Stockpiling and Price Index for Storable Goods

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Very very preliminary, comments welcome

Many goods are storable. This matters.

- Models usually assume perishable goods and neglect intertemporal substitution.
- Storable goods
  - perishable goods; durable goods
  - Even vegetable and milk can be stored.
- The key: a discrepancy between purchase and consumption
  - Household inventory
  - Temporary sales often increase purchase more than consumption.
    - Measurement of price elasticity
    - Effective price? Integration of sales and regular prices.
  - Price index
    - Chain drift (as an example of the importance of goods storability)
    - Measurement of the Cost of Living Index (COLI)

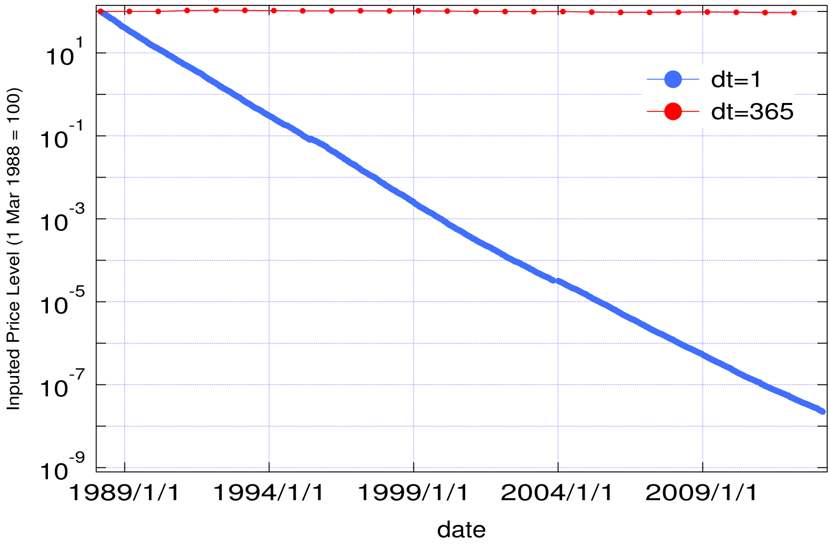
Chain Drift

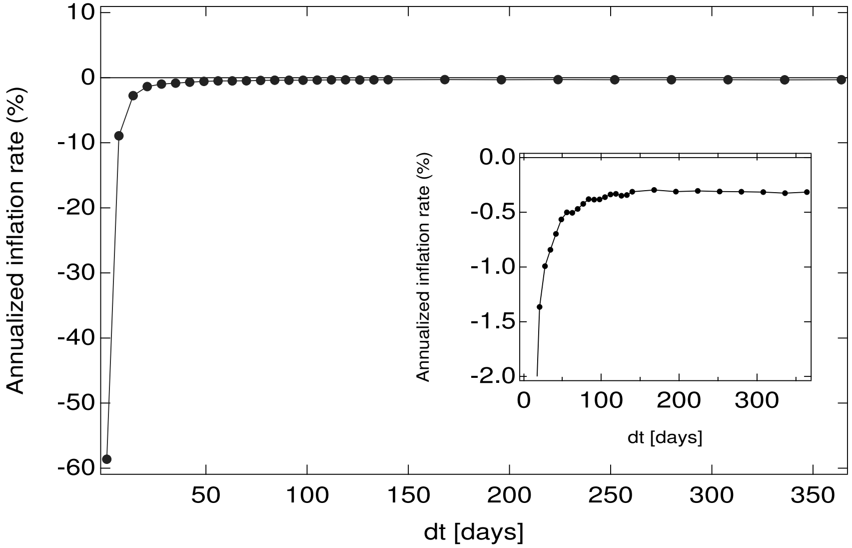
- It is known that the Törnqvist index is a good approximation of the COLI (superlative),
  - Changes in the chained price indexes based on Törnqvist are defined as

$$\pi_t^{C,T} = \sum_{k \in K_{t-1} \cap K_t} \frac{W_{t-1}^k(K_{t-1} \cap K_t) + W_t^k(K_{t-1} \cap K_t)}{2} \log\left(\frac{p_t^k}{p_{t-1}^k}\right), \tag{1}$$

    where the weight share $W_t^k(K_{t-1} \cap K_t) \equiv p_t^k x_t^k / \sum_{k' \in K_{t-1} \cap K_t} p_t^{k'} x_t^{k'}$.

- but, when we apply this to Japan's scanner daily data, ...


Inputed Price Level (1 Mar 1988 = 100)
10^1
10^-1
10^-3
10^-5
10^-7
10^-9
1989/1/1
1994/1/1
1999/1/1
2004/1/1
2009/1/1
date
dt=1
dt=365


Annualized inflation rate (%)
10
0
-10
-20
-30
-40
-50
-60
50
100
150
200
250
300
350
dt [days]
Annualized inflation rate (%)
0.0
-0.5
-1.0
-1.5
-2.0
0
100
200
300
dt [days]

Storability matters

- Chain drift
- Huge stockpiling at sales
  - Sales account for 20 to 30% of total revenues.
  - Big demand increase and subsequent drop when consumption tax rate rises
- Measurement of price elasticity
- Consumption unobservable from purchase data

What We Do

- Report stylized facts associated with storable goods
  - retailer and home scanner data
- Construct a quasi dynamic model for storable goods
  - incorporate stockpiling behavior by households
  - explain the facts
- Infer consumption/inventory
  - aim to propose the method to calculate the COLI
  - aim to see relations between stockpiling and business cycles/trends

What We Find

- Evidence of household inventory, consistent with theory
- Huge bias in the chained price index, consistent with theory
  - Caused mainly by stockpiling at sales
- Infer consumption/inventory
  - Bias is mitigated by consumption-based index, but not perfectly
  - More fundamentally, asymmetry due to the non-negative constraint of inventory and purchase
  - Our COLI mitigates the downward bias but not perfectly.

