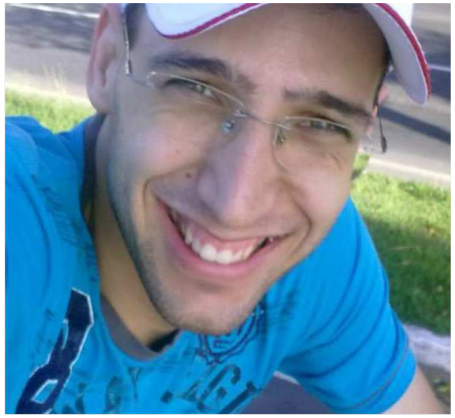
Workshop on Data Science - Emap/FGV

Spatio-Temporal Data Analytics via Graph Signal Processing

Prof. Luis Gustavo Nonato

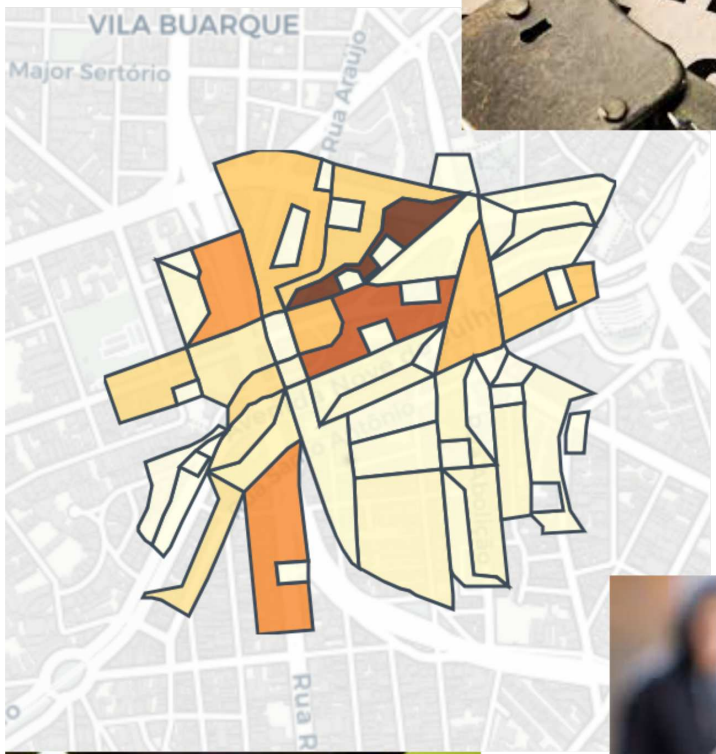
University of São Paulo - Brazil

Joint Work With

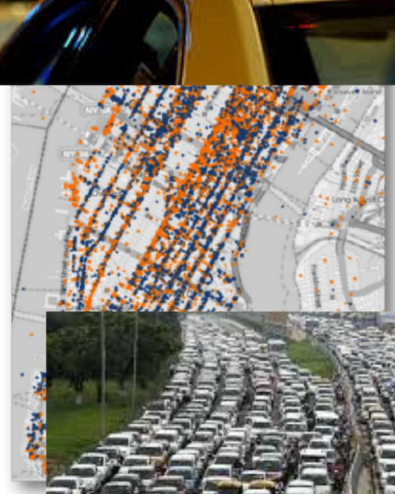


What is spatio-temporal data and why to understand it?

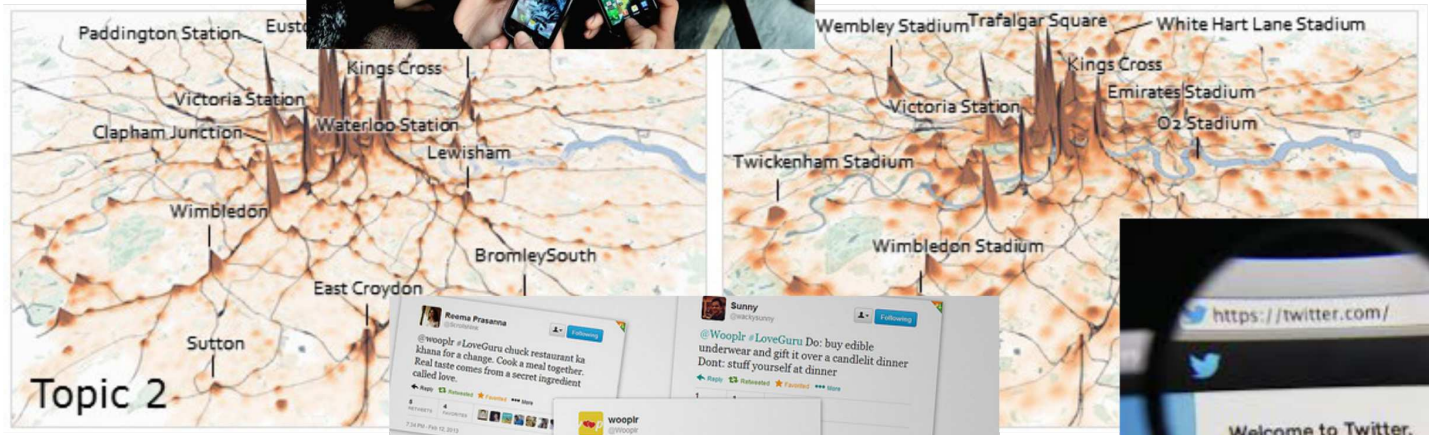
Spatio-Temporal Data Analysis



Spatio-Temporal Data Analysis

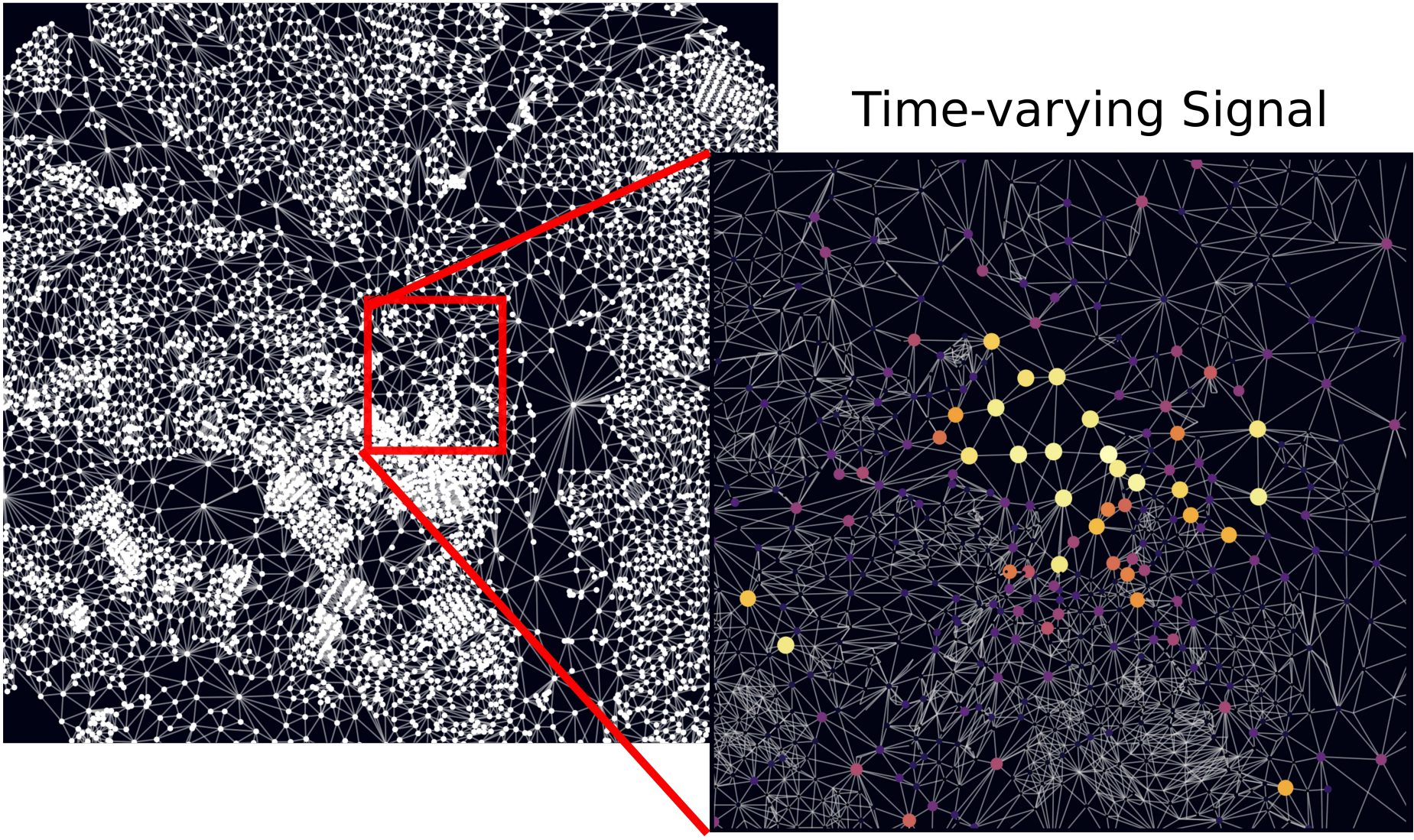


Spatio-Temporal Data Analysis



Graph

Time-varying Signal



Graph Signals Processing

How to analyze signals defined on graphs so as to uncover phenomena and identify patterns hidden in the data?

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The main idea of GSP is to adapt well stablished signal processing tools such as Linear Filtering, Fourier, and Wavelet transforms to the context of graphs.

Basic Concepts and Notation

The Graph Laplacian

The Graph Laplacian

A undirected weighted graph $G = (V, E, W)$ is defined by a set of nodes $V = \{\tau_i\}$, edges $E = \{e_{ij}\}$ linking pairs of nodes τ_i and τ_j, and a symmetric weight matrix W whose entries satisfies

$$\begin{cases} w_{ij} > 0 & \text{if } \exists\ e_{ij} \in E \\ w_{ij} = 0 & \text{otherwise} \end{cases}$$

w_{ij} indicates how strongly the nodes τ_i and τ_j are linked.

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Graph Laplacian

$$L = D - W$$

where D is a diagonal matrix with entries $d_{ii} = \sum_j w_{ij}$.

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L is a symmetric positive semidefinite matrix.

Spectral Decomposition

Eigenvectors of L

Eigenvalues of L

$$\underbrace{\begin{bmatrix} \vdots & \vdots & & \vdots \\ u_1 & u_2 & \cdots & u_n \\ \vdots & \vdots & & \vdots \end{bmatrix}}_{U} \qquad \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{\Lambda}$$

$$L = U \Lambda U^\top$$

Spectral Decomposition

Eigenvectors of L Eigenvalues of L

$$\underbrace{\begin{bmatrix} \vdots & \vdots & & \vdots \\ u_1 & u_2 & \cdots & u_n \\ \vdots & \vdots & & \vdots \end{bmatrix}}_{U} \quad \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{\Lambda}$$

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Eigenvectors are function defined on the vertices of G

$$u_i : V \to \mathbb{R}$$

The *Discrete Courant's Nodal Theorem*

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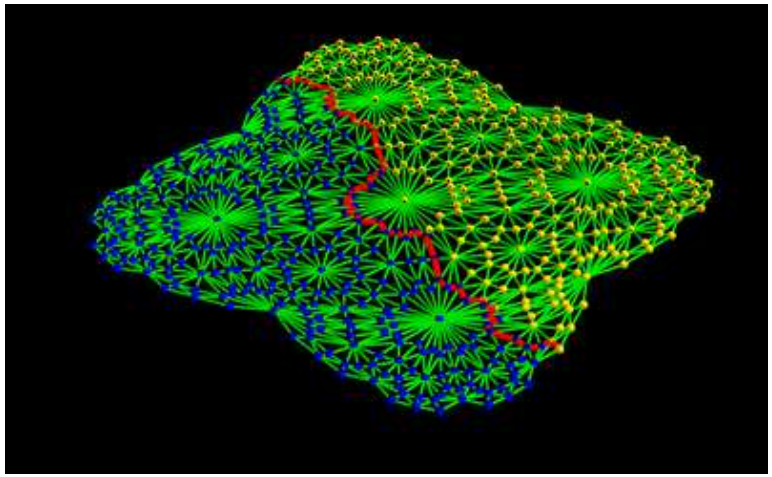

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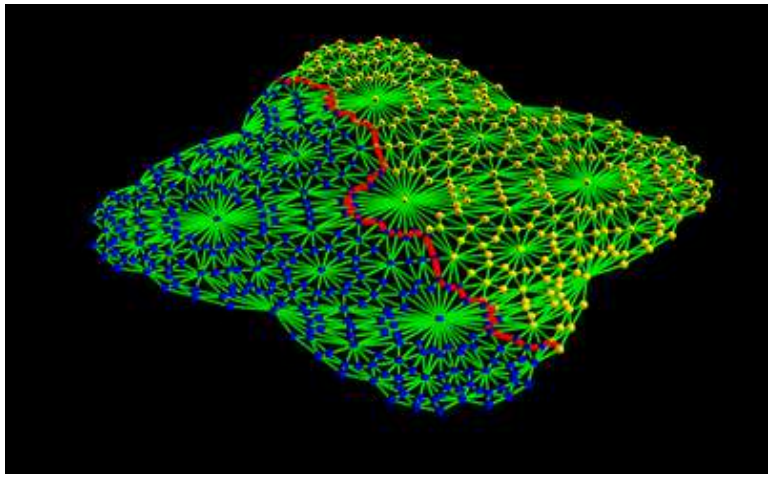
The *Discrete Courant's Nodal Theorem*



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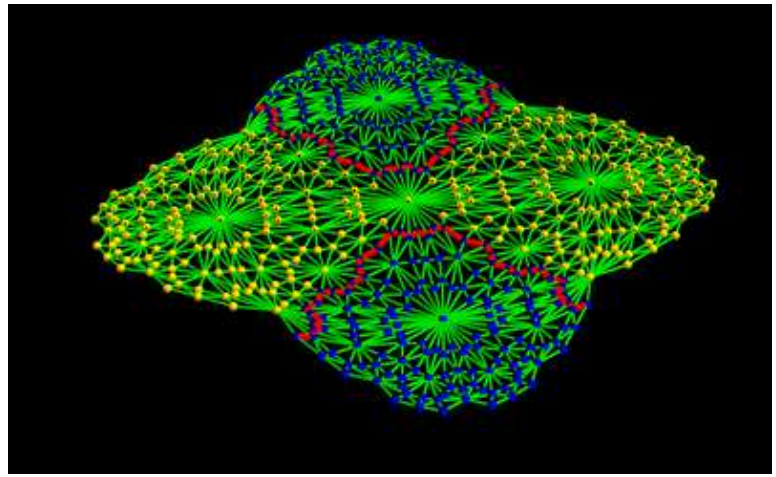

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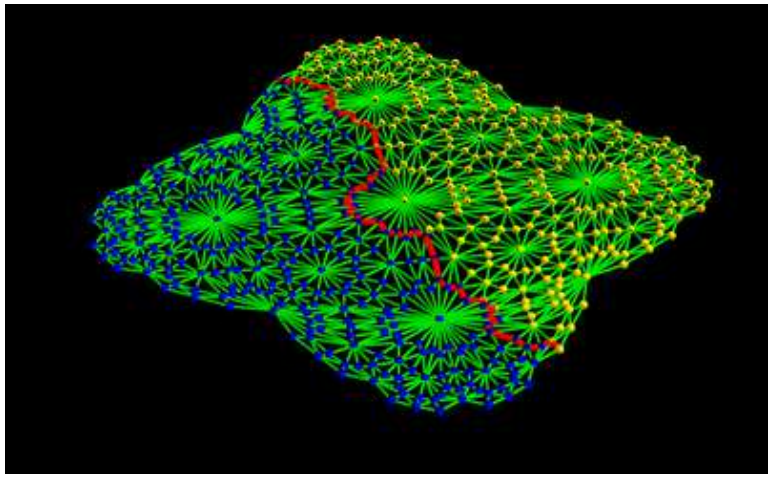
The *Discrete Courant's Nodal Theorem*



