Yield Curve Modeling and Infinite Dimensional Stochastic Volatility Process

Zeyu Cao, Raphael Douady

October 3, 2023

Abstract

We consider an infinite dimensional generalization of the SABR yield curve model, where both the yield curve and its volatility operator are modeled as solutions of some Hilbert space-valued stochastic differential equations. In particular, the volatility operator is modeled as a stochastic process in the space of Hilbert-Schmidt operators. We show that, if we add to the classical risk-neutral drift, a term of the form $D(x)\frac{\partial^2}{\partial x^2}$, which imposes mean-reversion to the second derivative of the yield curve, forcing it to be more regular, then a mild solution exists under certain technical conditions. With additional regularity conditions on the volatility process, we can show that this mild solution is also a strong solution. The primary goal is to handle complex dependencies of the stochastic volatility operator process on the underlying interest rate such as SABR-type models.

To achieve this goal, we propose a quite flexible model for stochastic operator-valued volatility processes which only requests that the square-norm of the interest rate dependent volatility operator process is bounded by any bounded square-norm of the underlying interest rate in the sense of expectation. The usual SABR model can be easily generalised to infinite dimensions, and regarded as a special case of our proposed model. In particular, we can show that when the parameter β is between 0 and 0.5, the mild solution exists globally while, in general, it only exists locally.

Our proof is based on Leray-Schauder fixed point theorem and on some priori inequalities on the stochastic operator processes we construct. Further on, we relaxed the assumptions and studied the regularity of the solutions to some specific volatility models in Hilbert spaces.

In the poster, we will

- Explain the problem of finding a proper stochastic operator-valued volatility process;
- State our proposed general model and show how the generalised SABR model can be regarded as a special case;
- Show the major steps of the proof of the existence of mild or weak solutions of our model;
- Discuss some further directions and on-going work.

This is a joint work with Prof. Raphael Douady.