Literature

- Storable goods; household inventory
  - Empiric: Boizot, Robin, and Visser (2001), Griffith et al (2009), Hendel and Nevo (2006a), Kano (2018)
  - Models: Boizot, Robin, and Visser (2001), Hendel and Nevo (2006a, 2006b), Cashin and Unayama (2016)
  - Structural estimation: Erdem, Imai, and Keane (2003), Hendel and Nevo (2006b), Osborne (2018)
- Sales for storable goods
  - Empirics: Blattberg and Neslin (1989), Neslin and Schneider Stone (1996), Hendel and Nevo (2003QME)
  - Models of stockpiling: Salop and Stiglitz (1982), Hong, McAfee, and Nayyar (2002), Hendel, Lizzeri, and Roketskiy (2014)
- Chain drift or the time aggregation problem in the chain index
  - Frisch (1936), Reinsdorf (1999), Feenstra and Shapiro (2003), ILO (2004) etc
  - Solutions proposed by Feenstra and Shapiro (2003), Ivancic, Diewert, and Fox (2011), Haan and van der Grient (2011)
- COLI
  - Static COLI: Konus (1924), Feenstra and Shapiro (2003), Chevalier and Kashyap (2018)
  - Dynamic COLI: Reis (2009), Gowrisankaran and Rysman (2012), Wang (2013), Osborne (2018)

Outline of the Talk

- Data
  - stylized facts
- Model
  - quasi dynamic model
  - consistency with stylized facts
- Empirical strategy to infer consumption/COLI
  - model
  - applying to data

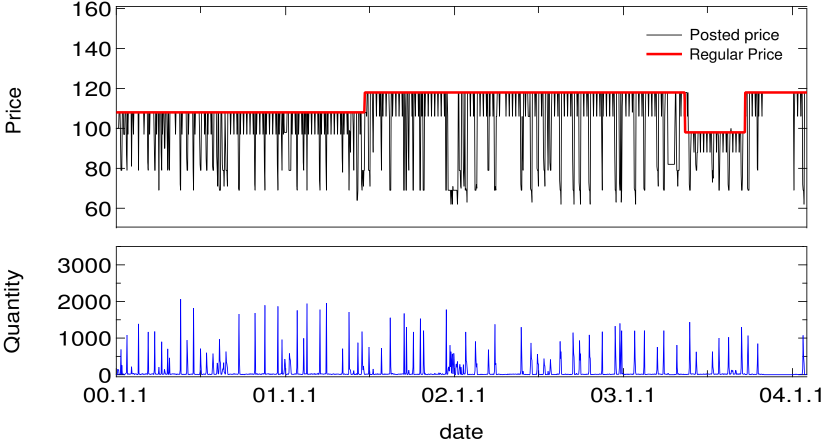
Facts from Japan's Scanner Data

Data

- Japan's scanner data: 2 kinds
- Retailer-side Nikkei POS (Point of Sales)
  - Daily, from March 1, 1988 to present
  - Processed food and domestic articles
    - 17 percent of household's expenditure
  - Quantity and sales for product i at retailer r on date t
  - Product i is identified by the Japanese Article Number (JAN) and the Nikkei's 3-digit product categories (such as salt, yogurt, and cup noodle; around 150)

Illustrative Figure of Nikkei POS

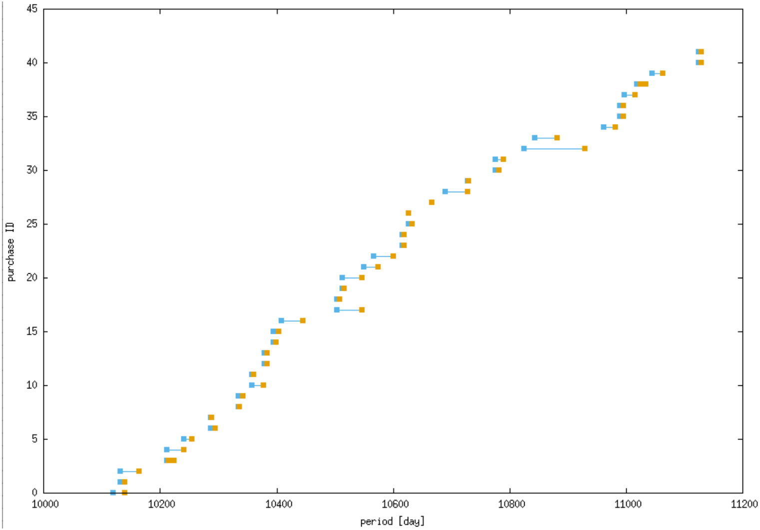
A cup noodle *i* sold at retailer *r*



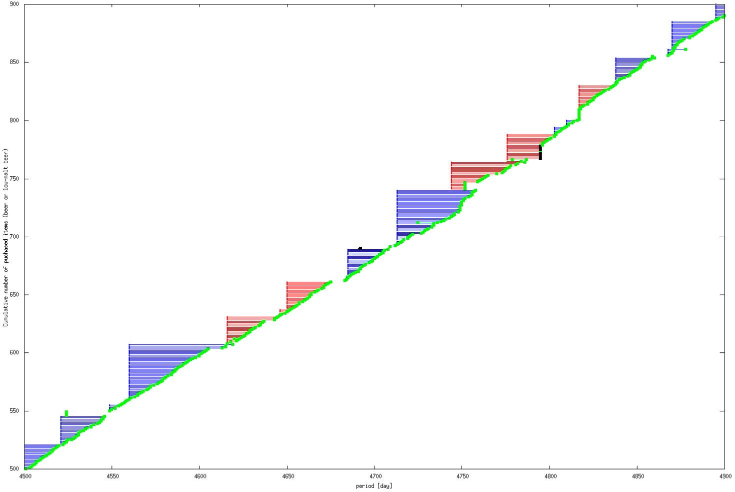
- Household-side Shoku-map (food map)
  - Respondents: mainly housewives, 4,000 households
  - Daily from Sep 1998 to present
  - Food only
  - Records
    - who, what, when and where purchased, when consumed, when consumption ends
    - no price information
    - not on how much (i.e. weights) consumed

Illustrative Figure of Shoku-map 1

Consumption pattern of items in a certain 3-digit category for a particular household: instant cup noodle (purchase in blue, the last use in brown)

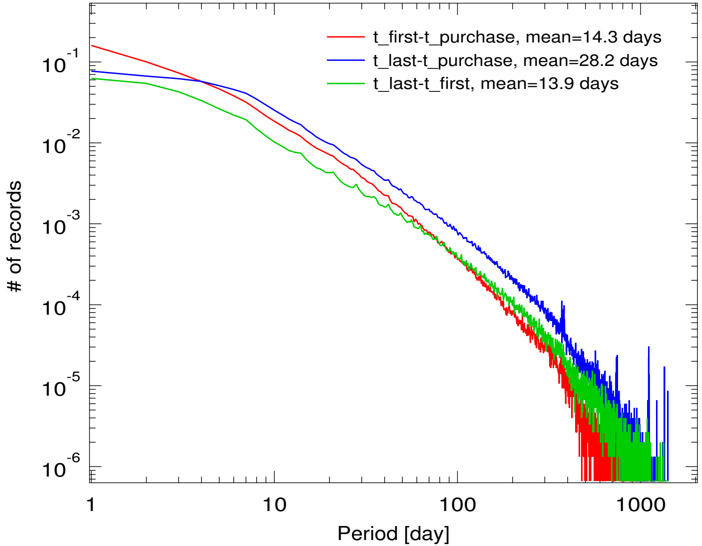


Consumption pattern of items in a certain 3-digit category for a particular household: beer