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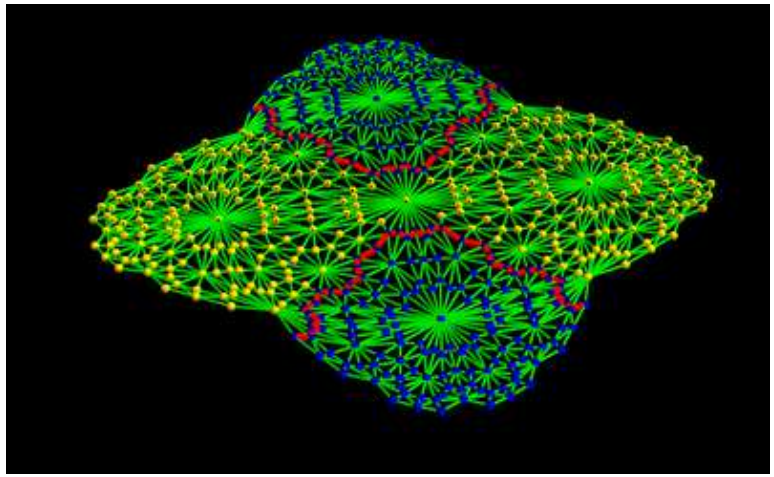

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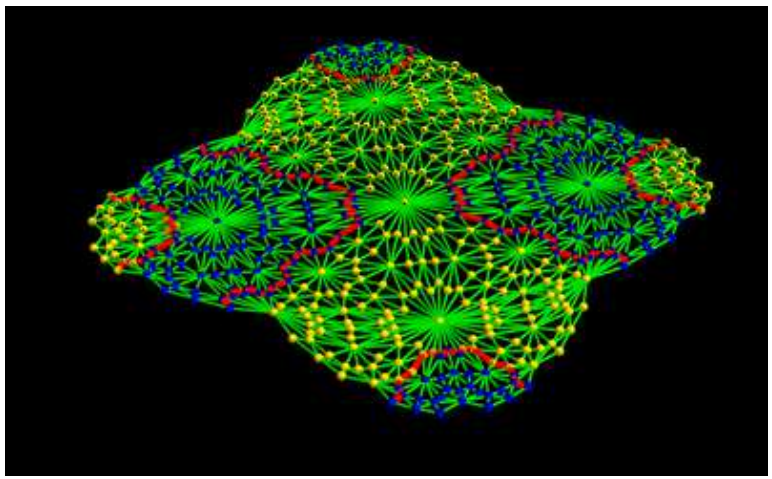

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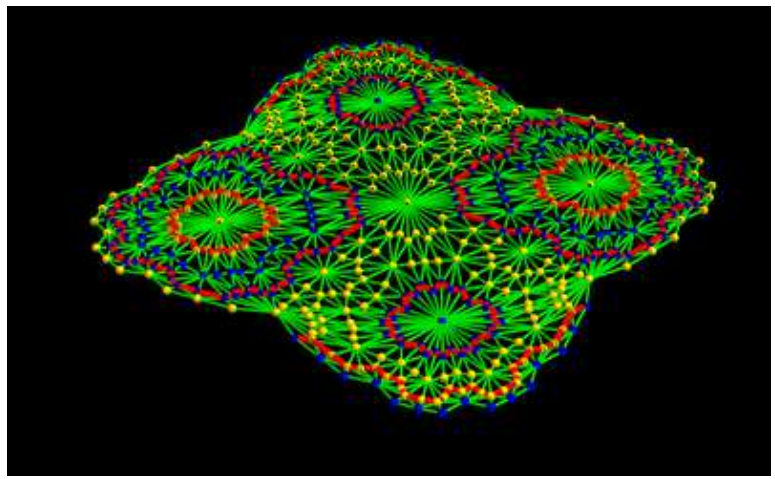

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Graph Laplacian

Eigenvectors of the graph Laplacian behaves similarly to Fourier basis, tending to increase oscillation as the corresponding eigenvalue increases.

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$$\begin{aligned} \lambda_l &\longrightarrow \text{frequencies} \\ u_l &\longrightarrow \text{basis function} \end{aligned}$$

Graph Fourier Transform

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Graph Fourier Transform (GFT)

The Graph Fourier Transform of f is defined as

$$GFT[f](\lambda_l) = \widehat{f}(\lambda_l) = < f, u_l > = \sum_{i=1}^{n} f(i) u_l(i) \rightarrow \boxed{\widehat{f} = U^\top f}$$

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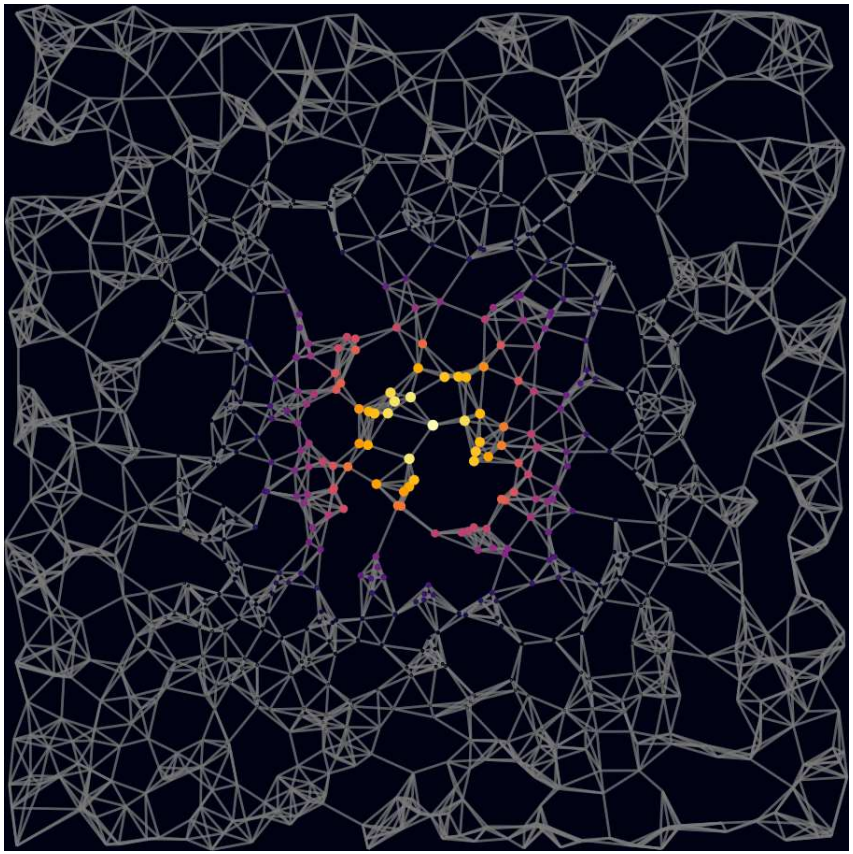
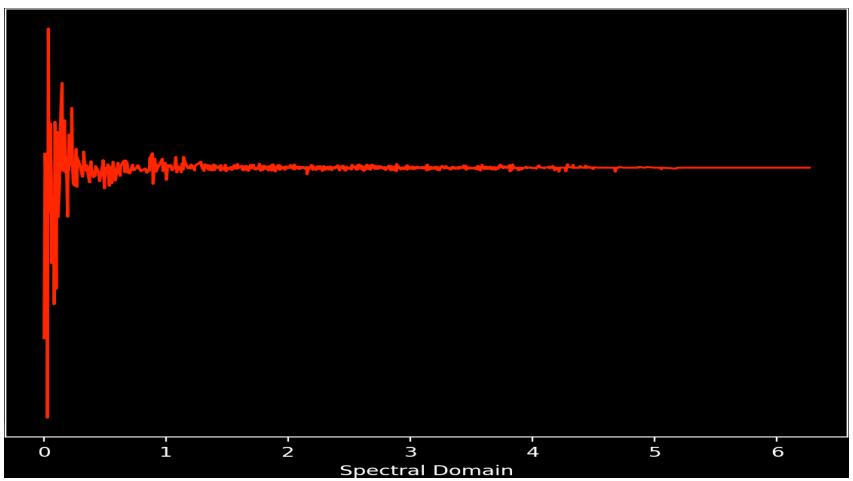
Inverse Graph Fourier Transform

The Inverse Graph Fourier Transform is given by

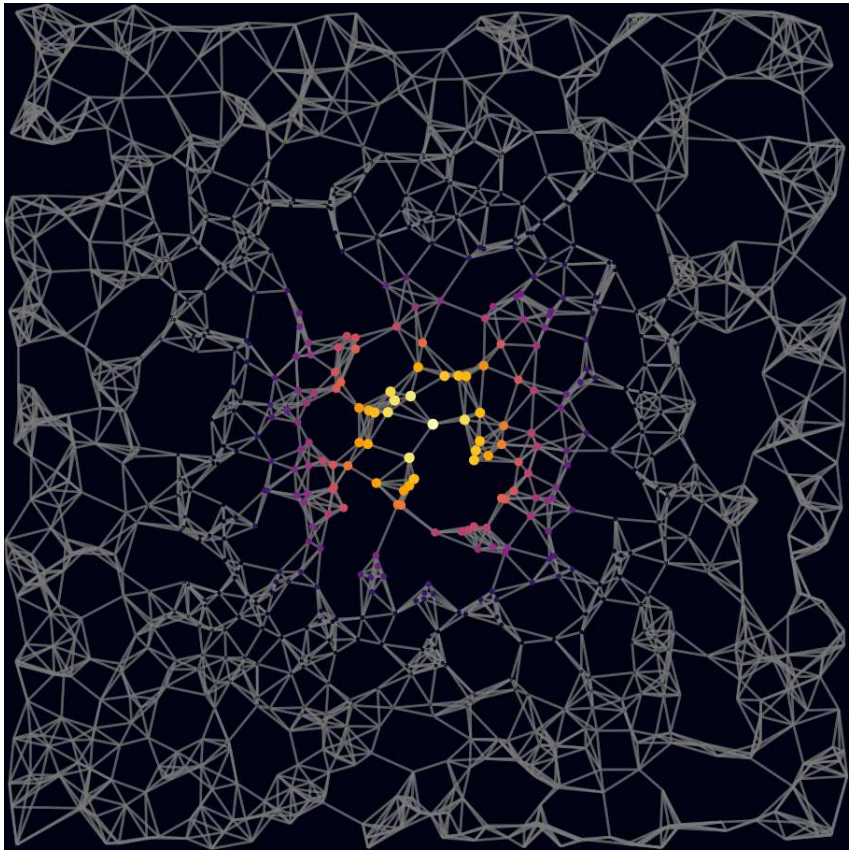
$$iGFT[\widehat{f}](i) = f(i) = \sum_{l=0}^{n-1} \widehat{f}(\lambda_l) u_l(i) \to \boxed{f = U\widehat{f}}$$

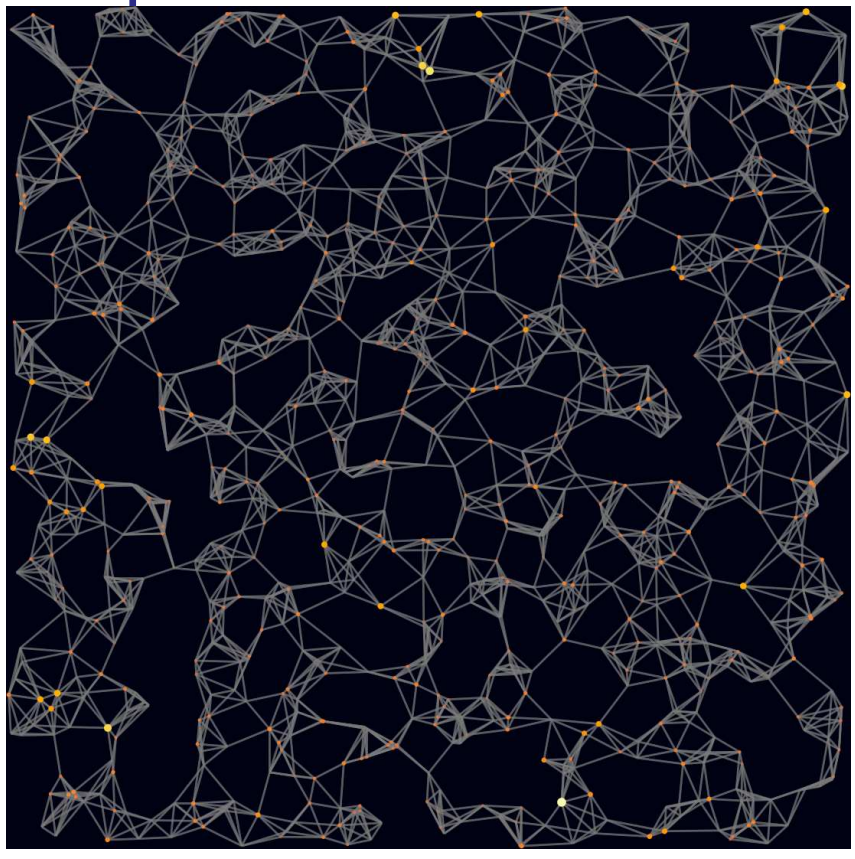
Graph Fourier Transform

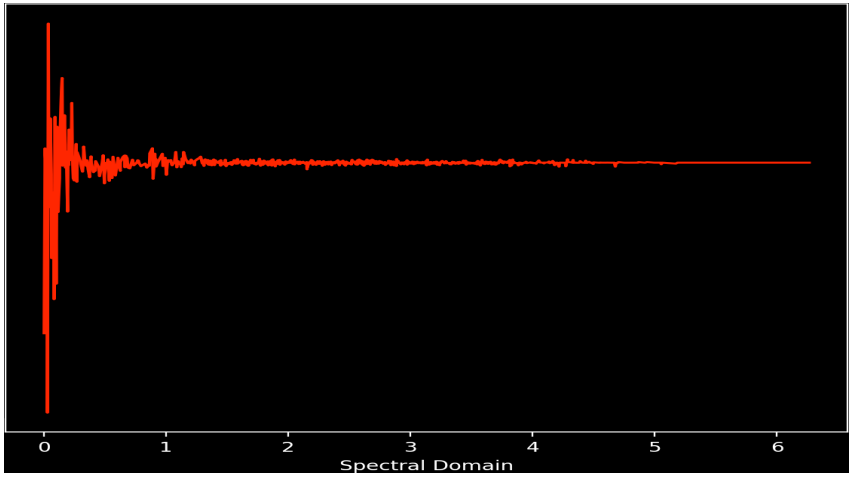


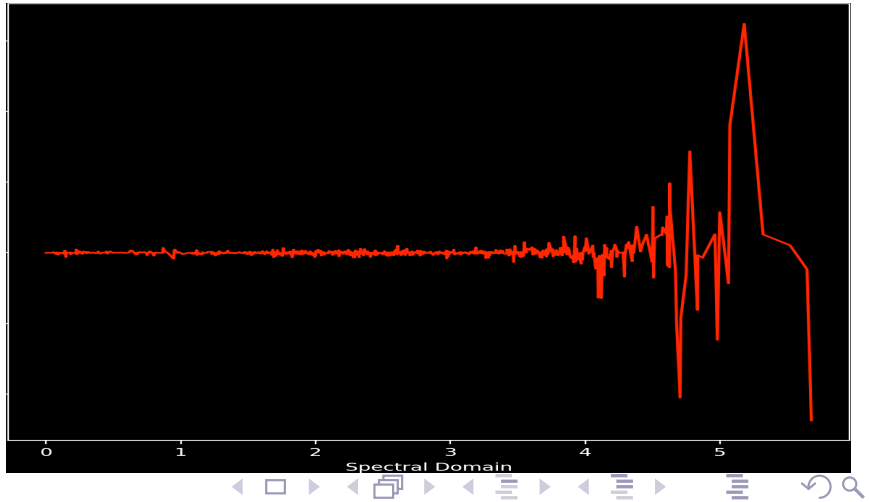


Graph Fourier Transform









Filtering Signals on Graphs

Classical Filtering and Convolution

Convolution

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x-t)g(t)dt$$

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Linear Time Invariant Filter

If $\mathcal{L}$ is a linear time invariant filter then there exists a function h such that

$$\mathcal{L}[f] = f * h$$

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The definition of convolution can not be directly applied to graphs because the shift operation $f(x-t)$ is not well defined on the graph domain.

