one month before expiration presents a better

risk-return ratio;

6 - As proposed, when setting the maximum cost, the insured floor followed the variation of market volatility, adjusting to the market's perception of risk, which, in turn, has been shown not to represent the real risk of an Extreme Negative Event; and

7 - After a significant drop, the cost of insurance remained at high levels, even with lower probabilities of Extreme Negative Events, demonstrating that market confidence was not restored to the same proportion as prices recovered.

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A. Additional tables and figures

This appendix has an additional table that contains the Descriptive statistics of the effective returns of the selected Insured Portfolio and SPY, similar to Table 4 but containing the results year by year and accumulated annually in the tested period. Values in bold highlight the best results comparatively.

It is observed that in years of low volatility, the cost of insurance is relatively low, and in years of greater volatility, the purchase of insurance brings considerably higher returns to the insured portfolio, in relation to the market return.

Table A1
Complete Descriptive Statistics of the Effective Returns of the Selected Insured Portfolio and SPY

Statistics of Returns of Portfolios					
		Insured Portfolio		SPY	
Coefficients	Year	By Year	Accumulated	By Year	Accumulated
Accumulated Returns (%)	2016	18.72%		**20.55%**	
	2017	17.69%	40.78%	**18.48%**	**43.91%**
	2018	**-4.85%**	**34.91%**	-7.01%	34.78%
	2019	28.26%	73.21%	**28.65%**	**73.57%**
	2020	**33.94%**	**134.10%**	15.09%	101.63%
	2021	**28.27%**	**196.26%**	27.14%	152.87%
ADR - Average Daily Return (%)	2016	0.0756%		**0.0824%**	
	2017	**0.0649%**	0.0716%	**0.0676%**	**0.0762%**
	2018	**-0.0198%**	**0.0411%**	**-0.0290%**	0.0409%
	2019	0.0988%	0.0560%	**0.1000%**	**0.0562%**
	2020	**0.1156%**	**0.0690%**	**0.0556%**	0.0568%
	2021	**0.1112%**	**0.0745%**	**0.1073%**	0.0636%
STD - Standard Deviation (%)	2016	**0.601%**		0.602%	
	2017	0.10%	0.568%	**0.09%**	**0.567%**
	2018	**0.373%**	0.134583%	**0.374%**	**0.134576%**
	2019	**2.34%**	**0.70%**	**2.35%**	0.72%
	2020	**0.41%**	**0.89%**	**0.43%**	0.90%
	2021	**0.610%**	**0.06%**	**0.613%**	0.07%
δ (ADR / STD)	2016	0.1258		**0.1368**	
	2017	**0.6380**	0.1261	**0.7910**	**0.1343**
	2018	**-0.0531**	**0.3053**	**-0.0775**	0.3043
	2019	**0.0423**	**0.0800**	**0.0426**	0.0778
	2020	**0.2794**	**0.0777**	**0.1300**	0.0630
	2021	**0.1822**	**1.1706**	**0.1748**	0.9727
Maximum drawdown (%)	2016	-6.28%		**-6.01%**	
	2017	-3.78%	-6.28%	**-3.03%**	**-6.01%**
	2018	**-19.15%**		-20.18%	
	2019	**-7.51%**	**-19.15%**	-6.62%	-20.18%
	2020	**-18.64%**	**-19.15%**	-34.10%	
	2021	**-5.31%**	**-19.15%**	-5.42%	-34.10%
Average Floor (%)	2016	**75.92%**		0.00%	
	2017	**81.61%**	**78.90%**		
	2018	**77.59%**	**78.45%**		
	2019	**78.65%**	**78.50%**		
	2020	**65.98%**	**75.94%**		
	2021	**65.29%**	**74.30%**		
δ Ratio	2016	0.9197		**1.0000**	
	2017	0.8065	0.9383	**1.0000**	
	2018	0.6850	**1.0033**	**1.0000**	1.0000
	2019	0.9928	**1.0281**	**1.0000**	1.0000
	2020	**2.1483**	**1.2323**	1.0000	
	2021	**1.0420**	**1.2035**	1.0000	
Business Days to Expiration on Rollover		46		-	
Maximum Daily Insurance Cost (%)		0.0020%		0.0000%	
Equivalent Monthly Cost (%)		0.04%		0.00%	
Equivalent Annual Cost (%)		0.51%		0.00%	

Each year refers to the following periods: (2016) from February 9th to December 30th; (2017) from January 3 to December 29; (2018) from January 2nd to December 31st; (2019) from January 2nd to December 31st; (2020) from January 2nd to December 31st; and (2021) from January 4th to November 19th.