Illustrative Figure of Shoku-map 2

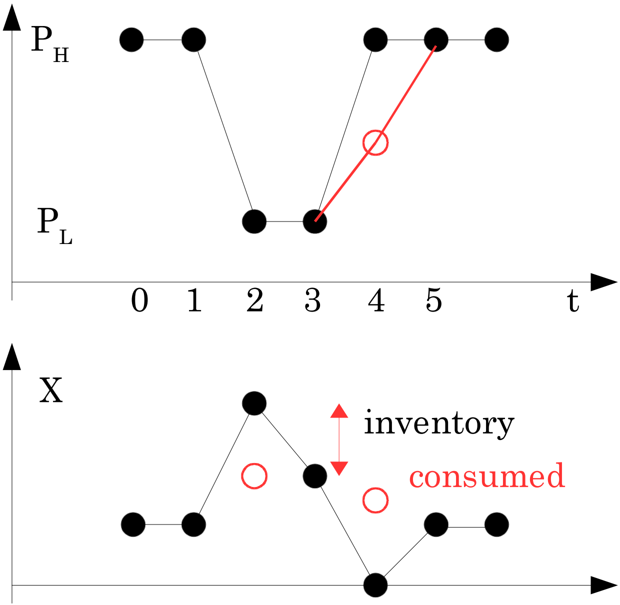
Density of consumption duration



3 Facts

- Still tentative (more analyses needed to test statistical significance)

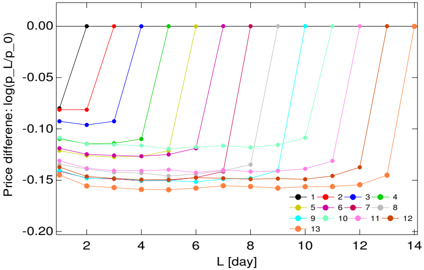
1. Changes in the quantity purchased in a sales event (stockpiling)
2. State dependent consumption
3. Chain drift

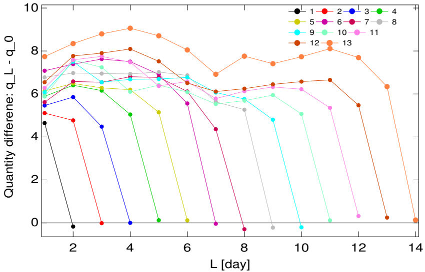

P_H
P_L
0 1 2 3 4 5 t
X
inventory
consumed

Fact 1. The quantity purchased just before sales tends to be greater than that purchased just after sales. The quantity purchased during the first half of sales tends to be greater than that purchased during the second half of sales.

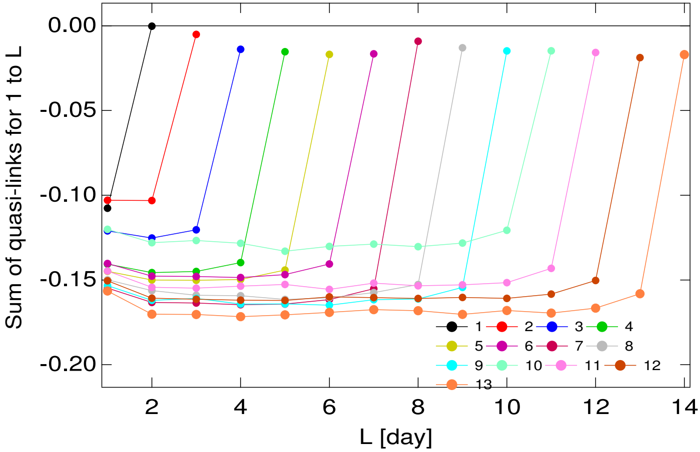
Movement of prices (left) and quantities (right) over a sales period. Comparison with the price/quantity just before sales. Unweighted average of all goods. Often zero purchase (no observation) after sales.

More analysis needed...





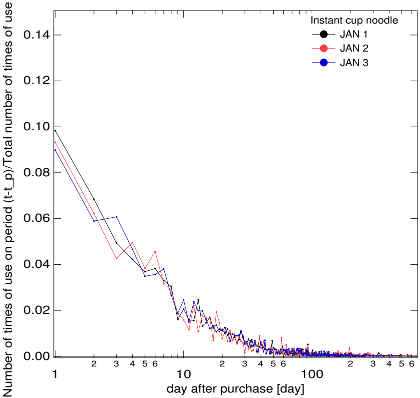
This causes the chain drift.

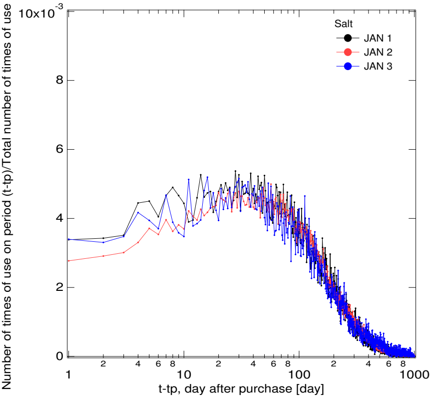


Fact 2. After purchase, consumption is decreasing over time.

Shoku-map: relative frequency of the number of use after purchase

Heterogeneity needs to be controlled by conditioning inventory durations/goods/households.





Price Index

Chained price indexes based on Laspeyres, Paasche, and Törnqvist:

$$\pi_t^{C,L} = \sum_{k \in K_{t-1} \cap K_t} W_{t-1}^k (K_{t-1} \cap K_t) \log \left(\frac{p_t^k}{p_{t-1}^k} \right), \tag{2}$$

$$\pi_t^{C,P} = \sum_{k \in K_{t-1} \cap K_t} W_t^k (K_{t-1} \cap K_t) \log \left(\frac{p_t^k}{p_{t-1}^k} \right), \tag{3}$$

$$\pi_t^{C,T} = \sum_{k \in K_{t-1} \cap K_t} \frac{W_{t-1}^k (K_{t-1} \cap K_t) + W_t^k (K_{t-1} \cap K_t)}{2} \log \left(\frac{p_t^k}{p_{t-1}^k} \right), \tag{4}$$

respectively, where the weight share $W_t^k (K_{t-1} \cap K_t) \equiv p_t^k x_t^k / \sum_{k' \in K_{t-1} \cap K_t} p_t^{k'} x_t^{k'}$. Note that weights are based on either purchase (observable) or consumption (unobservable), and we attach the asterisk $(*)$ for the latter case such as $\pi_t^{C,L*}$.