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$$(f * g)(i) = \sum_{l=1}^{n} \widehat{f}(\lambda_l)\widehat{g}(\lambda_l)u_l(i)$$

Graph Spectral Filtering

$$\mathcal{L}[f] = iGFT[\widehat{f} \cdot \widehat{h}]$$

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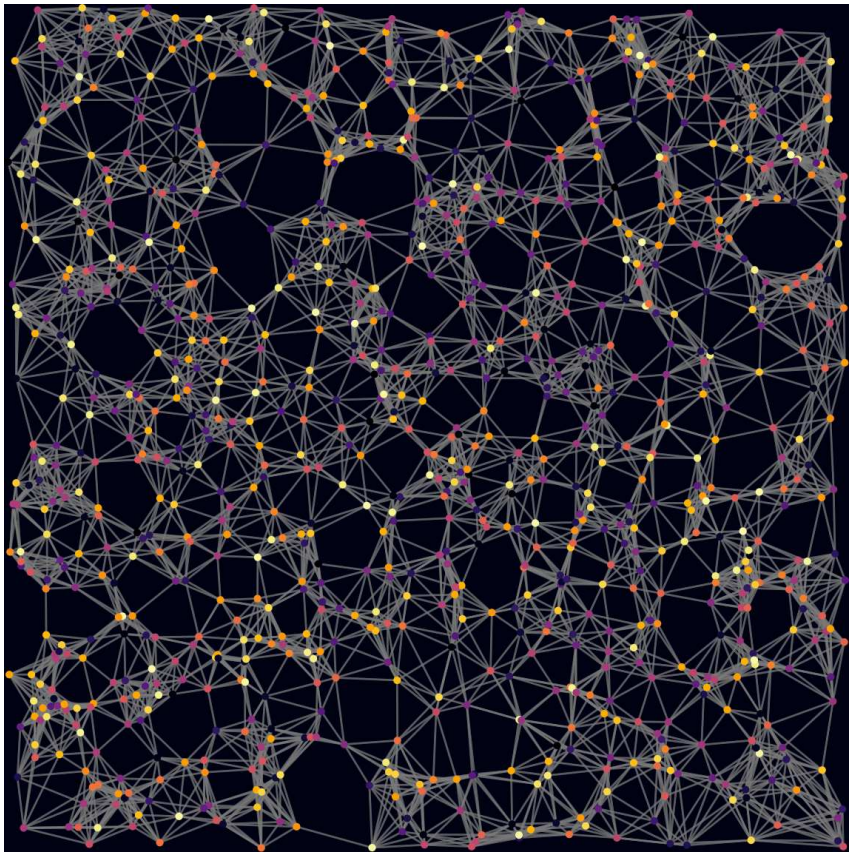
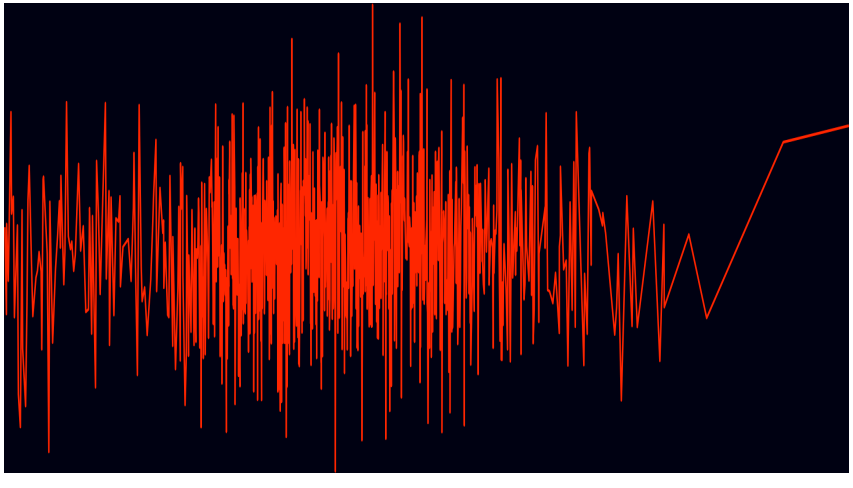
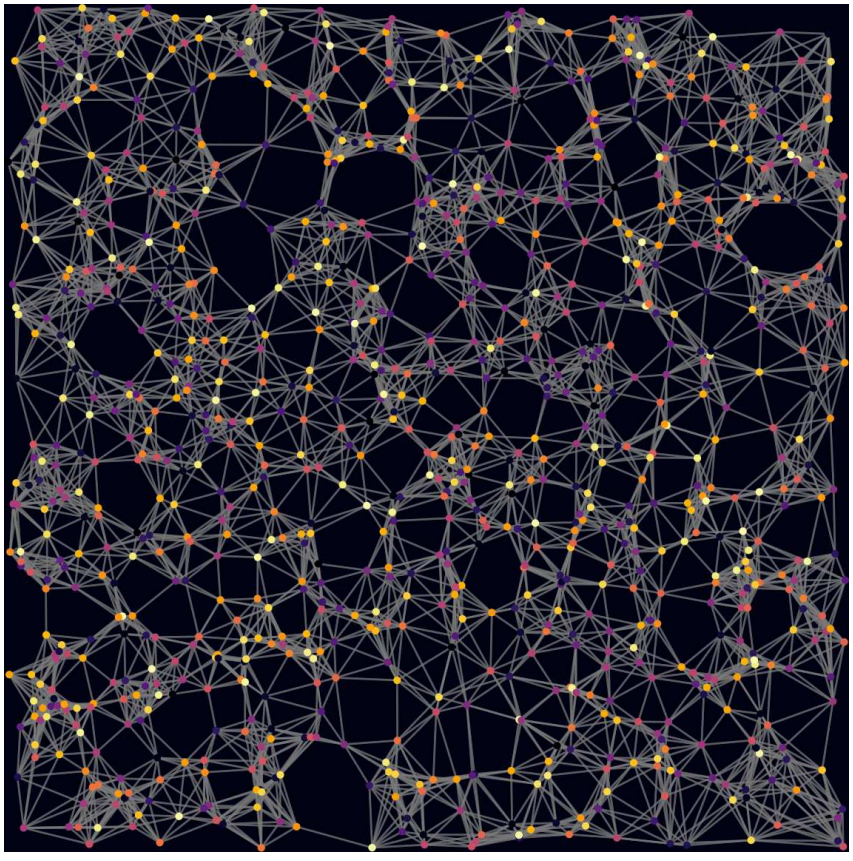
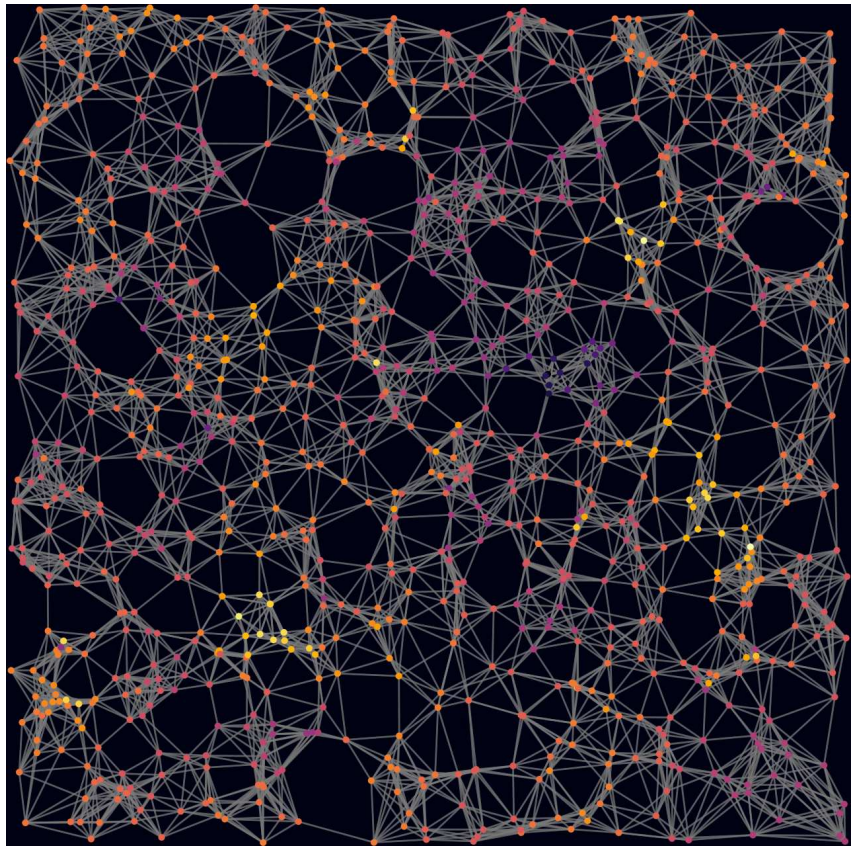
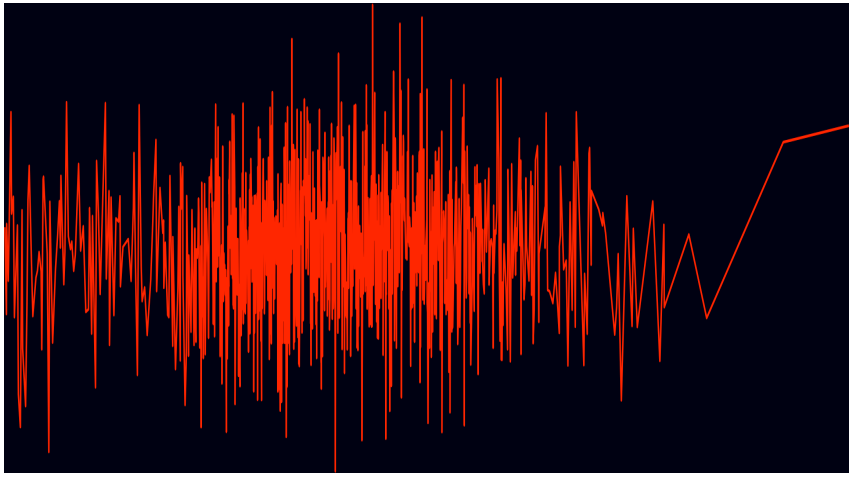
$$\mathcal{L}[f] = U \begin{bmatrix} \widehat{h}(\lambda_1) & & \\ & \ddots & \\ & & \hat{h}(\lambda_n) \end{bmatrix} U^\top f$$

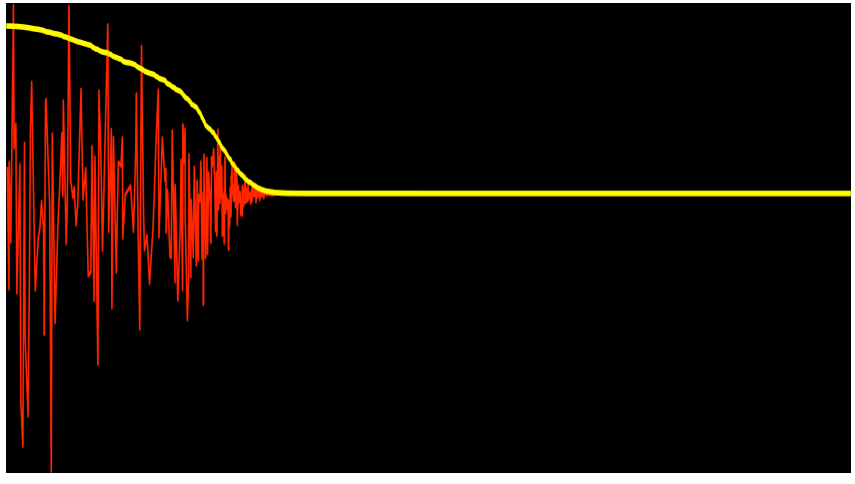












Filter Design

Graph Spectral Filtering: Smoothing

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Smoothing a function f:
Find h such that $g = \mathcal{L}[f] = \widehat{h}(L)f$ minimizes

$$\arg\min_{h}\{\|f-g\|^2 + \gamma g^\top L g\}$$

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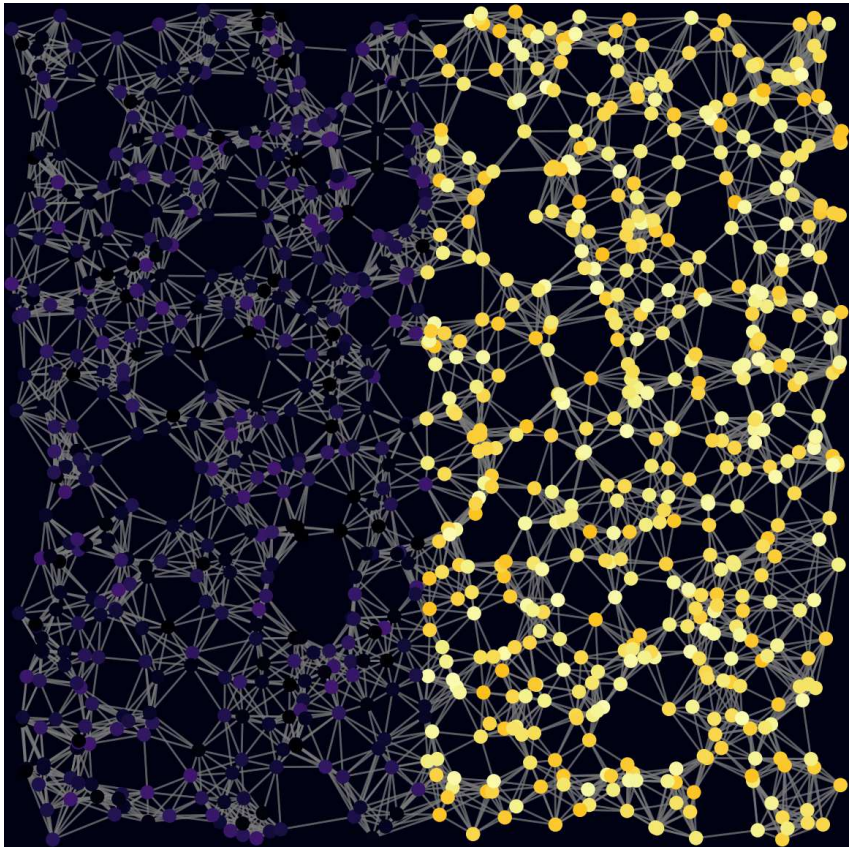
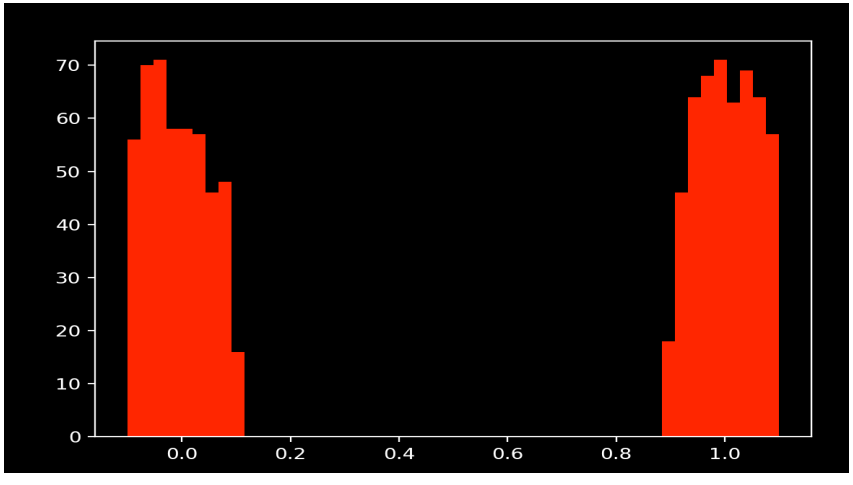
It can be shown that

$$\widehat{h}(\lambda) = \frac{1}{1+\gamma\lambda}$$

is the sought solution.