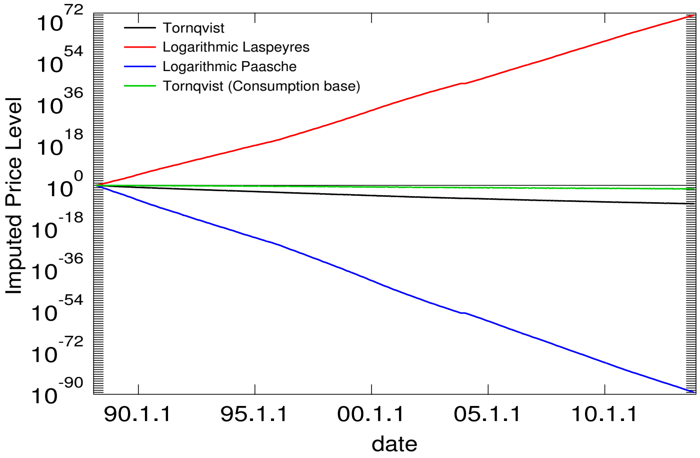
Fixed-based price indexes

$$\pi_t^{FB,L} = \sum_{k \in K_0 \cap K_t} W_0^k(K_0 \cap K_t) \log\left(\frac{p_t^k}{p_0^k}\right) - \sum_{k \in K_0 \cap K_{t-1}} W_0^k(K_0 \cap K_{t-1}) \log\left(\frac{p_{t-1}^k}{p_0^k}\right), \tag{5}$$

$$\pi_t^{FB,P} = \sum_{k \in K_0 \cap K_t} W_t^k(K_0 \cap K_t) \log\left(\frac{p_t^k}{p_0^k}\right) - \sum_{k \in K_0 \cap K_{t-1}} W_{t-1}^k(K_0 \cap K_{t-1}) \log\left(\frac{p_{t-1}^k}{p_0^k}\right), \tag{6}$$

$$\begin{aligned} \pi_t^{FB,T} &= \sum_{k \in K_0 \cap K_t} \frac{1}{2}\left(W_0^k(K_0 \cap K_t) + W_t^k(K_0 \cap K_t)\right) \log\left(\frac{p_t^k}{p_0^k}\right) \\ &\quad - \sum_{k \in K_0 \cap K_{t-1}} \frac{1}{2}\left(W_0^k(K_0 \cap K_{t-1}) + W_{t-1}^k(K_0 \cap K_{t-1})\right) \log\left(\frac{p_{t-1}^k}{p_0^k}\right). \end{aligned} \tag{7}$$

Fact 3. When weights are based on purchase (observable), the change in the price index has the following inequality: $\pi^{C,P} < \pi^{C,T} < 0 < \pi^{C,L}$. Chain drift. Consumption-based index to be explained later.



Model

Quasi Dynamic Model

- Partial equilibrium
- Storable goods
- Price: high (regular) or low (sales); stochastic and exogenous
- Warehouse firms
  - purchase goods from producers, hold inventory, and sell them to households
  - inventory cost; no depreciation; free entry
- A representative household
  - does not hold inventory.
  - Purchase equals consumption.
  - Purchase goods from warehouse firms and/or producers.
  - **The problem to solve the COLI is static.**

Household

No inventory. The household's cost minimization problem:

$$\min_{c_t^k} \left\{ \sum_{k \in K_t} r_t^k c_t^k + \lambda_t \left\{ U - \left[\sum_{k \in K_t} b_t^k \left(c_t^k \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \right\}, \tag{8}$$

where r_t^k represents the price of storable goods k in period t that the household purchases and $b_t^k = b^k + \varepsilon_t^k$.
The above problem is static, so the COLI is the same as the conventional one. All we need to know is the price, r_t^k. We extensively use the following equation with respect to the optimal quantity purchased:

$$c_t^k = \left(\frac{r_t^k / b_t^k}{r_{t'}^k / b_{t'}^k} \right)^{-\sigma} c_{t'}^k. \tag{9}$$

Warehouse Firms

There exist an infinite number of warehouse firms under free entry. Each warehouse firm maximizes the firm value:

$$V_t^k = \mathrm{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left\{ \sum_{k \in K_{t+j}} \left(r_{t+j}^k y_{t+j}^k - p_{t+j}^k x_{t+j}^k - C(i_{t+j}^k) \right) \right\} \right], \tag{10}$$

subject to

$$i_t^k = i_{t-1}^k - y_t^k + x_t^k. \tag{11}$$

The inventory as well as the purchase must be nonnegative:

$$x_t^k, \; i_t^k \geq 0. \tag{12}$$

$C(0) > 0$, $C(i)' > 0$ and $C(i)'' \geq 0$

The first-order conditions with respect to x_t^k and i_t^k are

$$0 = r_t^k - p_t^k + \psi_t^k, \tag{13}$$

$$C'(i_t^k) = \beta \mathrm{E}_t[r_{t+1}^k] - r_t^k + \mu_t^k. \tag{14}$$

Free entry: the zero value for the entrant firm with zero inventory,

$$V^k(i^k_{t-1} = 0, p^k_t) \leq 0. \tag{15}$$

The price of storable goods follows a Markov process. It takes either of the two, non-sale P_H or sale P_L $(P_H > P_L)$ and

$$\begin{aligned} \text{Prob}(P_L|P_H) &= \overline{q} \\ \text{Prob}(P_L|P_L) &= \underline{q}. \end{aligned} \tag{16}$$

The goods market clearing is

$$\int_0^{N_t} y^k_{t,j} dj + \int_0^{M_t} z^k_{t,j} dj = \int_0^1 c^k_{t,j} dj, \tag{17}$$

where $z^k_{t,j}$ represents the supply of storable goods k by producers.

COLI

The cost minimization problem subject to constant utility leads to the optimal quantity purchased equal to

$$c_t^k = \left(r_t^k / b_t^k \right)^{-\sigma} \lambda_t^\sigma U. \tag{18}$$

The COLI, $\lambda_t = \mathcal{C}(r_t)$, is given by

$$\mathcal{C}(r_t) = \sum_{k \in K_t} r_t^k c_t^k = \left[\sum_{k \in K_t} \left(b_t^k \right)^{\sigma} \left(r_t^k \right)^{1-\sigma} U \right]^{1/(1-\sigma)}. \tag{19}$$

In order to calculate the COLI, we need to know two variables in every period: the price r_t^k and the share $r_t^k c_t^k$.

Equilibrium Property

Omit the superscript of k for simplicity. Denote the aggregate inventory at the end of period $t-1$, by $I_{t-1} \equiv \int_0^{N_t} i_{t-1,j} dj$.