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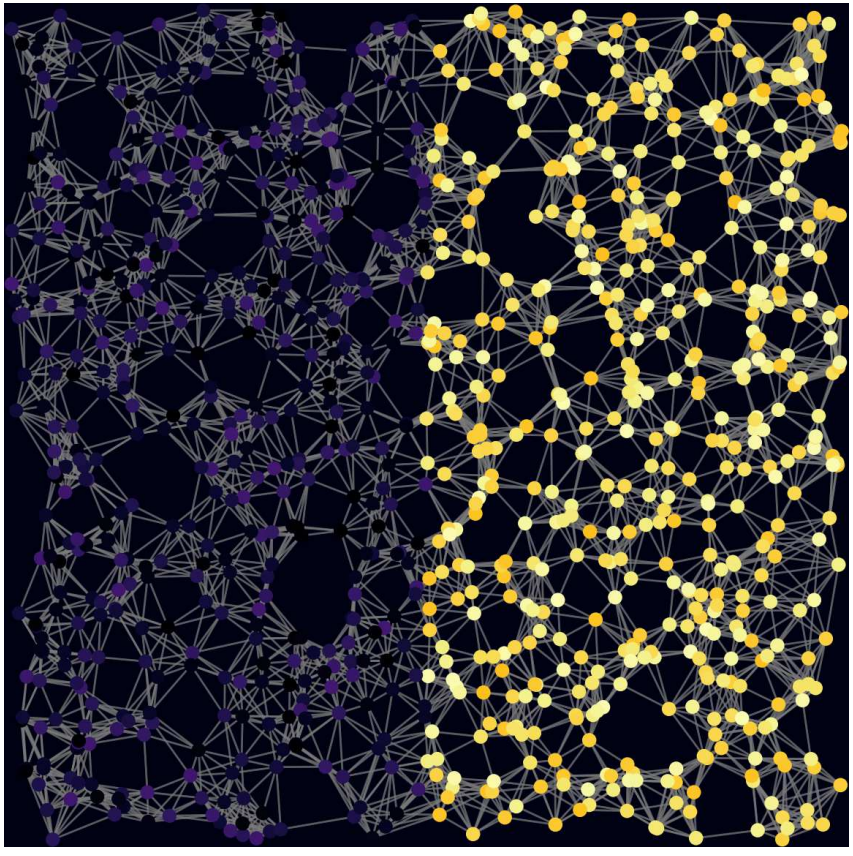

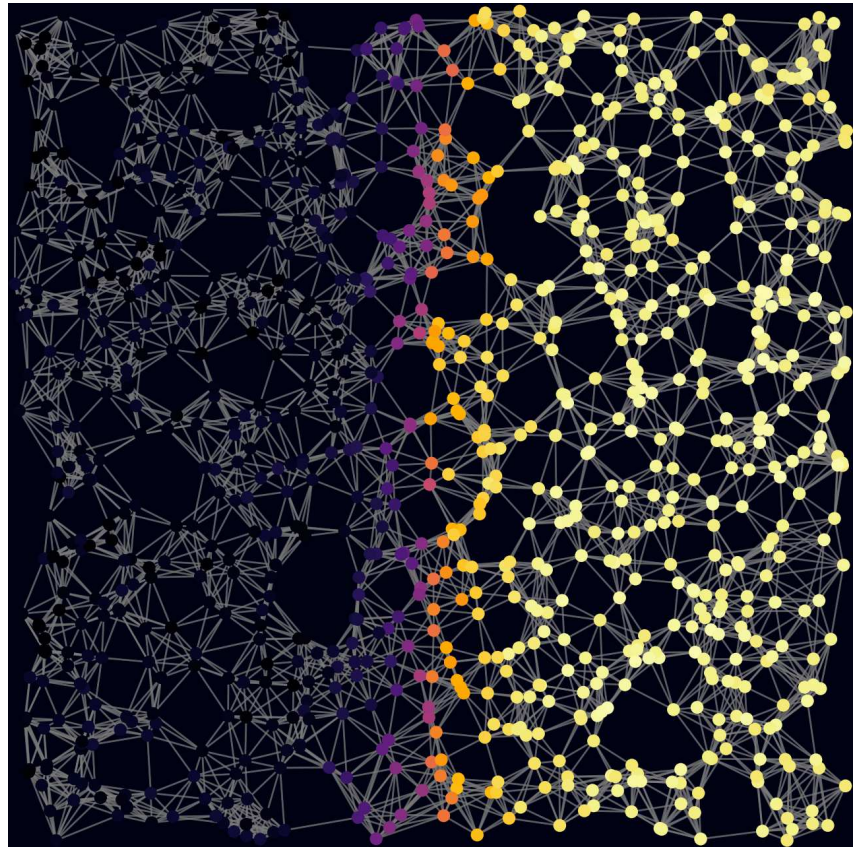

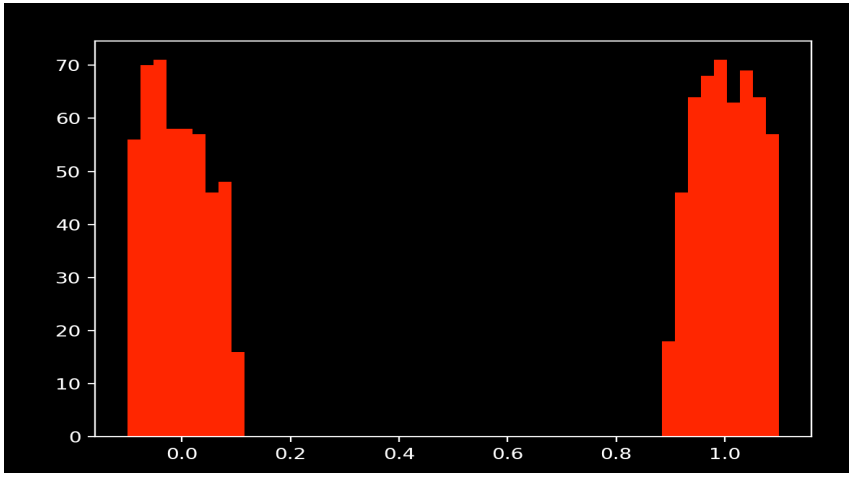
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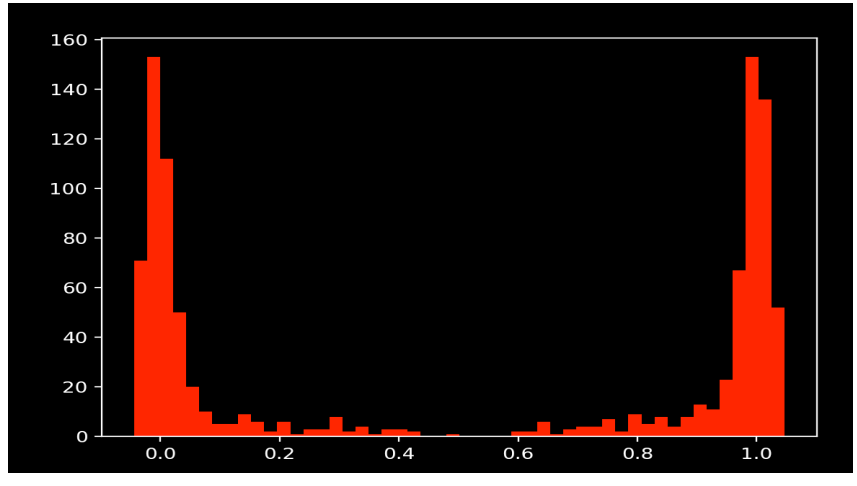
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Graph Spectral Filtering: GLoG

Laplacian of Gaussian (LoG)

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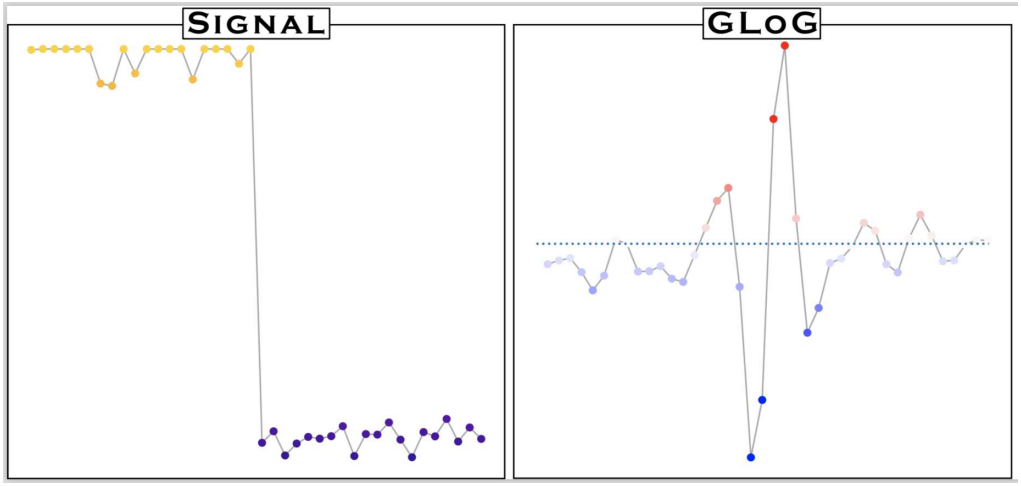
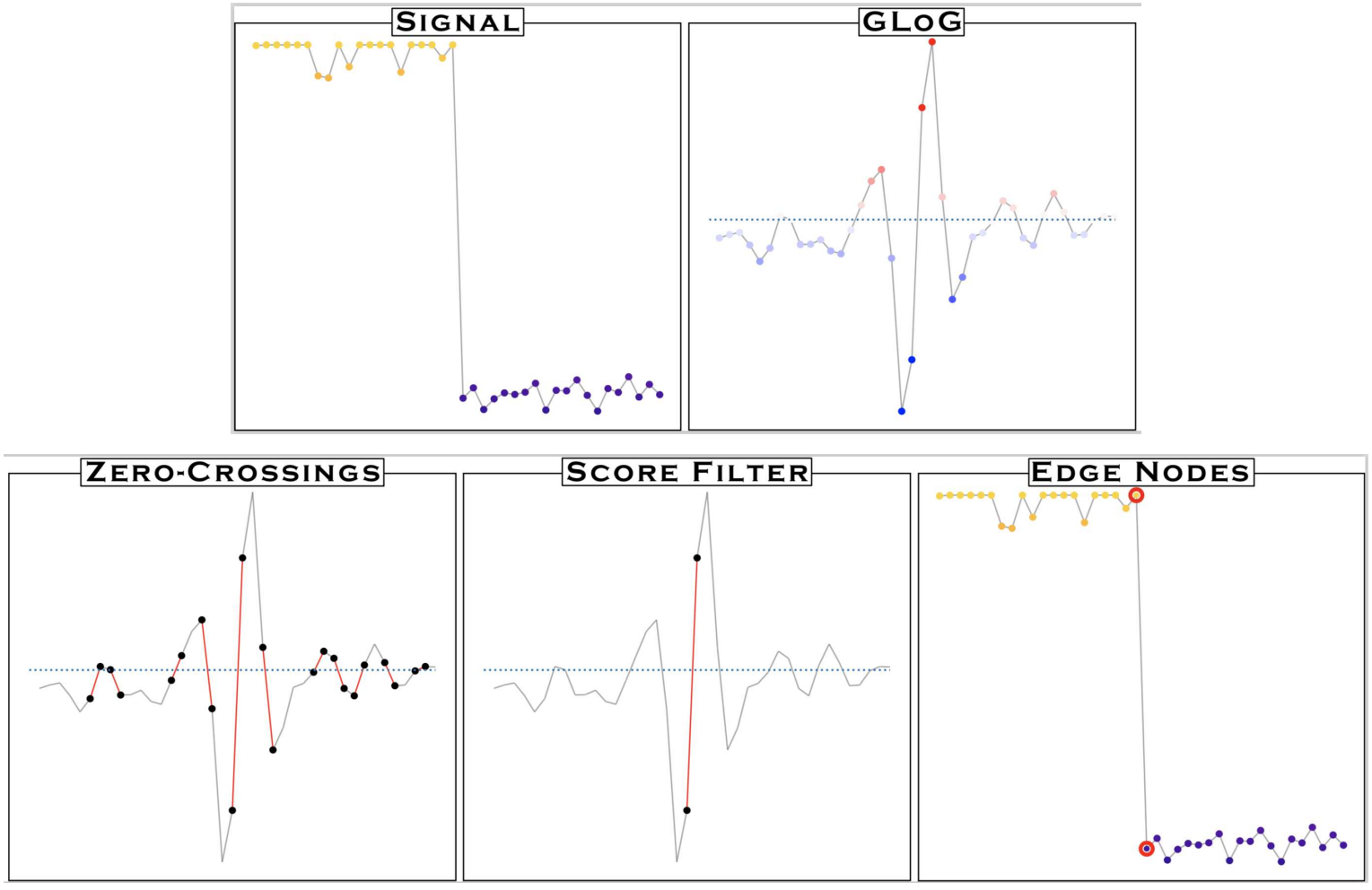


Graph Spectral Filtering: GLoG

Laplacian of Gaussian (LoG)



$$LoG(f) = \nabla^2[G * f], \quad G \text{ is a Gaussian function}$$

Graph Spectral Filtering: GLoG

Laplacian of Gaussian (LoG)

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$$GLoG(f) = \widehat{h}(L)f$$

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Laplacian of Gaussian (LoG)

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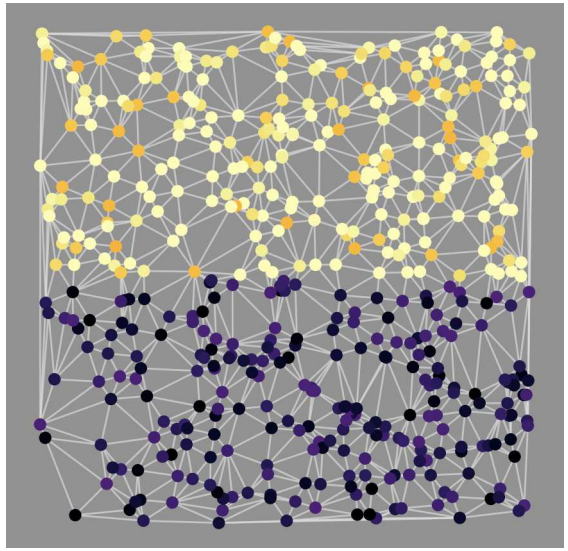
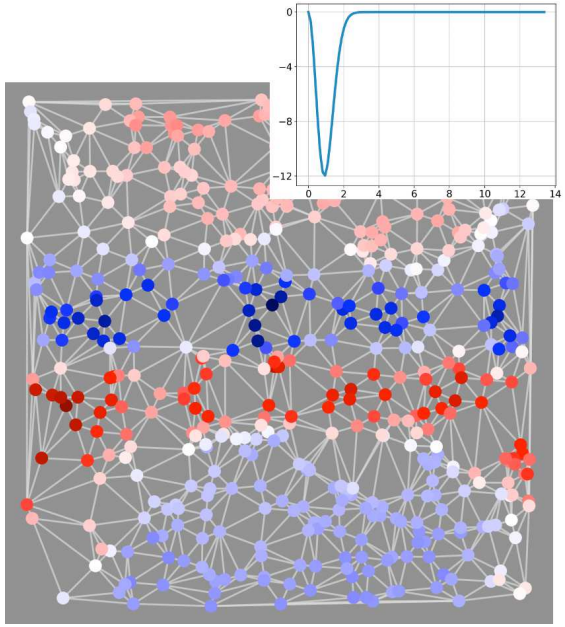
$$GLoG(f) = \widehat{h}(L)f$$

The solution is given by

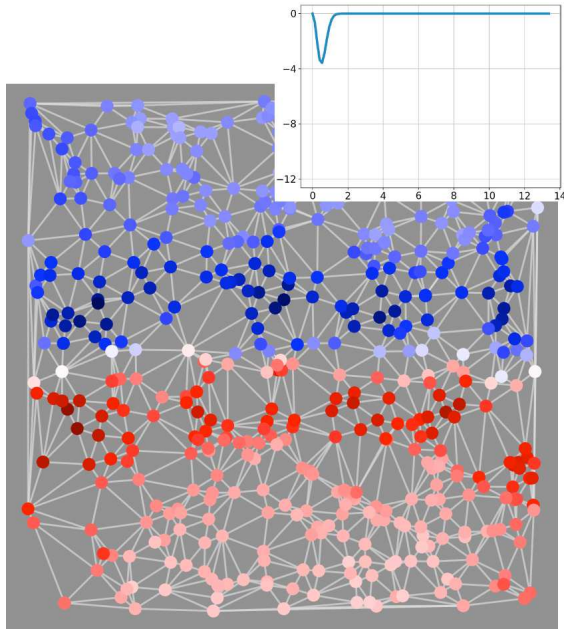
$$\widehat{h}(\lambda_l) = -4\pi^2\lambda_l^2 \exp(\sigma^2\lambda_l^2/2)$$

In the 2D case, the $GLoG$ is equivalent to the LoG filter.