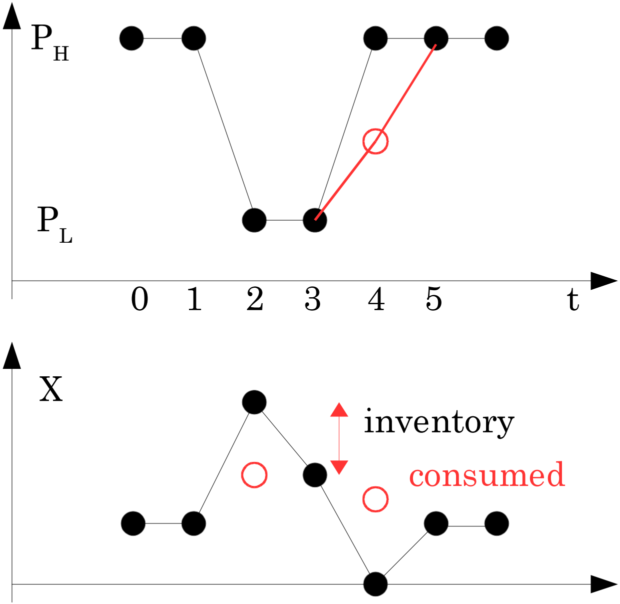
Lemma 1

The price between warehouse firms and households, r_t, satisfies $0 < r_t \leq P_H$.
Suppose no big unexpected shock to b_t. Then, r_t satisfies $P_L \leq r_t \leq P_H$ and, when $p_t = P_L$, $r_t = P_L$. Also, $r_t = r(I_{t-1}, p_t, b_t)$ is non-decreasing in p_t and b_t and non-increasing in I_{t-1}.

Lemma 2

Suppose no big unexpected shock to b_t. If $p_t = P_H$, warehouse firms do not purchase, that is, $x_t = 0$. If $p_t = P_{L,}$ warehouse firms purchase and hold inventory i_t, which is independent of i_{t-1}, I_{t-1}, and b_t.

i.e. At sales, the household purchases directly from producers at $r_t = P_L$. Warehouse firms hold inventory to sell goods at a higher price.


P_H
P_L
0
1
2
3
4
5
t
X
inventory
consumed

Equilibrium Property Consistent with Fact 1

Lemma 3

Suppose no shock: $b_t = b$.

Then, the quantity purchased by warehouse firms and households just before sales is greater than or equal to that just after sales.

The quantity purchased by warehouse firms and households at the first day of sales is greater than or equal to that at the final day of sales.

Remark: If the duration of sales is known ex ante, the quantity purchased by warehouse firms and households at the first day of sales is smaller than or equal to that at the final day of sales.

Equilibrium Property Consistent with Fact 2

Lemma 4

Suppose no shock: $b_t = b$. Consumption is decreasing with r_t. After sales end, r_t and consumption are non-decreasing and non-increasing over time, respectively.

Equilibrium Property Consistent with Fact 3

Let m represent the degree of stockpiling, which indicates how long inventory remains after sales end.

Lemma 5

Consider one sales event for storable goods k such that $p_t = p_{t+T+1} = P_H$ and $p_{t+j} = P_L$ for $j = 1, \cdots, T$ $(T \geq 1)$. Suppose that the effect of goods k is so small that the price and quantity of other goods are unchanged; $\sum_{k' \in K_0 \cap K_t} p_t^{k'} x_t^{k'} = 1$; the price before and after sales is P_H for a sufficiently long duration (i.e., $p_{t-j} = p_{t+T+1+j} = P_H$ for $j = 0, 1, \cdots, T_H$, where T_H is sufficiently large compared with m); and $b_t^k = b^k$.

If $\sigma > (<) 1$ and $m \geq 1$, the change in the price index from t to $t + T + 1 + T_H$ satisfies $\pi^{COLI} = 0$; $\pi^{C,L} > (<) 0$; $\pi_t^{C,T} < 0$; $\pi^{C,T*} < (>) 0$; and $\pi^{FB} = 0$. If $\sigma = 1$ or $m = 0$, all the above change in the price index is zero except for $\pi_t^{C,T} < 0$.

Furthermore, if $\sigma > 1$, $\pi^{C,P} < \pi^{C,T} < 0 < \pi^{C,L}$. If $\sigma > 1$ and $m = 1$, $\pi^{C,T} < \pi^{C,T*} < 0$.

- It is known that the Törnqvist index is a good approximation of the COLI (superlative) up to the second order, but...
- Use of consumption instead of purchase does not eliminate the chain drift.
- Key: systematic asymmetry due to the non-negative constraint of inventory and purchase
    - Purchase asymmetry between price decrease and price increase
    - Consumption based approach mitigates this asymmetry,
    - but effective price r is asymmetric: big price decrease and small price increase
    - Third order comes in.
    - Need for a better index such as superlative with the consideration of σ.

Order r Superlative Indices

Suppose that the consumer's utility is expressed as

$$\mathcal{C}(r_t) = \left[\sum_{i \in K} \sum_{k \in K} \alpha^{ik} \left(r_t^i\right)^{(1-\sigma)} \left(r_t^k\right)^{(1-\sigma)} \right]^{1/\{2(1-\sigma)\}} \tag{20}$$

where $\alpha^{ik} = \alpha^{ki}$.

Define P_r as

$$P_r(r_0, r_1, c_0, c_1) = \frac{\left\{ \sum_{k \in K} s_0^k \left(\frac{r_1^k}{r_0^k} \right)^{(1-\sigma)} \right\}^{1/\{2(1-\sigma)\}}}{\left\{ \sum_{k \in K} s_1^k \left(\frac{r_0^k}{r_1^k} \right)^{(1-\sigma)} \right\}^{1/\{2(1-\sigma)\}}}, \tag{21}$$

where s_t^k represents the consumption share of product k at t.

Lemma 6

Given the unit cost function of (20), P_r equals $\mathcal{C}(r_1)/\mathcal{C}(r_0)$.

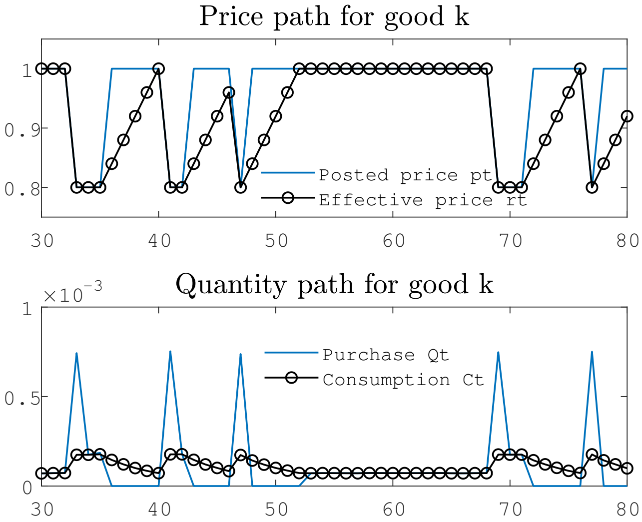
Similar index: Lloyd–Moulton index (P_{LM} and P_{LM*})