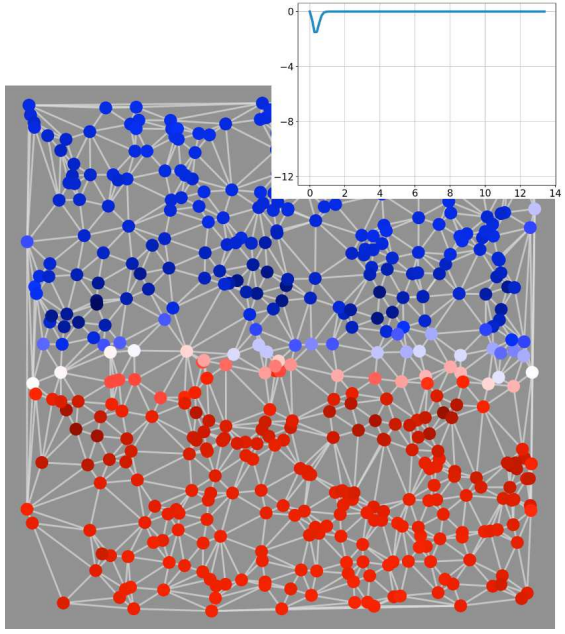




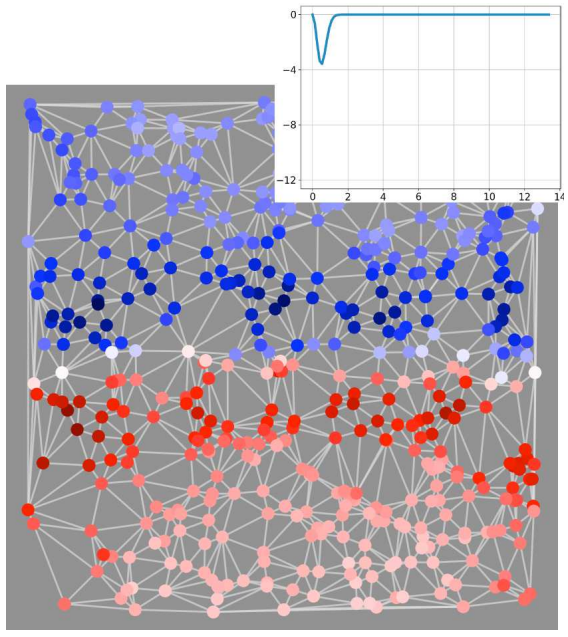
$\sigma = 1$



$\sigma = 2$



$\sigma = 3$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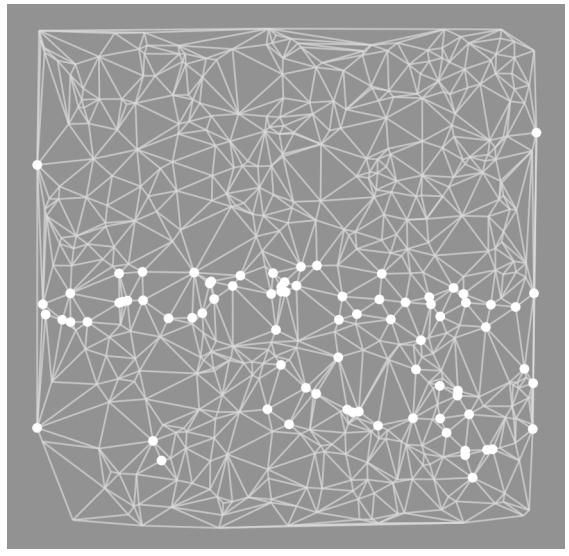
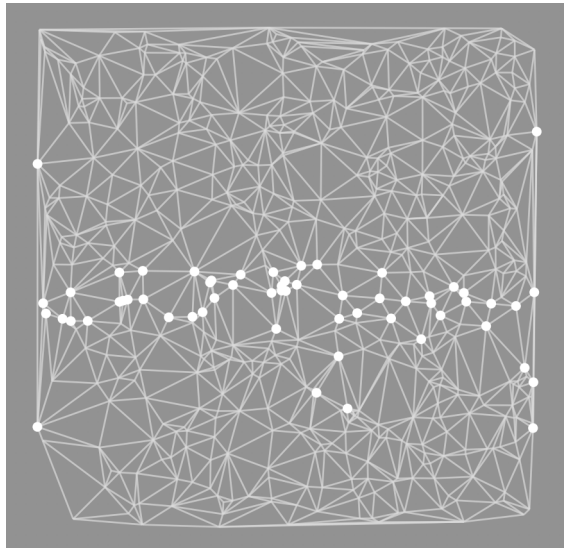
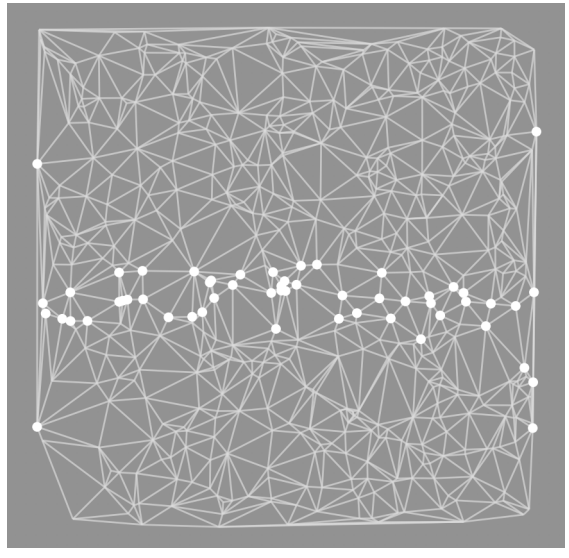
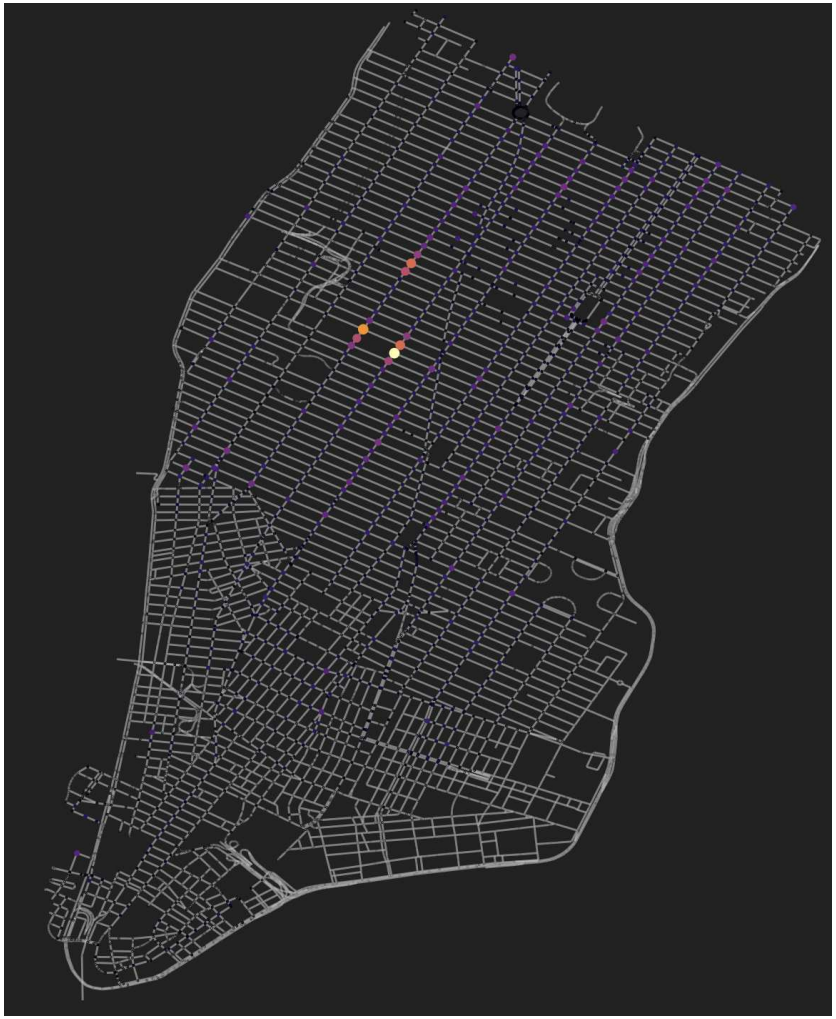




Visual Enhancement

Taxi Pick ups in Downtown Manhattan



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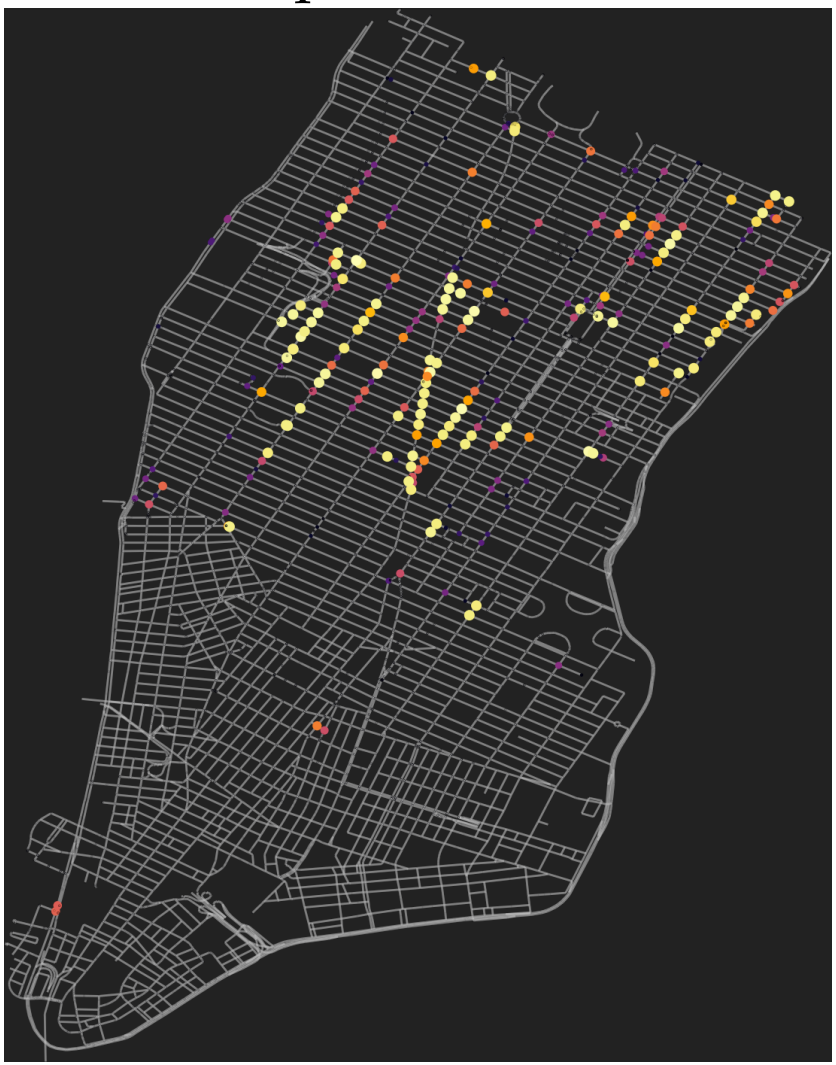

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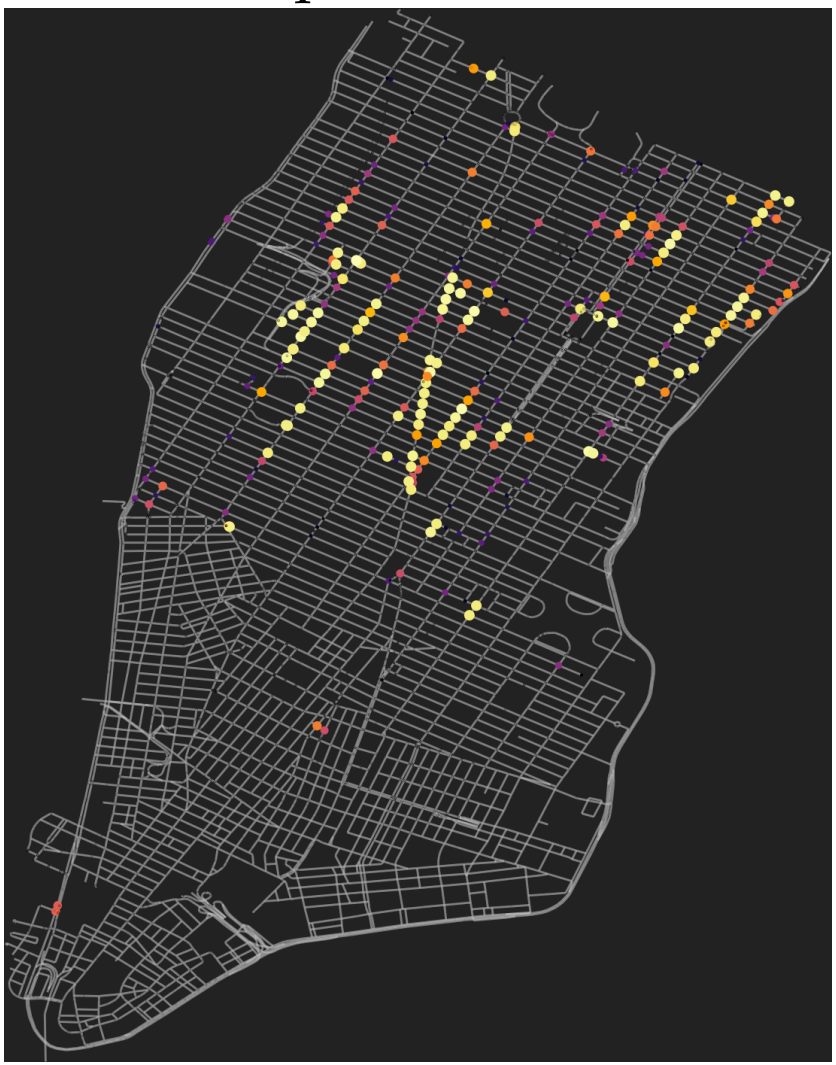

Spatio-Temporal Data Analysis via GLoG

Spatio-Temporal Data Analysis

$$f : V \times T \to \mathbb{R}, \qquad T = \{t_1, t_2, \ldots, t_m\}$$

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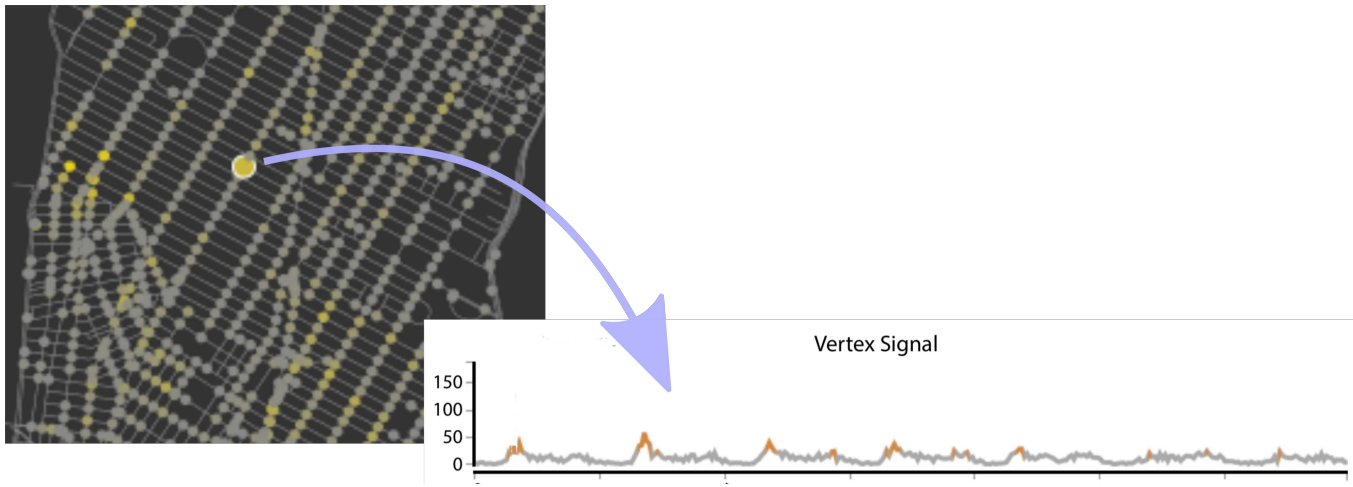

Edge nodes: adjacent nodes where GLoG changes sign

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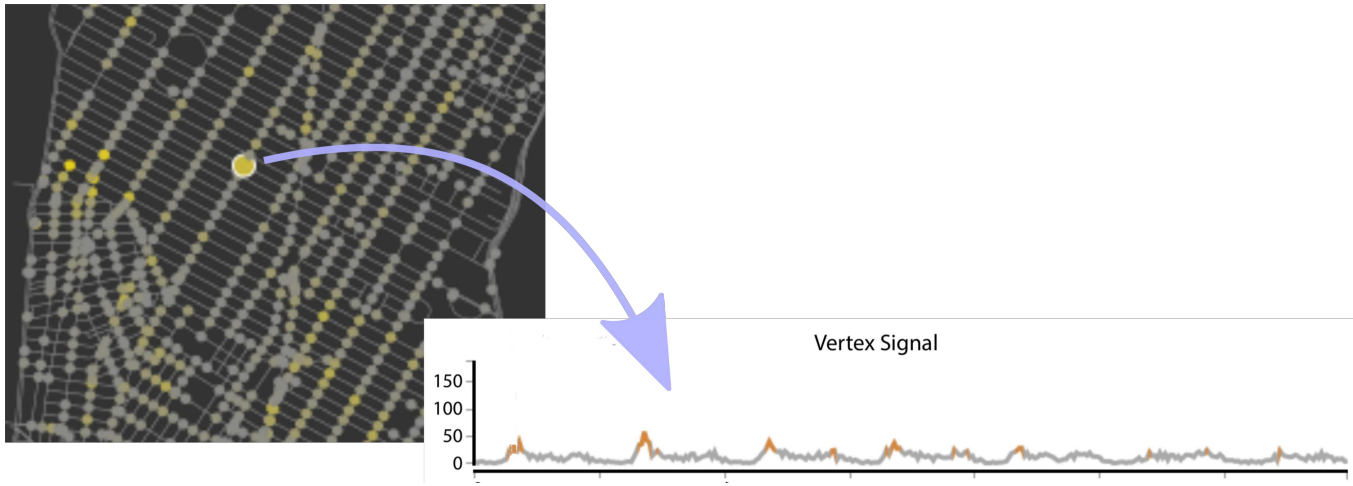