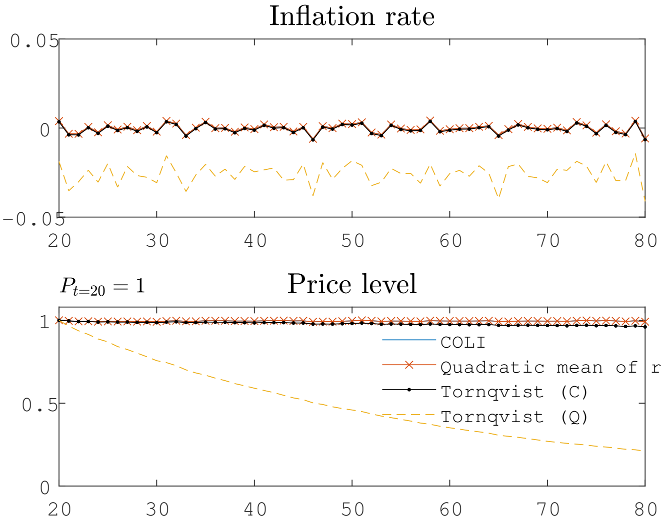
$$P_{LM}(r_0, r_1, c_0, c_1) = \left\{ \sum_{k \in K} s_0^k \left(\frac{r_1^k}{r_0^k} \right)^{1-\sigma} \right\}^{1/(1-\sigma)}, \tag{22}$$

$$P_{LM*}(r_0, r_1, c_0, c_1) = \left\{ \sum_{k \in K} s_1^k \left(\frac{r_0^k}{r_1^k} \right)^{1-\sigma} \right\}^{1/(\sigma-1)}. \tag{23}$$

Numerical Simulation

- $N = 1000$: cross section samples
- $\overline{q} = 0.1$: prob of sales given regular
- $\underline{q} = 0.5$: prob of sales given sales
- $P_L / P_H = 0.8$: ratio of sales prices to regular prices
- $\sigma = 4$: elasticity of substitution
- Furthermore, we assume a linearly increasing path for effective price r_t after sales end (see later in details)
  - $m = 5$: the degree of stockpiling (effective sales duration after sales end)


Price path for good k
1
0.9
0.8
30
40
50
60
70
80
Posted price pt
Effective price rt
Quantity path for good k
×10^-3
1
0.5
0
30
40
50
60
70
80
Purchase Qt
Consumption Ct



Inflation rate
0.05
0
−0.05
20
30
40
50
60
70
80
$P_{t=20} = 1$
Price level
1
0.5
0
COLI
Quadratic mean of r
Tornqvist (C)
Tornqvist (Q)

Inflation rates (100 iterations)

	COLI	Quadratic mean of r	Tornqvist (C)	Tornqvist (Q)	Laspeyres (Q)	Paasche (Q)
Benchmark	3.37e-06	3.37e-06	-5.28e-04	-0.026	0.063	-0.114
	(7.13e-05)	(7.13e-05)	(7.20e-05)	(3.10e-04)	(9.07e-04)	(8.43e-04)
Low m	5.92e-07	5.92e-07	6.53e-07	-0.012	0.027	-0.051
$(m = 1)$	(7.19e-05)	(7.19e-05)	(7.30e-05)	(1.33e-04)	(3.52e-04)	(4.68e-04)
High m	-7.98e-08	-7.98e-08	-3.52e-04	-0.037	0.088	-0.162
$(m = 10)$	(6.54e-05)	(6.54e-05)	(6.66e-05)	(4.44e-04)	(1.21e-03)	(8.08e-04)
Low σ	3.07e-06	3.07e-06	-5.22e-05	-0.026	0.053	-0.104
$(\sigma = 2)$	(6.75e-05)	(6.75e-05)	(6.76e-05)	(3.14e-04)	(8.57e-04)	(8.81e-04)
Low P_L/P_H	1.14e-05	1.14e-05	-1.70e-02	-0.066	0.273	-0.405
$(P_L/P_H = 0.5)$	(2.37e-04)	(2.37e-04)	(2.70e-04)	(7.82e-04)	(3.01e-03)	(2.38e-03)
High $\overline{q}$	5.04e-06	5.04e-06	-5.59e-04	-0.033	0.090	-0.156
$(\overline{q} = 0.2)$	(6.14e-05)	(6.14e-05)	(6.16e-05)	(2.46e-04)	(8.00e-04)	(5.69e-04)
High $\underline{q}$	-8.24e-06	-8.24e-06	-3.93e-04	-0.029	0.027	-0.084
$(\underline{q} = 0.8)$	(7.41e-05)	(7.41e-05)	(7.53e-05)	(2.81e-04)	(5.36e-04)	(7.28e-04)

Empirical Strategy to Calculate Consumption/COLI

Motivation

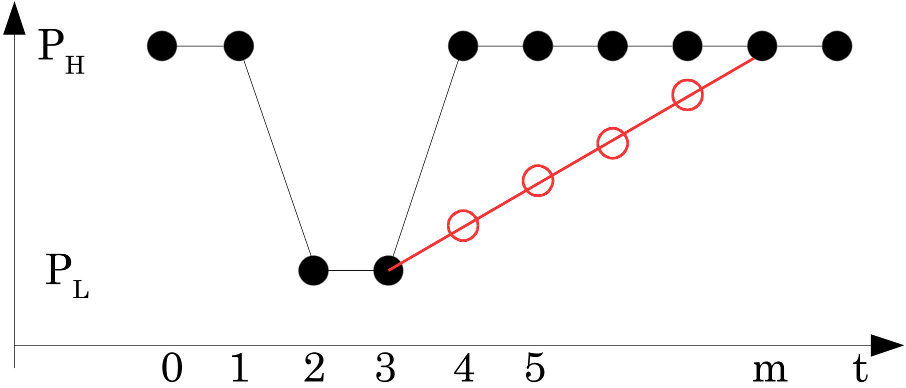
- We want to know
  - consumption (c_t), the effective price (r_t) and the degree of stockpiling (m)
  - also price elasticity (σ)
    - ★ without the form of inventory costs, discount factor, the probability of sales, the size of sales, ...
  - necessary for the COLI
- Hereafter, we impose restrictive conditions.

Suppose $b_t = b$. Consider the path of $r_H(I_{t-1}) \equiv r(I_{t-1}, P_H)$, where $r_H(I_{t-1}) \geq r(I_{t-1}, P_L) = P_L$. When $p_t = P_H$, warehouse firms optimize as

$$C'(i_H; I_{t-1}) = \beta \{(1-\overline{q}) r_H(I_t) + \overline{q} P_L\} - r_H(I_{t-1}) + \mu_t, \qquad (24)$$

where μ_t ($\mu_t \geq 0$): the Lagrange multiplier associated with i.

- If $C'(i_H; I_{t-1}) = C > 0$; $\beta = 1$; $\overline{q} = 0$; $\mu_t = 0$, then $r_H(I_t) - r_H(I_{t-1})$ is positive constant $\rightarrow$ The expectation of a *linear* price increase prevents inventory from falling to zero instantaneously.
  - If $\beta < 1$ and/or $\overline{q} > 0$, $r_H(I_t) - r_H(I_{t-1})$ is increasing with t.
  - If $C''(i_H; I_{t-1}) > 0$, $r_H(I_t) - r_H(I_{t-1})$ is decreasing with t.
- Thus, we assume a linearly increasing path of r_H.
  - Corresponding consumption is calculated from $(r_H/P_L)^{-\sigma} X_L^*$.
  - After inventory duration $m \geq 1$, inventory is zero, which leads to $m = \frac{P_H - P_L}{P_L} \frac{\sigma - 1}{1-(P_H/P_L)^{-\sigma+1}} \frac{I_L}{X_L^*}$ (in continuous time)



P_H
P_L
0
1
2
3
4
5
m
t

- We infer σ, X_L^*, and I_L from observations.
  - V shape filter to identify sales
  - for each sales observation for each goods, obtain
    - ★ P_H and X_H : price and quantity purchased (equal to C) just before sales
    - ★ P_L^1 and X_L^1: the average price and quantity purchased (not necessarily equal to C) at the first half spell of sales
    - ★ P_L^2 and X_L^2: the average price and quantity purchased (not necessarily equal to C) at the first latter spell of sales
    - ★ $P_L = P_L^1$ and $X_L = X_L^1$ if $X_L^1 < X_L^2$ and $P_L = P_L^2$ and $X_L = X_L^2$ otherwise
  - calculate σ
    - ★ by calculating the log ratio of quantity consumed at sales to that at regular divided by the log ratio of price at sales to that at regular

$$-\frac{\log\left(X_L/X_H\right)}{\log\left(P_L/P_H\right)} \tag{25}$$

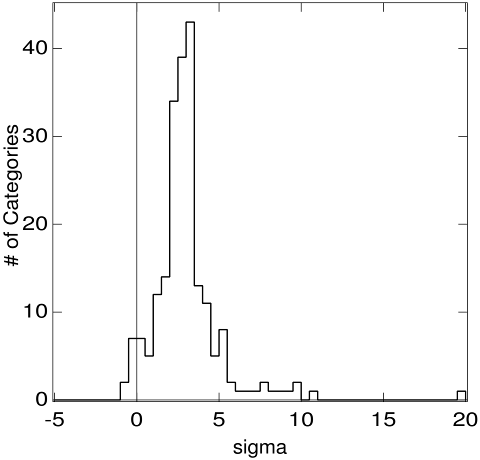
    - ★ and taking the unweighted average for each 3-digit product category.
  - calculate C at sales as $X_L^* = \left(P_L/P_H\right)^{-\sigma} X_H$
  - calculate inventory at the end of sales as $I_L = \Sigma_{j=1}^{T} X_{t+j} - TX_L^*$, where sales period is from $t+1$ to $t+T$.

Algorithm to Calculate Price Indexes

1. For each product k at retailer r, use the V-shape filter to identify sales.
   - $K = L = 42$; larger than or equal to 2 yen; set the quantity at zero and the price at the regular price, if the quantity and, thus, the price are not observed after sales.
   - Let a sales period be $t_s + 1$ to $t_s + T$.
2. Infer X_L^* and I_L for each sales event and then σ for each 3-digit product category.
   - Calculate m and $r_{H,j} = (j/m)(P_H - P_L) + P_L$ for each sales
3. Calculate price (r_t) and quantity consumed (c_t)
   - At t_s, $r_t = P_H$ and $c_t = X_H$.
   - If sales at t, then $r_t = P_L$ and $c_t = X_L^* = (P_L/P_H)^{-\sigma} X_H$.
   - If regular at t,
     - ★ Unless the next sales begin by $t = t_s + T + m - 1$, then $r_t = r_{H,j}$ $(j = t - t_s - T)$ and $c_t = (r_{H,j}/P_L)^{-\sigma} X_L^*$.
     - ★ Otherwise, $r_t = p_t$ and $c_t = X_t$.

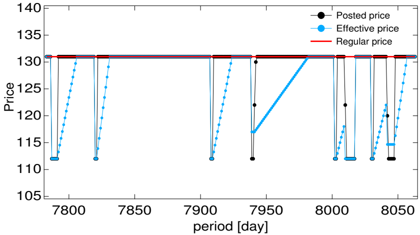
Price elasticity σ

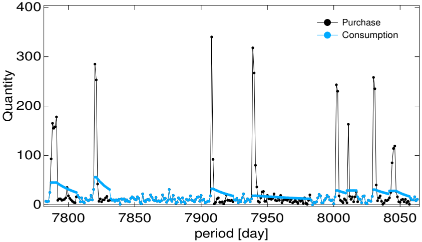
How different when neglecting storability (simply take the slope between sales and regular)???



The price and quantity path

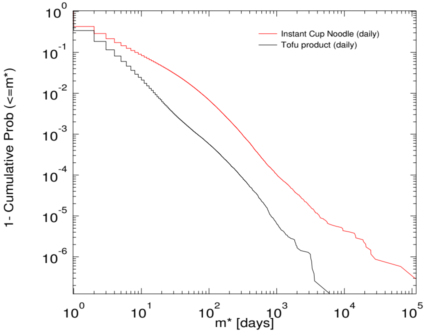
An instant cup noodle(?) at one retailer

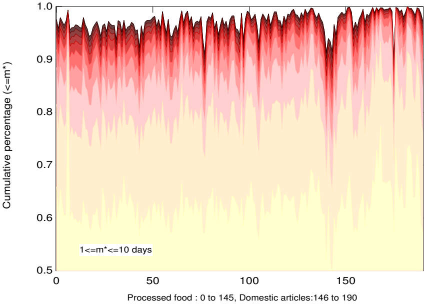




Degree of stockpiling m

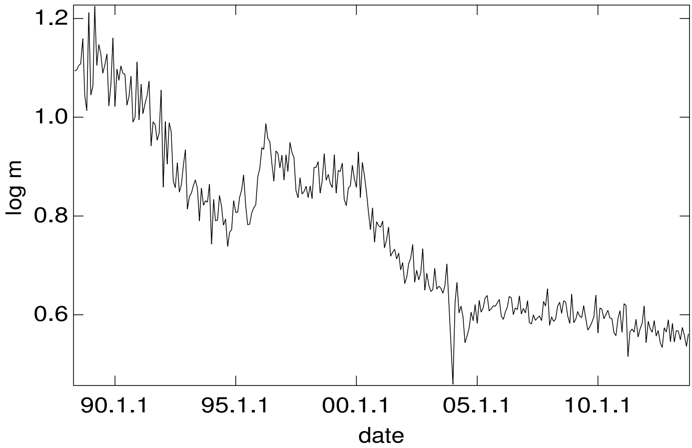
Calculated m is mostly a few days.