Edge nodes: adjacent nodes where GLoG changes sign
Probability of edge nodes

$$I(\tau_i, t_k) = \begin{cases} 1 & \text{if } \tau_i \text{ is an edge node in time slice } t_k \\ 0 & \text{otherwise} \end{cases}$$

$$p_e(\tau_i) = \frac{1}{m} \sum_{k=1}^{m} I(\tau_i, t_k)$$

Probability of an observed node configuration:

$$p(I(\tau_i, t_k)) = \begin{cases} p_e(\tau_i) & \text{if } I(\tau_i, t_k) = 1 \\ 1 - p_e(\tau_i) & \text{if } I(\tau_i, t_k) = 0 \end{cases}$$

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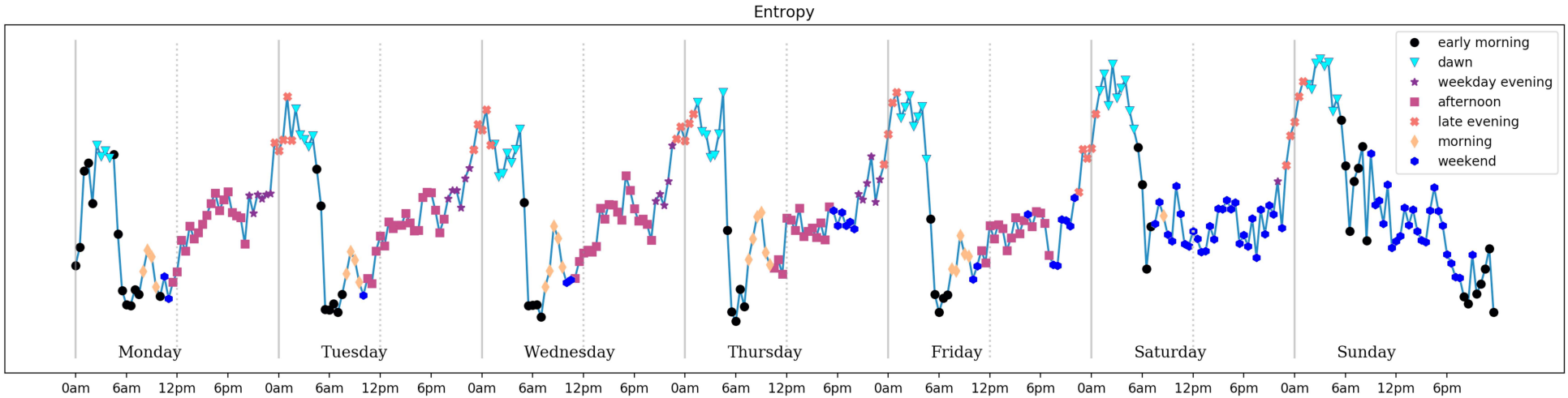
Time Slice Entropy

$$E(t_k) = -\sum_{i=1}^{n} p(I(\tau_i, k)) \log p(I(\tau_i, k))$$

Spatio-Temporal Data Analysis

Analyzing Taxi Pick ups in Downtown Manhattan



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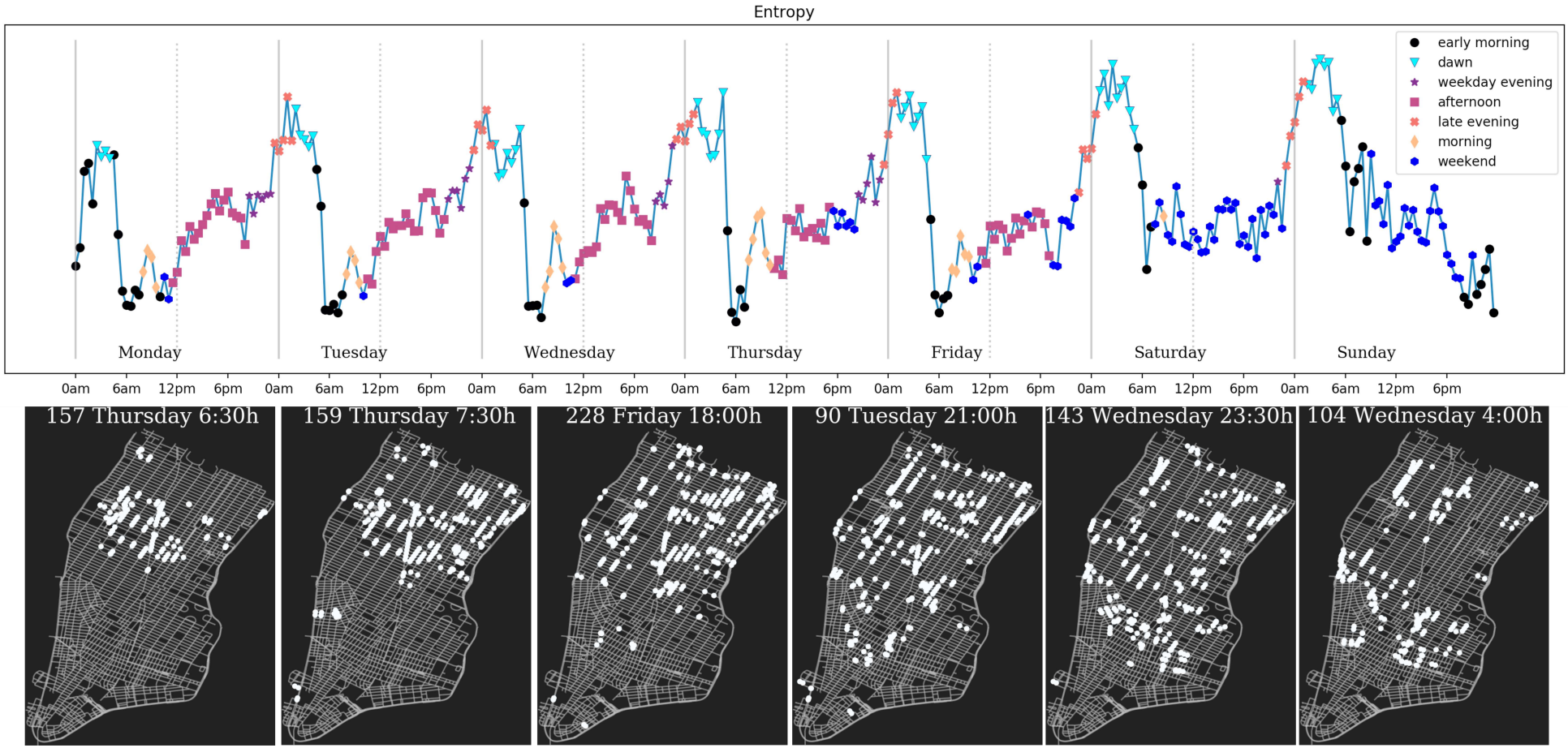

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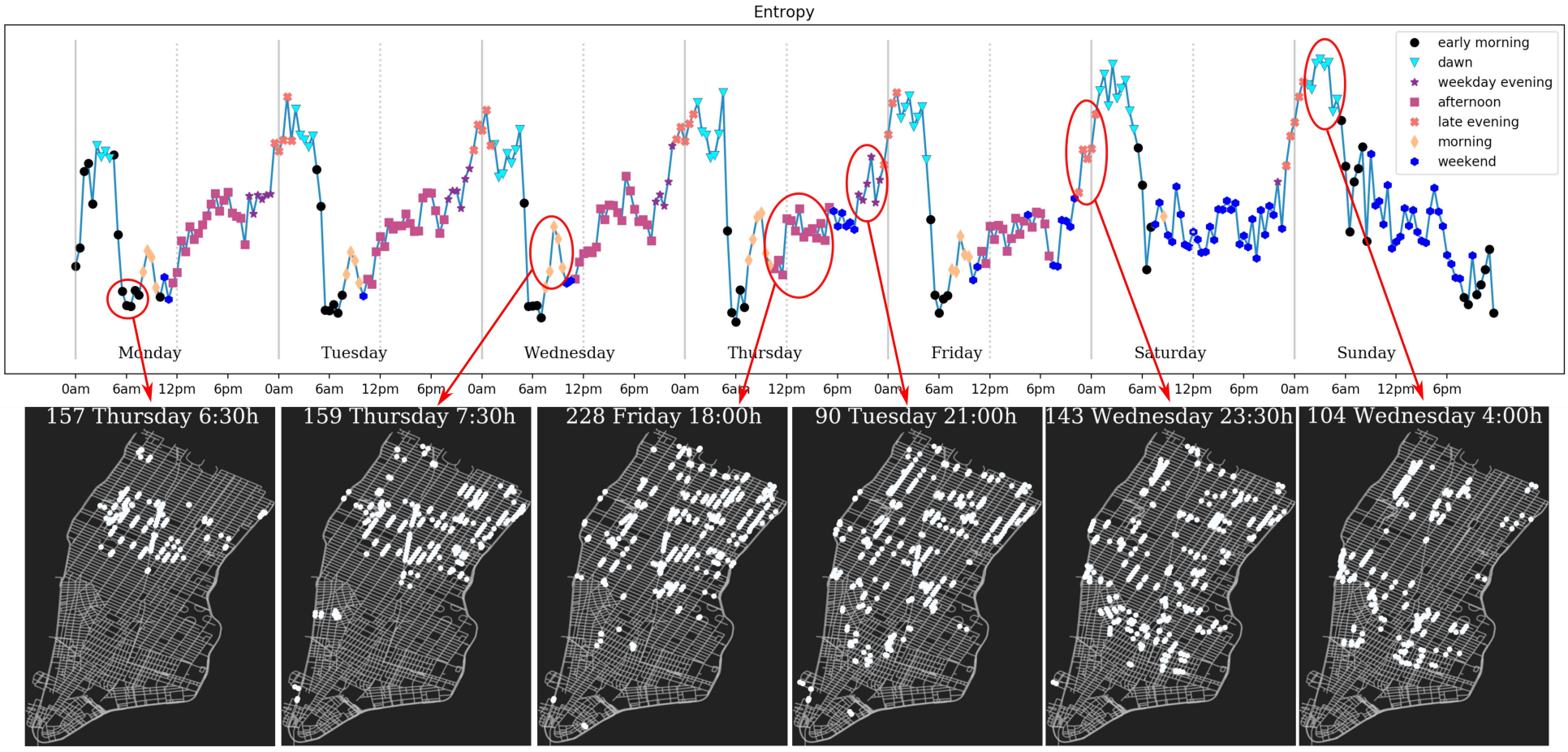
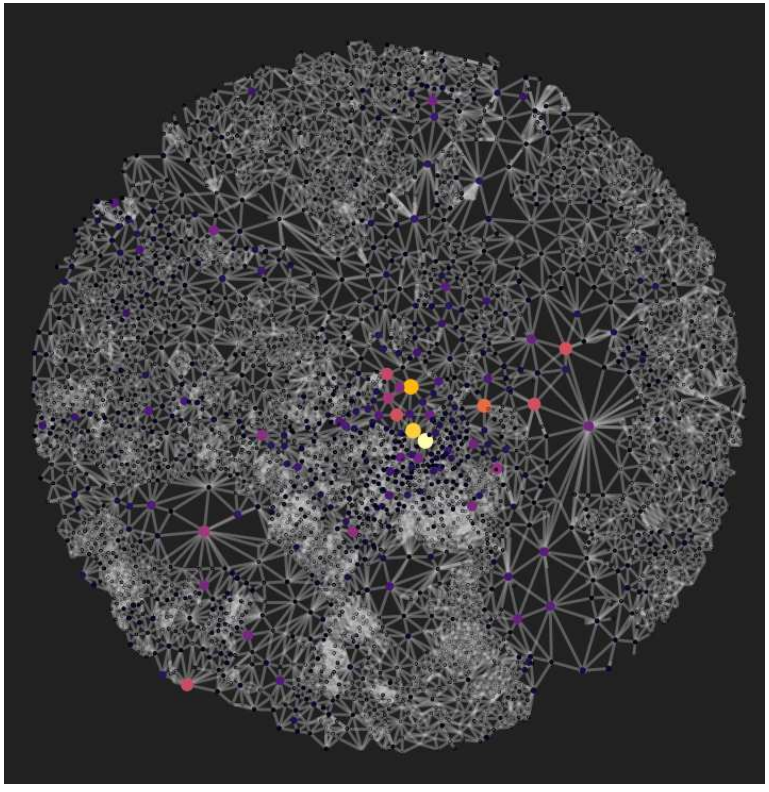
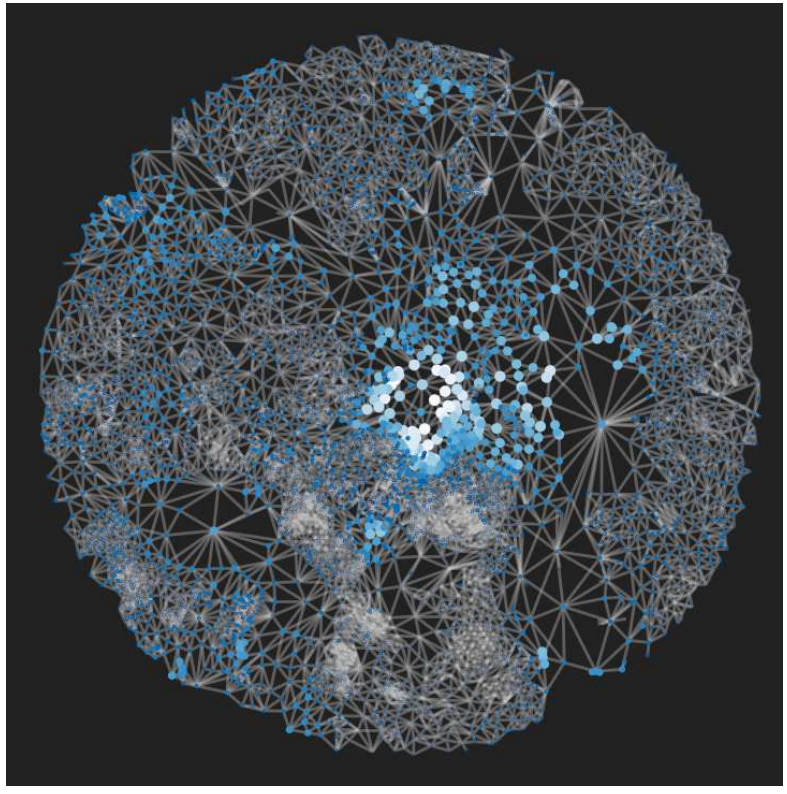


Spatio-Temporal Data Analysis

Analyzing Passerby Robbery in São Paulo City





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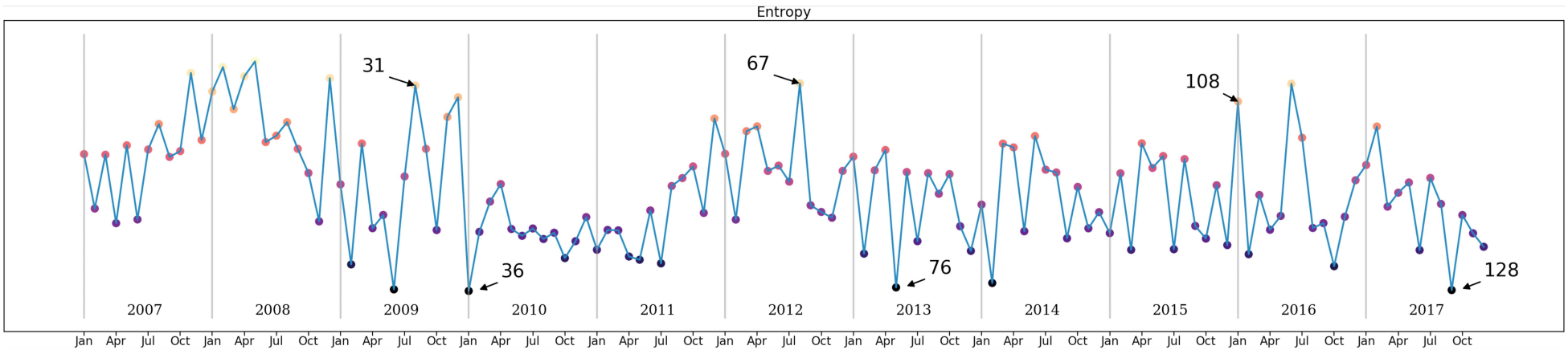

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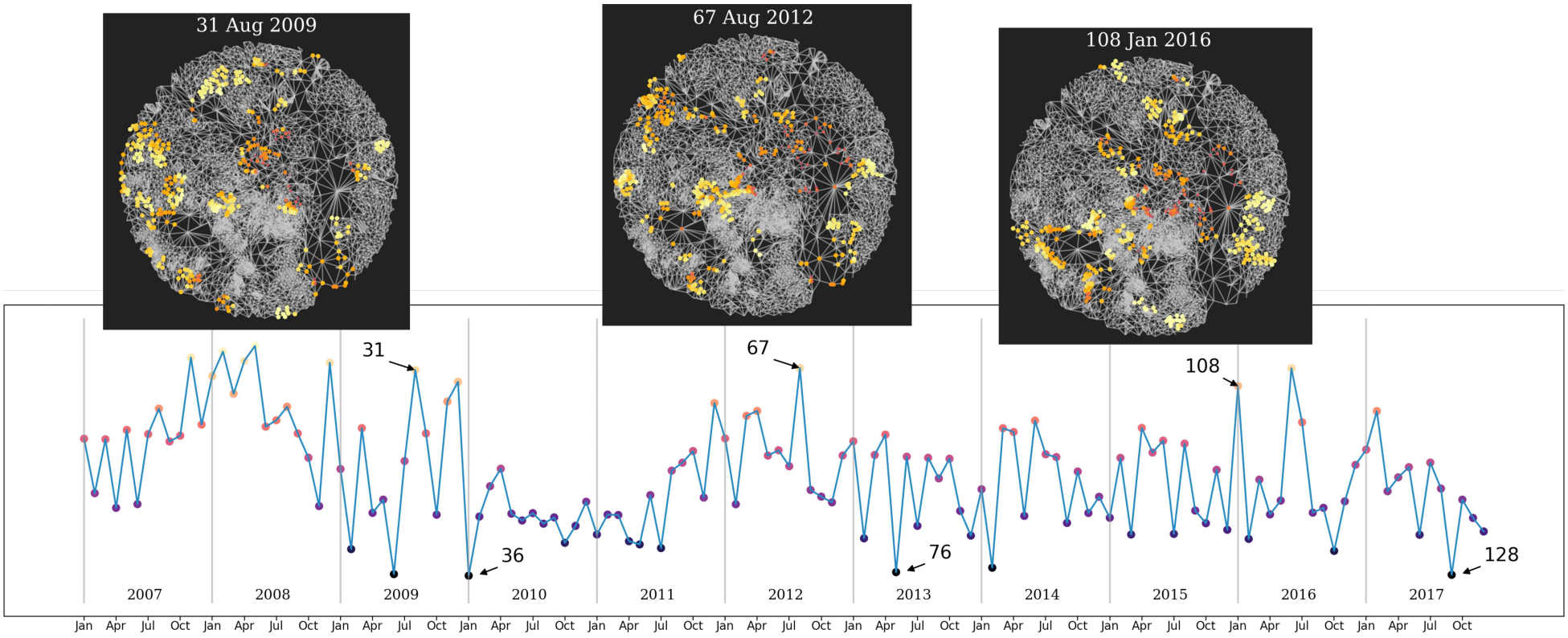

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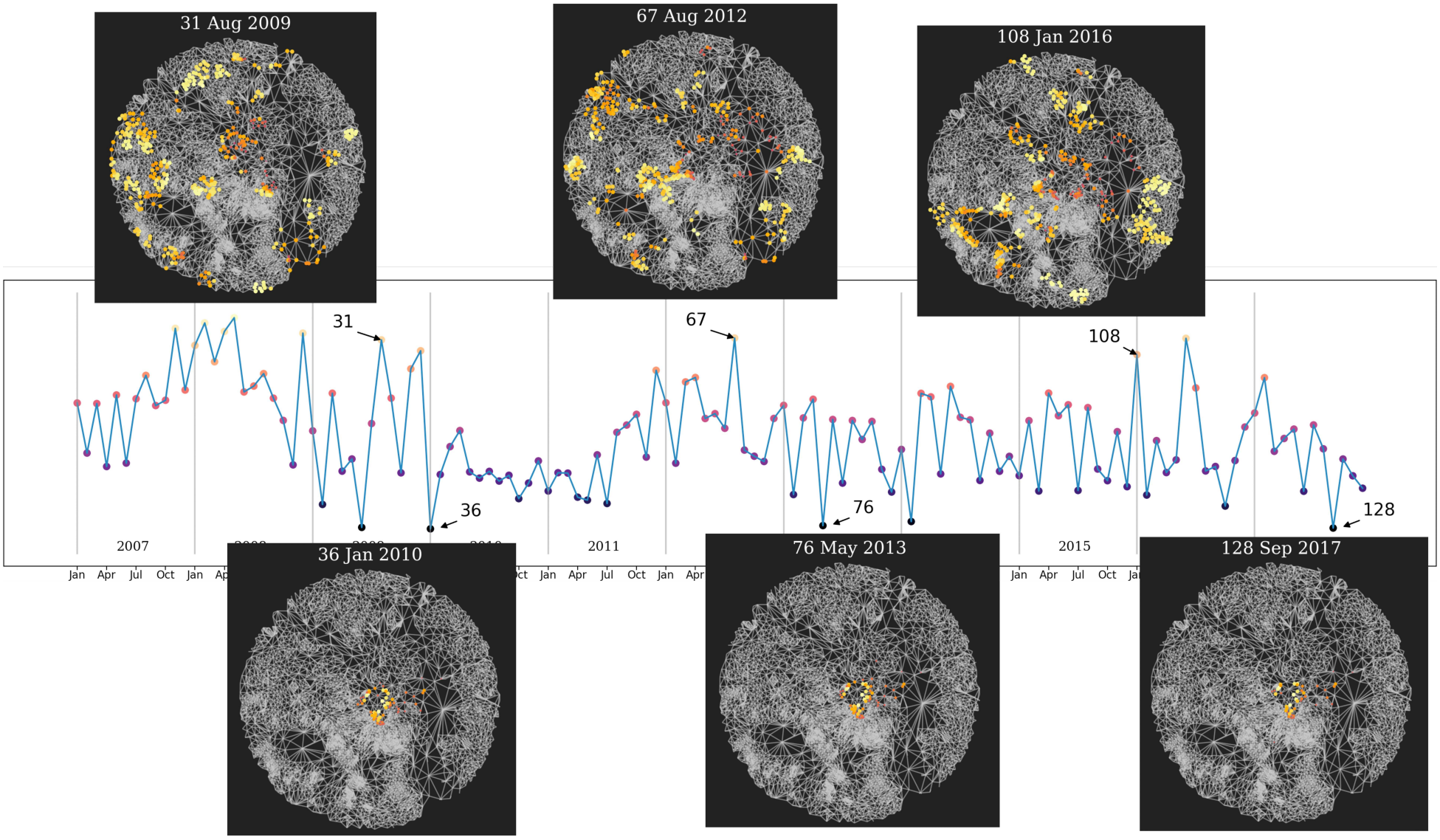

Graph Wavelet Transform

Continuous Wavelets Transform

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$$\psi^{s,t}(x) = \frac{1}{s}\psi\left(\frac{x-t}{s}\right) \quad \longrightarrow \quad \mathcal{W}[f](s,t) = \left\langle f, \psi^{s,t}(x) \right\rangle$$

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$$\int \frac{|\widehat{\psi}(\lambda)|^2}{|\lambda|} d\lambda < \infty \quad \longrightarrow \quad |\widehat{\psi}(0)|^2 = 0 \text{ (band-pass filter)}$$

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Moreover, the sought graph wavelet basis must be band pass filters with particular properties.

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The normalizing constant $\sqrt{n}$ ensures that the translation preserves the mean of the signal.

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Notice that $\widehat{f}(s\lambda_l)$ might not be in the range $[0, \lambda_n]$.

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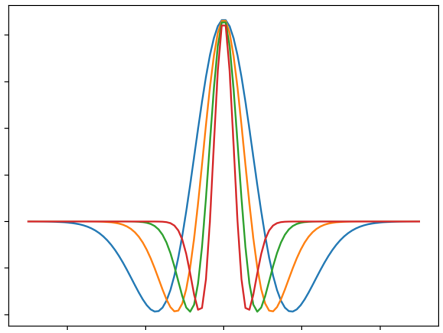
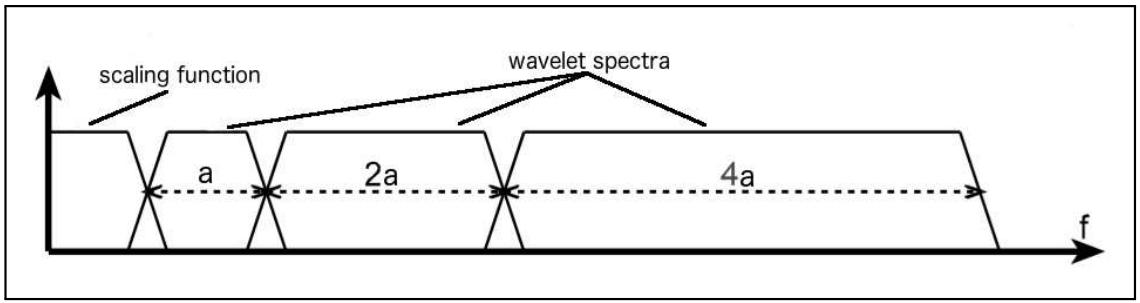
implies that stretching a band-limited function in the spatial domain shrinks and shifts its spectrum.





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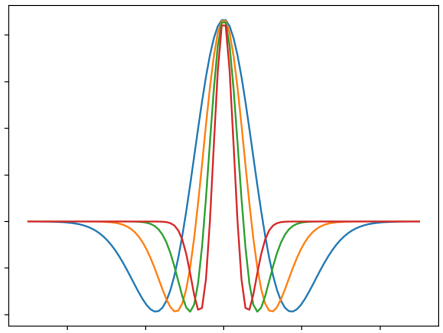

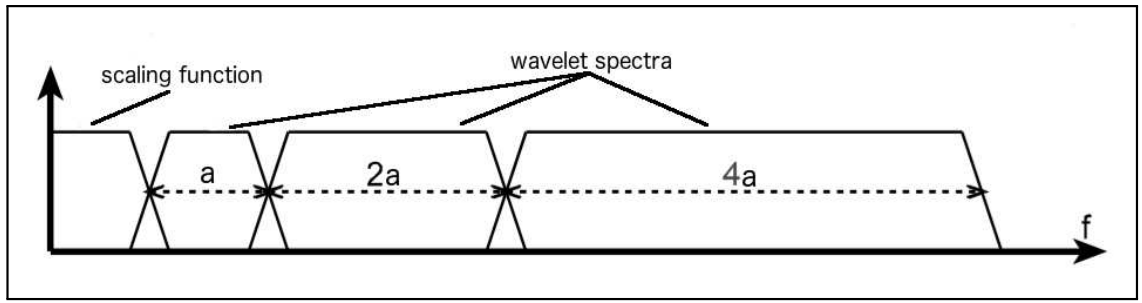

A complete representation demands to complete the basis with a low-pass scaling function.

Hammond et al. have shown that the following set of filters satisfy the shrink and shift property:

$$g(x) = \begin{cases} x^2 & \text{for } x < 1 \\ -5 + 11x - 6x^2 + x^3 & \text{for } 1 \leq x \leq 2 \\ 4x^{-2} & \text{for } 2 < x \end{cases}.$$

and that

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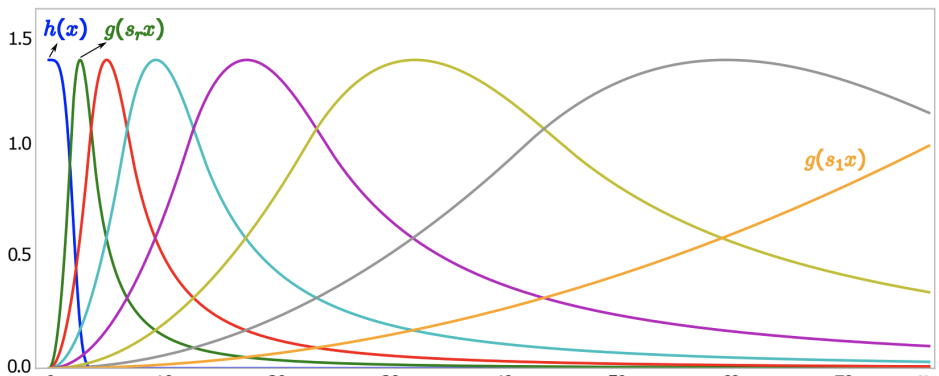
$g(s\lambda)$ and $h(\lambda)$



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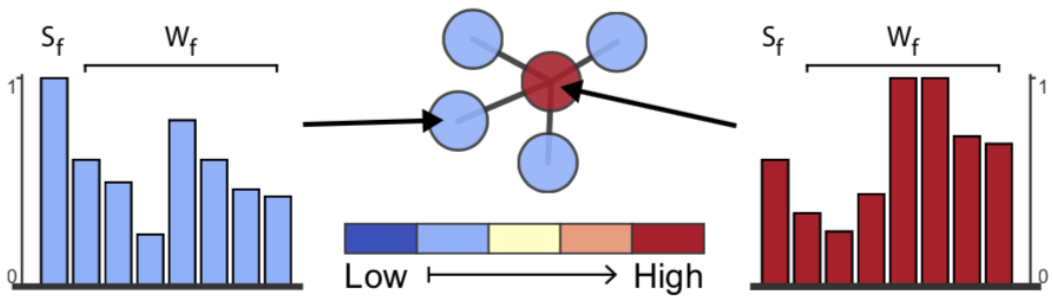
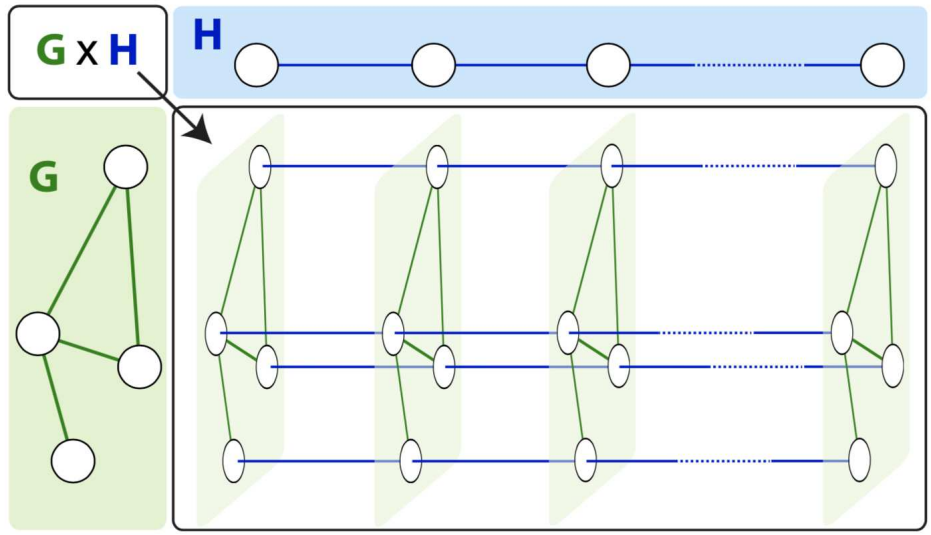


GWT for Spatio-Temporal Data Analysis

Joint Space and Time Analysis



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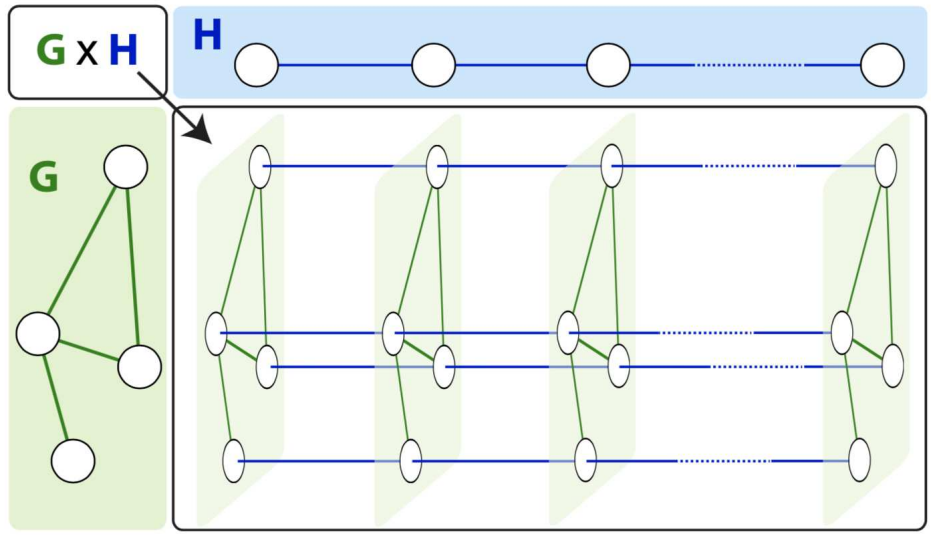

$$\begin{cases} (u_l, \lambda_l) \text{ eigenpair of } G \\ (v_k, \mu_k) \text{ eigenpair of } H \end{cases} \Rightarrow (u_l \otimes v_k, \lambda_l + \mu_k) \text{ eigenpair of } G \times H$$

where $\otimes$ is the Kronecker product.