Aggregated time series m

We will examine why m decreases later.



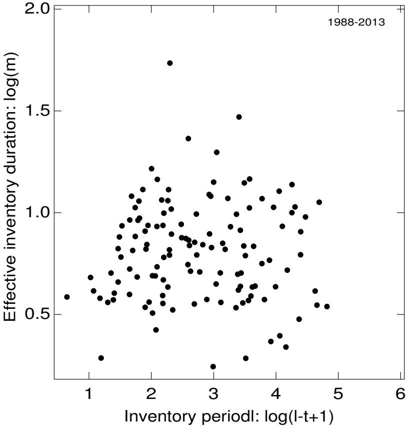
Model Justifications

- Examine a key variable m
  - Relation with shokumap data (inventory duration and stockpiling)
  - Episode of consumption tax increase on April 1, 2014

Relation between shokumap data and the estimate of m

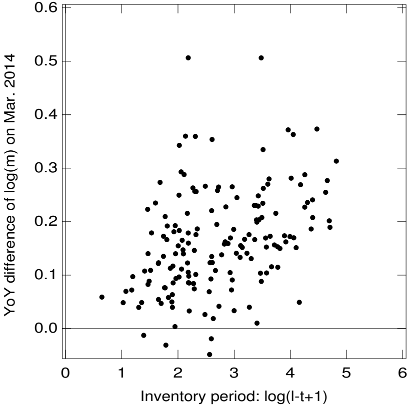
No correlation between log(inventory period) and $\log m$ over the entire observation periods: 0.020

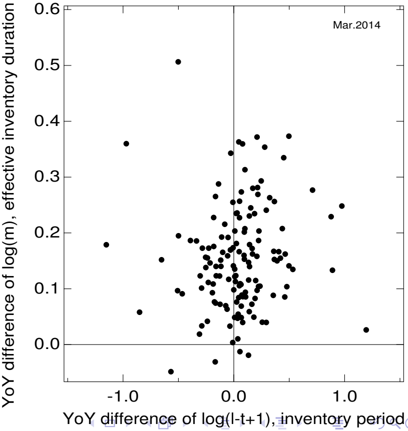
$\rightarrow$ m does not necessarily represent storability.



Relation between shokumap data and the estimate of m

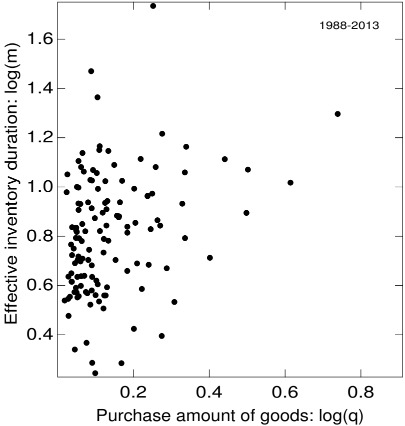
Positive correlation with the change in log$m$ in Mar. 2014 (one month before the consumption tax increase): 0.414 (left) and 0.156 (right)
$\rightarrow$ Goods with a long inventory duration tend to be stockpiled more before the consumption tax increases.

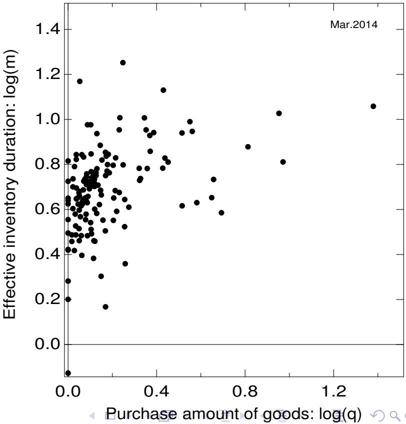




Relation between shokumap data and the estimate of m

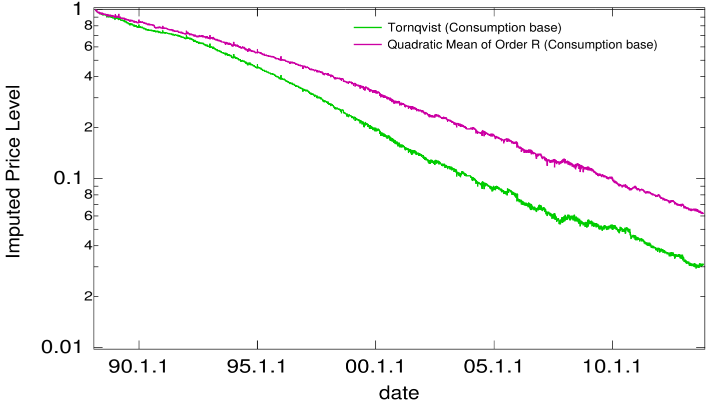
Positive correlation between log(the quantity of purchase) and $\log m$ over the entire observation periods: 0.313 (left) and 0.452 (right)
$\rightarrow$ m captures stockpiling when price decreases (or is expected to increase).





Price Indexes (Consumption based)

Milder chain drift, but not completely disappeared (roughly −5% annually)



Determinants of m

Variable m captures the degree of stockpiling (inventory duration).
When $p_t = P_L$, warehouse firms optimize as

$$C'(i_L; I_{t-1}) = \beta \left\{ (1-\underline{q}) r_H(I_t) + \underline{q} P_L \right\} - P_L + \mu_t. \tag{26}$$

Suppose $C'(i_t; I_{t-1}) = C > 0$, then we should have

$$C = \beta(1-\underline{q}) r_{H,1} - (1-\beta\underline{q}) P_L,$$

which becomes

$$C = \beta(1-\underline{q}) \left\{ \frac{1}{m} \frac{P_H - P_L}{P_L} + 1 \right\} P_L - (1-\beta\underline{q}) P_L,$$

$$m = \beta(1-\underline{q}) \frac{P_H - P_L}{P_L} \left(1 - \beta + \frac{C}{P_L} \right)^{-1}. \tag{27}$$

Thus, m depends on the probability of sales at t conditional on sales at $t-1$ negatively, the size of sales discounts positively, and the cost of inventory negatively. More generally, $r_{H,1}$ depends on $\overline{q}$, so the above m should depends on $\overline{q}$, too.

Determinants of m

- Panel regression of m on $\overline{q}$, $\underline{q}$, and $\frac{P_H - P_L}{P_L}$ with product-category and time dummies.
- Panel regression of m on $\underline{q}$ and $\frac{P_H - P_L}{P_L}$ with product-category and time dummies.
- Time series of the time dummy
- Relation to business cycles

Concluding Remarks

- Remaining tasks. Comments welcome
  - Imperfect elimination of the chain drift
  - Any important variables to look at?
    - Price stickiness, consumption smoothing, elasticity, relation to business cycles...