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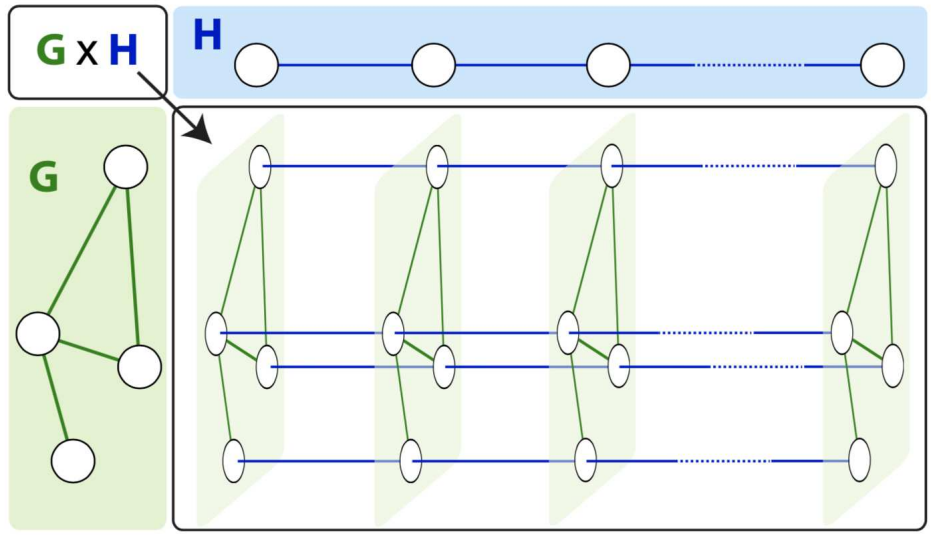

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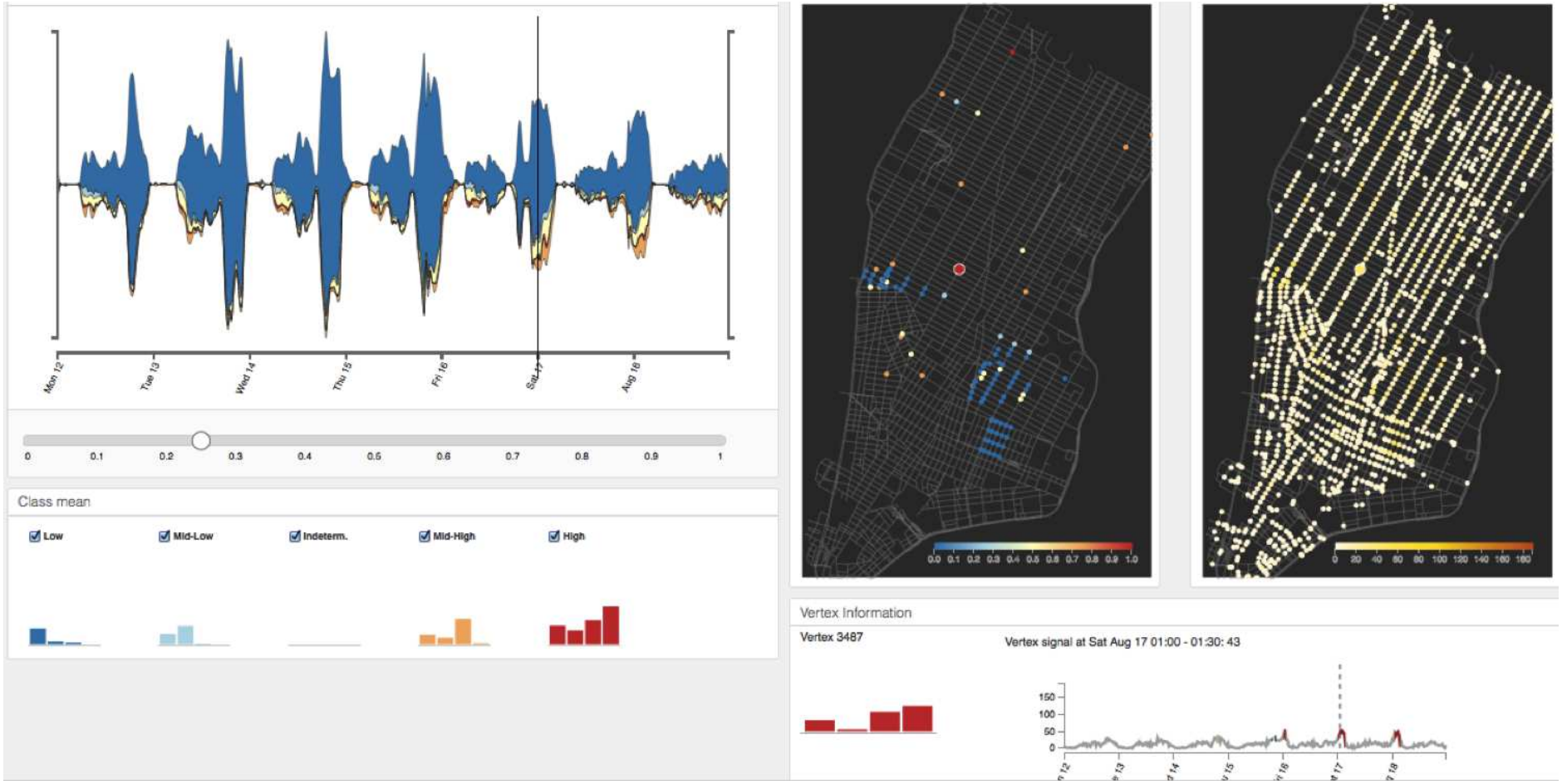
This property is extremely useful to make the joint space and time analysis computationally feasible.

Analyzing taxi pickups



More than 1.5 million nodes and 2 million edges.

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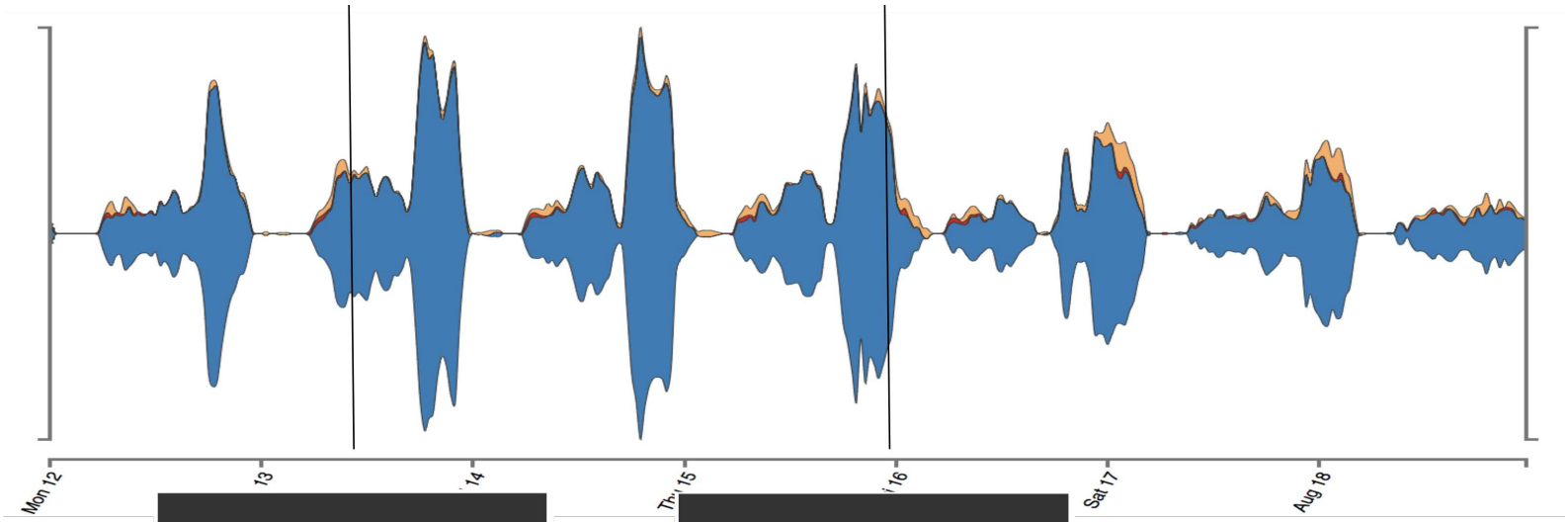

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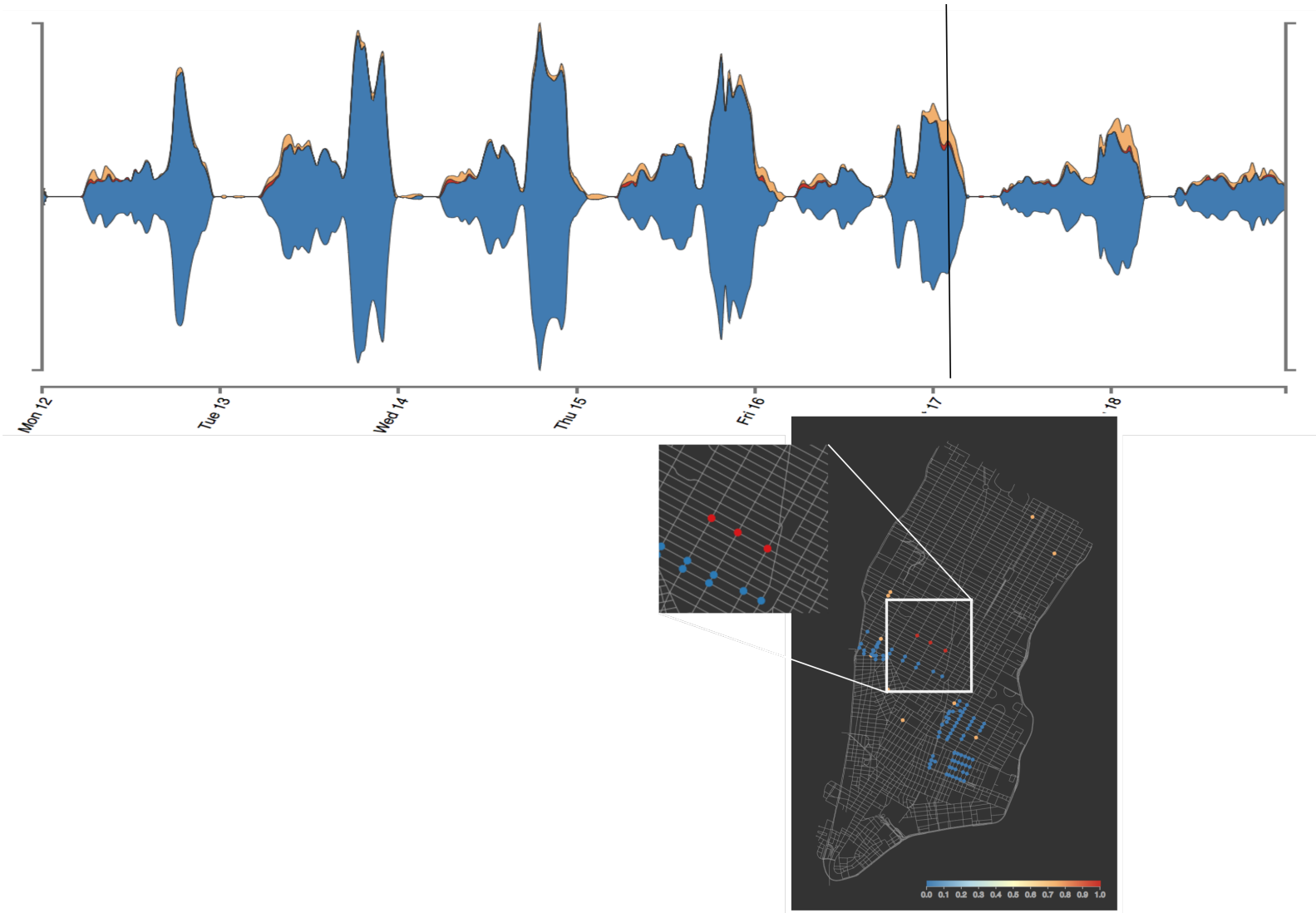

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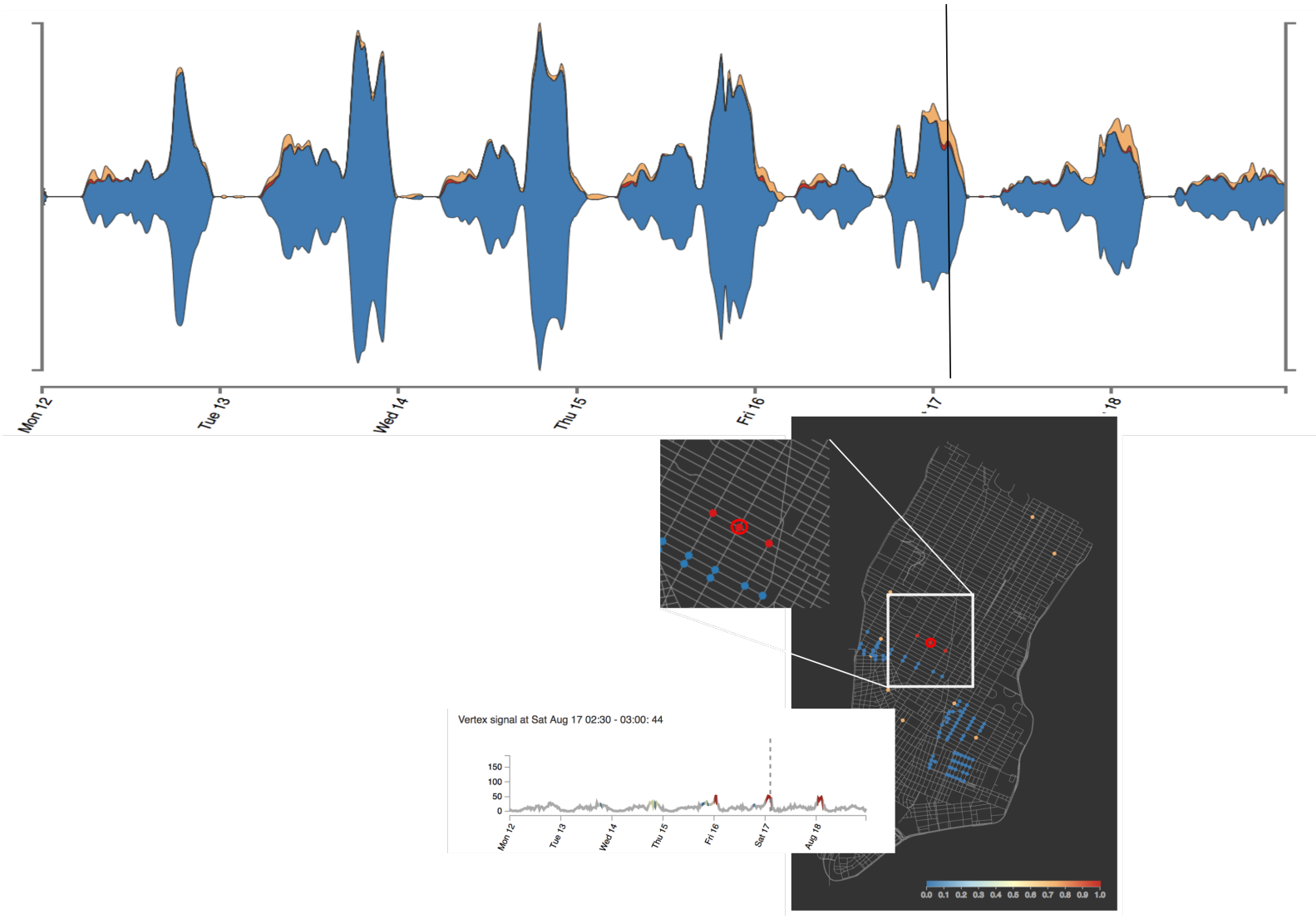

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  - very hot topic is the interplay between GSP + Deep Learning

THANKS !!